MATH 151 [A01]
Test 1 (Version A)
Thursday October 15, 2015

Instructor: Dr. Jill Simmons

Last Name, First Name: Solutions

Student Number: V00_____

TO BE ANSWERED ON THE EXAM
DURATION: 50 minutes

SCORE: /24

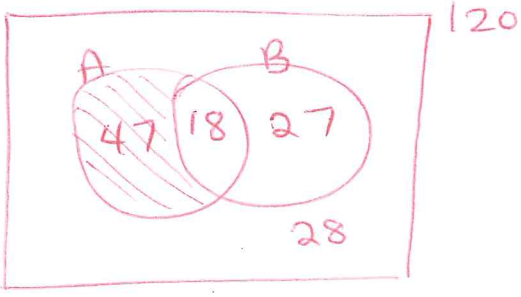
THIS EXAM HAS 4 PAGES, PLUS COVER.

Instructions:

- The only calculator permitted is the Sharp EL-510R or Sharp ELl-510RNB. No other electronic devices are permitted.
- Write out your solutions carefully and completely on the question paper and simplify your final answer as much as possible. Marks will not be given for final answers that are not supported by appropriate work.
- If extra space is required, you may use the backs of the test pages, but be sure to indicate where you have done so. No outside paper of any kind is allowed.
- Once you have answered each question, enter your *simplified* answer in the space provided. Final answers should be given as whole numbers or decimals rounded to 4 decimal places.

1. (3 marks) Two sets, A and B, have elements in some universe $\mathcal{U}$ with $n(\mathcal{U}) = 120$. If $n(A) = 65$, $n(B) = 45$, and there are 28 elements that are in neither of the two sets, how many elements are only in A?

Answer: 47



$n(A \cup B) = 92 = 65 + 45 - n(A \cap B) \Rightarrow n(A \cap B) = 18$

2. Consider the linear arrangements of the 10 letters in the word M A N A G E M E N T.

   (a) (1 mark) If there are no restrictions, how many different arrangements are there?

M A N G E T
M A N E

Answer: 226,800

$$\frac{10!}{2!2!2!2!}$$

   (b) (2 marks) How many of the arrangements have the two As separated by at least one other letter (i.e. the As are not side-by-side)?

First arrange M, M, N, N, G, E, E, T

$$\frac{8!}{2!2!2!}$$

Answer: 181,440

Then place As: $\binom{9}{2}$

$$\frac{8!}{2!2!2!}\binom{9}{2}$$

3. An English teacher wants her students to each write a book report, and each student can choose whichever book they want from a list of 4 books.

   (a) (1 mark) How many different ways are there for a class of 8 students to each choose which book they want to write about?

   Answer: 65,536

   $\underbrace{4 \cdot 4 \cdot 4 \cdots 4}_{8 \text{ times}} = 4^8$

   (b) (2 marks) How many of the ways for the 8 students to make their selection would result in 2 reports on each of the 4 books?

   Answer: 2520

   Choose 2 people for each book:

   $\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}$

4. (3 marks) Alexia wants to select a password that is 12-characters long and is a palindrome (reads the same forwards as backwards). She wants half of the characters to be letters that are not O (any of the other 25 letters are allowed) and the other half to be digits selected from the digits 1-9 (not 0). An example of such a password is HX233AA332XH. How many such passwords are possible?

   Answer: 227,812,500

   _ _ _ _ _ _ ← only need to decide 1st 6 spaces

   3 letters, 3 digits

   $\binom{6}{3} \cdot 25^3 \cdot 9^3$

   ↑ choose where letters go

5. (2 marks) Independent events E and F of an experiment have $Pr(E) = 0.35$ and $Pr(F|E) = 0.65$. Determine $Pr(E \cup F)$.

$Pr(F) = .65$

Answer: .7725

$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E) \cdot Pr(F)$

$= .35 + .65 - (.35)(.65)$

$= .7725$

6. (4 marks) Consider the experiment of rolling a balanced 6-sided die 10 times and observing the result. Let E be the event that at least one 6 is rolled and let F be the event that no 4s are rolled.

You do not need to make any calculations to answer the following questions:

(a) Are the events E and F mutually exclusive? Explain. A single sentence is enough.

No It is possible to roll 6s and no 4s.

(b) Are the events E and F independent? Explain. One or two sentences is all that is needed.

No If it is known that no 4s were rolled, this increases the probability that 6s were rolled.

7. (3 marks) A survey was conducted of 200 sports fans, and the people surveyed were categorized by their favourite sport. Each was asked whether they ever attend live games of their favourite sport. The number of people who answered YES and NO is shown for each category:

Favourite Sport	Number of YES responses	Number of NO responses
Hockey	7	63
Soccer	8	32
Tennis	3	17
Baseball	9	21
Other	16	24
		157

Suppose one of the 200 surveyed people is selected at random.

If the selected person does not attend live games of their favourite sport, what is the probability their favourite sport is a sport other than hockey?

Pr(not hockey | NO)

Answer: .5987

$= \frac{32+17+21+24}{157} = .5987$

8. (3 marks) When Ahmed puts his socks away he just shoves them into a drawer without sorting them. In the drawer on a particular day are 8 white socks, 6 black socks, 4 green socks, and 4 blue socks. If Ahmed randomly grabs 4 socks out of the drawer, what is the probability he will end up with at least one matching pair (i.e. at least 2 of the same colour)?

Answer: .8950

$1 - Pr(\text{no matching socks}) = 1 - \frac{8 \cdot 6 \cdot 4 \cdot 4}{\binom{22}{4}}$

- END -

MATH 151 [A01]
Test 1 (Version B)
Thursday October 15, 2015

Instructor: Dr. Jill Simmons

Last Name, First Name: Solutions

Student Number: V00_____

TO BE ANSWERED ON THE EXAM
DURATION: 50 minutes

SCORE: /24

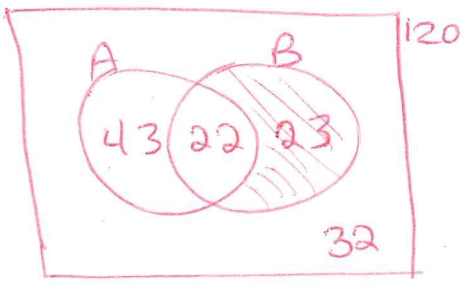
THIS EXAM HAS 4 PAGES, PLUS COVER.

Instructions:

- The only calculator permitted is the Sharp EL-510R or Sharp ELl-510RNB. No other electronic devices are permitted.
- Write out your solutions carefully and completely on the question paper and simplify your final answer as much as possible. Marks will not be given for final answers that are not supported by appropriate work.
- If extra space is required, you may use the backs of the test pages, but be sure to indicate where you have done so. No outside paper of any kind is allowed.
- Once you have answered each question, enter your *simplified* answer in the space provided. Final answers should be given as whole numbers or decimals rounded to 4 decimal places.

1. (3 marks) Two sets, A and B, have elements in some universe $\mathcal{U}$ with $n(\mathcal{U}) = 120$. If $n(A) = 65$, $n(B) = 45$, and there are 32 elements that are in neither of the two sets, how many elements are only in B?

Answer: 23



$n(A \cup B) = 88 = 65 + 45 - n(A \cap B) \Rightarrow n(A \cap B) = 22$

2. Consider the linear arrangements of the 10 letters in the word M A N A G E M E N T.

   (a) (1 mark) If there are no restrictions, how many different arrangements are there?

Answer: 226,800

M A N G E T
M A N E

$$\frac{10!}{2!2!2!2!}$$

   (b) (2 marks) How many of the arrangements have the two As directly next to each other?

Answer: 45,360

Glue As: (AA) One way

Then arrange (AA), M, M, N, N, G, E, E, T: $\frac{9!}{2!2!2!}$

3. An English teacher wants her students to each write a book report, and each student can choose whichever book they want from a list of 3 books.

   (a) (1 mark) How many different ways are there for a class of 9 students to each choose which book they want to write about?

Answer: 19,683

$\underbrace{3 \cdot 3 \cdot 3 \cdots 3}_{9 \text{ times}} = 3^9$

   (b) (2 marks) How many of the ways for the 9 students to make their selection would result in 3 reports on each of the 3 books?

Answer: 1680

Choose 3 people for each book:

$\binom{9}{3}\binom{6}{3}\binom{3}{3}$

4. (3 marks) Alexia wants to select a password that is 10-characters long and is a palindrome (reads the same forwards as backwards). She wants 4 of the characters to be letters that are not O (any of the other 25 letters are allowed) and 6 to be digits selected from the digits 1-9 (not 0). An example of such a password is H233XX332H. How many such passwords are possible?

Answer: 4556250

_ _ _ _ _ _ ← only need to decide 1st 5 spaces

2 letters, 3 digits

$\binom{5}{2} \cdot 25^2 \cdot 9^3$

↑ decide where letters go

5. (2 marks) Independent events E and F of an experiment have $Pr(E) = 0.35$ and $Pr(F|E) = 0.55$. Determine $Pr(E \cup F)$.

$Pr(F) = .55$

Answer: .7075

$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E)Pr(F)$

$= .35 + .55 - (.35)(.55)$

$= .7075$

6. (4 marks) Consider the experiment of flipping a fair coin 8 times and observing the result. Let E be the event that at least 4 Heads are flipped and let F be the event that at least 4 Tails are flipped.

You do not need to make any calculations to answer the following questions:

(a) Are the events E and F mutually exclusive? Explain. A single sentence is enough.

No It is possible to flip 4 Heads and 4 Tails.

(b) Are the events E and F independent? Explain. One or two sentences is all that is needed.

No If it is known that at least 4 Heads were flipped, that greatly reduces the probability of F.

7. (3 marks) A survey was conducted of 200 sports fans, and the people surveyed were categorized by their favourite sport. Each was asked whether they ever attend live games of their favourite sport. The number of people who answered YES and NO is shown for each category:

Favourite Sport	Number of YES responses	Number of NO responses
Hockey	7	63
Soccer	8	32
Tennis	3	17
Baseball	9	21
Other	16	24

157

Suppose one of the 200 surveyed people is selected at random.

If the selected person does not attend live games of their favourite sport, what is the probability their favourite sport is a sport other than Soccer?

Answer: .7962

$\Pr(\text{not soccer} \mid \text{NO})$

$= \frac{63+17+21+24}{157}$

8. (3 marks) When Ahmed puts his socks away he just shoves them into a drawer without sorting them. In the drawer on a particular day are 10 white socks, 6 black socks, 6 green socks, and 2 blue socks. If Ahmed randomly grabs 4 socks out of the drawer, what is the probability he will end up with at least one matching pair (i.e. at least 2 of the same colour)?

Answer: .9322

$1 - \Pr(\text{no matching socks})$

$= 1 - \frac{10 \cdot 6 \cdot 6 \cdot 2}{\binom{24}{4}}$

- END -