

Question 1. [10 marks]

The city wanted to evaluate the effect of new turning restrictions on the reduction of automobile traffic through a residential neighbourhood. It collected data on the number of cars during the peak evening rush hour on a sample of days before and on a sample of days after the restrictions were implemented. Appendix A shows some graphs of the data and some basic summaries.

- a. To evaluate the effectiveness of the turning restrictions, should you do an independent samples test or a paired sample test? Explain briefly.

[1] independent samples, since each day before the change is not linked to a day after the change.

- b. Test (using p-value) at the 0.05 level of significance whether the data support the city's claim that there was an average reduction of more than 50 cars per minute.

[5] Two-sample T for before vs after

	N	Mean	StDev	SE Mean
before	60	104.0	11.4	1.5
after	60	50.55	7.16	0.92

Difference = μ (before) - μ (after)

Estimate for difference: 53.45

95% lower bound for difference: 50.56

T-Test of difference = 50 (vs >): T-Value = 1.98 P-Value = 0.025 DF = 99

-Ho: $\mu_1 - \mu_2 = 50$, Ha: $\text{diff} > 50$

-SE is $\sqrt{11.4^2/60 + 7.16^2/60} = 1.74$

- $t = (53.45 - 50) / 1.74 = 1.98$, regardless of equal/unequal variance;

-p-value = $\text{Prob}(t > 1.98) = P(Z > 1.98) = 0.025$

-Reject the null H since p-value < 0.05; conclude average reduction more than 50 (0.5 each)

1 mark each above; accept pooled stdev = 9.537, with same t-statistic

- c. Calculate an appropriate 1-sided confidence interval for the reduction in the average number of cars during the peak evening rush hour.

[2] at least $53.45 - 1.645 * \sqrt{11.4^2/60 + 7.16^2/60}$ or $53.45 - 1.645 * 1.74$
or $53.45 - 2.86 = 50.59$; interval is (50.6, infinity)

- d. What key distributional assumption(s) is(are) absolutely necessary to support your statistical test?

[1] Need to assume the population distributions are not extremely skewed.

- e. Is(are) this(these) assumption(s) warranted? Explain with respect to specific boxplots.

[1] The first boxplot is only slightly skewed and the second is symmetric.

Question 2. [7 marks]

Newly developed 'Application Specific Integrated Circuits' or ASICs generally have low 'yield ratios' meaning that only 60% of these chips were useful. In the past, the yield ratio was only 60%. To determine if there has been an improvement, a new sample of 200 ASICs was taken and tested. It was found that 130 of them were useful.

- a. Test, using the p-value approach, whether the yield ratio has improved. Assume a level of significance of 5%.

[4] -Ho: $p = 0.60$, Ha: $p > 0.6$
- $Z = (0.65 - 0.6) / \sqrt{(0.6 * 0.4 / 200)} = 0.05 / 0.03464 = 1.44$
-p-value is 0.075
-Do not reject Ho, conclude no improvement

1 mark each above.

Test of $p = 0.6$ vs $p > 0.6$

Sample	X	N	Sample p	95%		
				Lower Bound	Z-Value	P-Value
1	130	200	0.650000	0.594524	1.44	0.074

- b. Calculate the appropriate asymmetric or one-sided confidence interval and explain why it confirms the conclusions you reached in either part 'a' above.

[3] -at least $0.65 - 1.645 * 0.0346$ or at least $0.65 - 0.057 = 0.593$ (2 marks)

-We do not reject Ho since the interval covers the hypothesized value of 0.60 (1 mark)

Question 3. [2 marks]

In the current election campaign, all the polls indicate that the major political parties are hovering around 30% of popular vote. Suppose you wanted to estimate a 95% confidence interval for the percentage support for each of the major parties, using a margin of error of plus-or-minus 1%. What sample size would be required?

$N = 0.3 * 0.7 * (1.96 / 0.01)^2 = 8067$ Deduct 1 mark if $p=q=0.5$ used as this is unrealistic.

Question 4. [8 marks]

Campaign managers are always looking at their own internal polls to figure out if their campaigns are achieving their goals. Here is a brief summary of the various polling numbers a campaign manager obtained 'This Week' and 'Four Weeks Back' for the candidate running in the current election in a certain riding.

When?	Four weeks back	This week
# In favour of candidate	110	163
Sample size	200	250

In all the following sub-questions, use 5% as the level of significance.

- a. Test if the improvement in the level of support exceeds 2%.

[4] -Ho: $p_1 - p_2 = 0.02$, Ha: $p_1 - p_2 > 0.02$ (here p_1 is the more recent population proportion and $p_1 - p_2$ is the improvement)

-SE is $\sqrt{(11/20 * 9/20 / 200 + 163/250 * 87/250 / 250)} = 0.0463$ (pooled proportion unnecessary) *0.5 marks*

-Z = $\{(163/250 - 11/20) - 0.02\} / 0.0463 = (0.102 - 0.02) / 0.0463 = 1.77$

-Reject Ho if $Z > 1.645$ *0.5 marks*

-Decide to reject Ho, conclude improvement exceeds 2%

A common mistake was to calculate

$$Z = \{(0.55 - 0.652) - 0.02\} / 0.0463 = -0.122 / 0.0463 = -2.51$$

This requires Ha: $p_2 - p_1 < -0.02$ (here $p_2 - p_1$ must be a "negative" drop in support), where the appropriate Z is $\{(0.55 - 0.652) - (-0.02)\} / 0.0463 = -0.082 / 0.0463 = -1.77$ with rejection region $Z < -1.645$.

Then the CI in (b) to estimate the drop would be an UB of $-0.102 + 0.076 = -0.026$ or $[-1, -0.026]$, which does not cover the hypothesized value of -0.02 .

It is not a LB of -0.026 and a CI of $[-0.026, 1]$, which of course covers 0.02 .

Clearly it is much more straightforward to think of a (positive) improvement.

Test and CI for Two Proportions

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Sample    X    N  Sample p
1         163  250  0.652000
2         110  200  0.550000
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95% lower bound for difference: 0.0258185

Test for difference = 0.02 (vs > 0.02): Z = 1.77 P-Value = 0.038

- b. Calculate an asymmetric or one-sided 95% confidence interval to estimate the improvement in the level of support.

[2] at least $0.102 - 1.645 * 0.0463$ or at least $0.102 - 0.076$ or 0.026

- c. What is the minimum value of this improvement? Does your interval support your conclusion in part b? Explain.

[2] Since the minimum is 2.6%, the confidence interval does not cover the hypothesized value of 2%; therefore, we reject the null hypothesis.

Question 5. [8 marks]

Appendix B shows comparison data on the return on equity (ROE) in 2007 and 2008 for a sample of financial institutions. You are interested in seeing if the data support a claim that the average ROE declined from 2007 to 2008.

- a. Explain whether an independent samples or a paired sample test is more appropriate.

[1] Paired test since we have ROEs in two years for the same financial institutions (note mention of a sample of)

- b. Given your answer in part a, test at the 0.05 level of significance whether the data support your claim.

[5] Paired T for ROE07 - ROE08

	N	Mean	StDev	SE Mean
ROE07	27	12.9035	17.6623	3.3991
ROE08	27	9.6128	15.0869	2.9035
Difference	27	3.29067	9.31277	1.79224

95% lower bound for mean difference: 0.23378

T-Test of mean difference = 0 (vs > 0): T-Value = 1.84 P-Value = 0.039

-Ho: $\mu(\text{diff}) = 0$, Ha: $\mu(\text{diff}) > 0$

- $T = 3.29/1.79 = 1.84$ (2-sample $t=0.74$, with $s_p = 15.56$, $SE=4.47$ and 50 df)

-Reject Ho if $T > 1.71$ (1.68 for 2-sample t) or since p-value between 0.025 and $0.05 < \alpha$

-Reject Ho and conclude ROE has declined (0.5 mark each point)

1 mark each above.

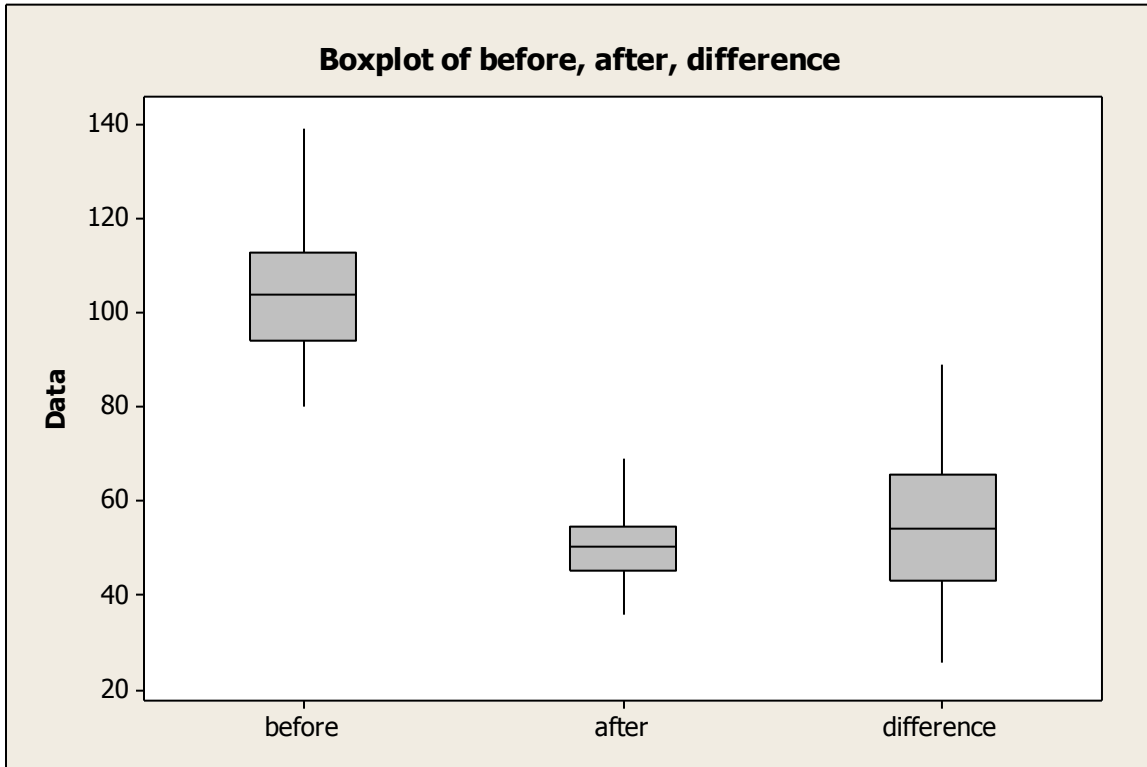
- c. Given your test in part b, what assumptions do you need to make regarding the underlying population distribution(s)? Explain with reference to specific boxplot(s) whether these assumptions are warranted.

[2] -Since the sample of differences is small, we need to assume that the population of changes is normally distributed.

-Since this is a paired test, we only look at the boxplot of differences (*must mention specific boxplot*)

-The assumption is warranted since the boxplot is relatively symmetric and there are no outliers.

Appendix A



Paired T-Test and CI: before, after

Paired T for before - after

	N	Mean	StDev	SE Mean
before	60	104.000	11.431	1.476
after	60	50.550	7.158	0.924
Difference	60	53.4500	13.3714	1.7262

T-Test of mean difference = 50 (): T-Value = _____ P-Value = _____

Two-Sample T-Test and CI: before, after

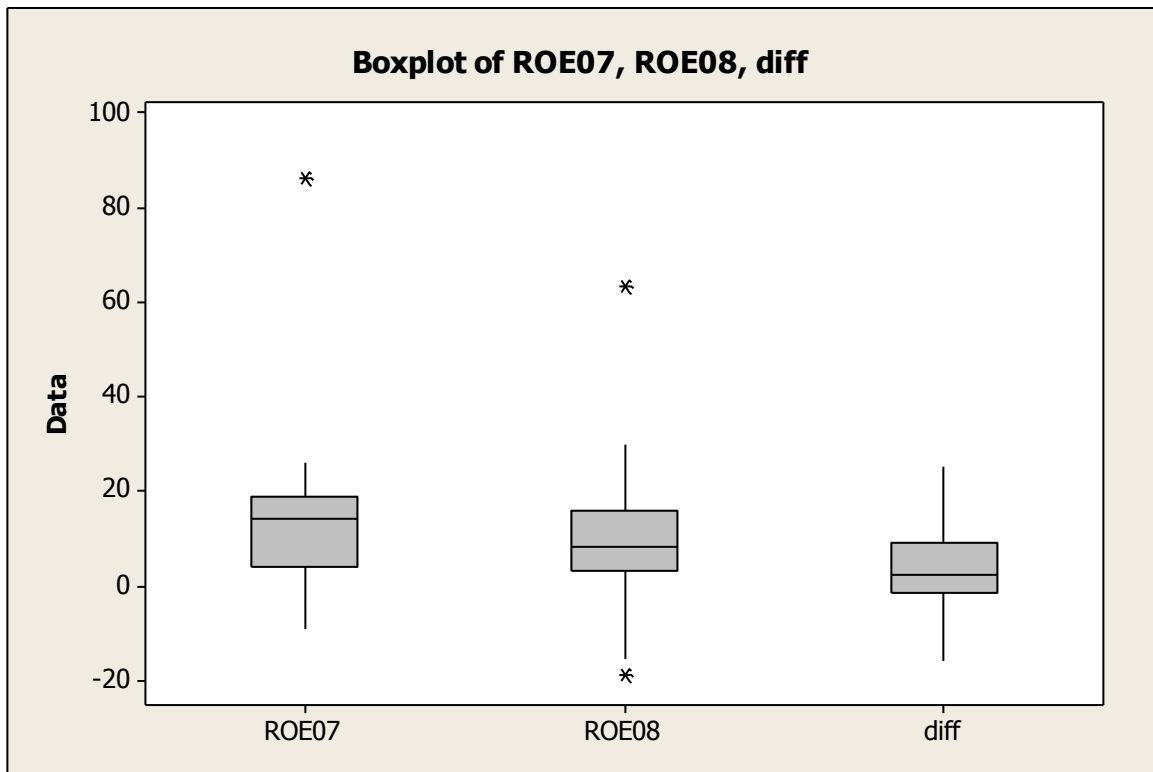
Two-sample T for before vs after

	N	Mean	StDev	SE Mean
before	60	104.0	11.4	1.5
after	60	50.55	7.16	0.92

Difference = μ (before) - μ (after)
 Estimate for difference: 53.4500

T-Test of difference = 50 (): T-Value = _____ P-Value = _____ DF = 99

Appendix B



Two-Sample T-Test and CI: ROE07, ROE08

Two-sample T for ROE07 vs ROE08

	N	Mean	StDev	SE Mean
ROE07	31	13.9	16.7	3.0
ROE08	31	10.2	14.3	2.6

Difference = μ (ROE07) - μ (ROE08)

Estimate for difference: 3.71487

T-Test of difference = 0 (____): T-Value = _____ P-Value = _____ DF = 58

Paired T-Test and CI: ROE07, ROE08

Paired T for ROE07 - ROE08

	N	Mean	StDev	SE Mean
ROE07	31	13.8862	16.7199	3.0030
ROE08	31	10.1713	14.3046	2.5692
Difference	31	3.71487	8.79247	1.57917

T-Test of mean difference = 0 () : T-Value = _____ P-Value = _____