MATH 1007C+M **Term test #1 – solution** **25 September 2015**

This test was written in room: ________________ Total marks: 32 marks

This test is closed book. **No calculators** or electronic aids are permitted. **Please supply your answers on this sheet.**

PLEASE PRINT ______________________ First name Last name ____________ Student number

Please show your work where appropriate! TA's have extra paper if you need it. Test duration: 50 minutes.

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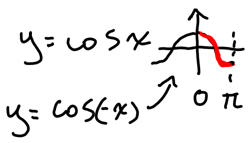
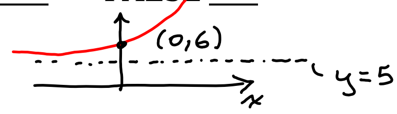
1. Fill in the blanks: $\cos(x-\pi/2) = \cos(-(\pi/2 - x)) = \cos(\pi/2 - x) = \sin x$



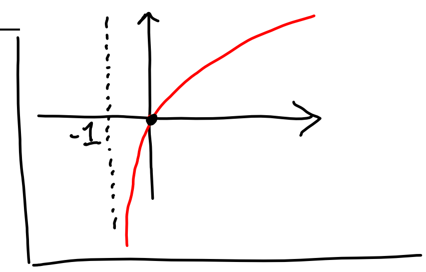
a. [1] $\cos(x - \pi/2) = \sin x$ ... TRUE ✓ FALSE ___

b. [1] The curve $y = \cos(-x)$ is decreasing on $[0, \pi]$ ... TRUE ✓ FALSE ___

c. [1] The range of the function $f(x) = (1.1)^x + 5$ is $(5, \infty)$



d. [1] The domain of the function $f(x) = \ln(x+1)$ is $(-1, \infty)$



2. **Simplify** as much as possible.

a. [2] $\dfrac{\frac{a}{b} - \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}} = \dfrac{\frac{a^2-b^2}{ab}}{\frac{a-b}{ab}} = \dfrac{a^2-b^2}{a-b} = \dfrac{\cancel{(a-b)}(a+b)}{\cancel{(a-b)}} = \boxed{a+b}$

b. [2] $\sqrt[3]{4}^{\,(\log_2(27))} =$

$$\ldots = \left(\left(2^2\right)^{1/3}\right)^{\log_2 3^3} = \left(2^{2/3}\right)^{\log_2 3^3} = 2^{\frac{2}{3}\cdot\log_2 3^3} = 2^{\log_2 (3^3)^{2/3}} =$$

$$\ldots = \left(3^3\right)^{2/3} = 3^{\left(3\cdot\frac{2}{3}\right)} = 3^2 = \boxed{9}$$

3. [2] Convert the expression $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}}}$ into the form $\left(\frac{a}{b}\right)^x$, where x is a rational number (a fraction).

$$\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}}} = \left(\frac{a}{b}\left(\frac{b}{a}\right)^{1/2}\right)^{1/2} = \left(\left(\frac{a}{b}\right)^{1/2}\right)^{1/2} = \boxed{\left(\frac{a}{b}\right)^{1/4}}$$

4. [2.5] Determine the inverse of the function $y = f(x) = 5(x-1)^3 - 2$.

- $y = 5(x-1)^3 - 2$

$y + 2 = 5(x-1)^3$

$\dfrac{y+2}{5} = (x-1)^3$

$x - 1 = \sqrt[3]{\dfrac{y+2}{5}}$

$x = \sqrt[3]{\dfrac{y+2}{5}} + 1 = f^{-1}(y)$

$$\boxed{\therefore\ y = f^{-1}(x) = \sqrt[3]{\frac{x+2}{5}} + 1}$$

5. [2.5] Solve the inequality $|2x - 5| \geq 7$, expressing the solution in the form of an interval or a union of sub-intervals.

NOTE: $|A| \geq b$ means $A \geq b$ or $A \leq -b$ $(b > 0)$

so: either $2x - 5 \geq 7$ OR $2x - 5 \leq -7$

... this stems from the 2 cases: $2x-5 \geq 0$ OR $2x - 5 < 0$

CASE 1: $2x - 5 \geq 0 \Rightarrow |2x-5| = 2x - 5$

$|2x-5| \geq 7 \Rightarrow 2x - 5 \geq 7 \Rightarrow \boxed{x \geq 6}$

CASE 2: $2x - 5 < 0 \Rightarrow |2x - 5| = -(2x-5)$

$|2x - 5| \geq 7 \Rightarrow -(2x-5) \geq 7$, $2x - 5 \leq -7$ $\Rightarrow \boxed{x \leq -1}$

SOLUTION is $(-\infty, -1] \cup [6, \infty)$

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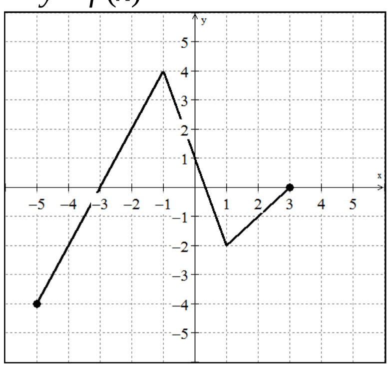
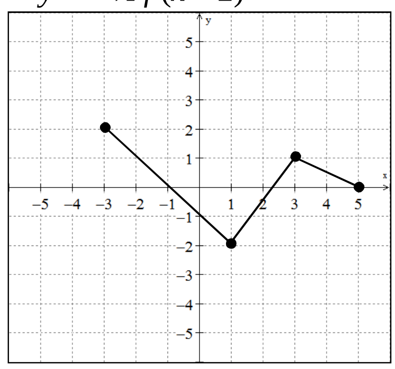
6. [3] The graph of f is plotted. Plot the graphs of $y = -\tfrac{1}{2} f(x-2)$ and $y = f(-x-2) = f(-(x+2))$

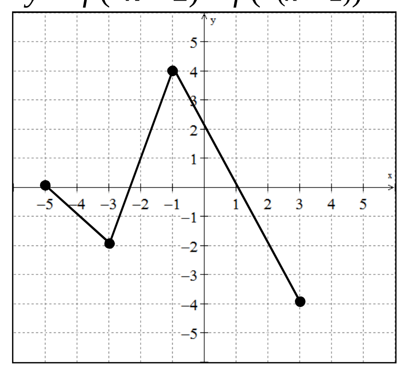
$y = f(x)$



$y = -\tfrac{1}{2} f(x-2)$



$y = f(-x-2) = f(-(x+2))$



7. Let $\cos\theta = \tfrac{2}{5}$, with $\tfrac{3\pi}{2} \le \theta \le 2\pi$

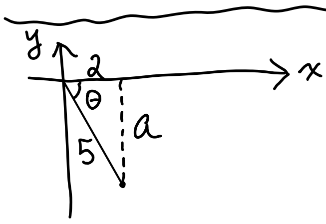
a. [1] Complete the sentence: θ is situated in quadrant 4.

b. [2.5] Determine $\sin\theta$

$\sin\theta < 0$ in Q4, so: $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-(2/5)^2} = \boxed{-\dfrac{\sqrt{21}}{5}}$

$\dfrac{25}{25} - \dfrac{4}{25}$

OR



$a = \sqrt{5^2 - 2^2} = \sqrt{21}$

$\therefore \sin\theta = -\dfrac{a}{5} = \boxed{-\dfrac{\sqrt{21}}{5}}$

c. [2] Determine $\tan\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\sqrt{21}/5}{2/5} = \boxed{-\dfrac{\sqrt{21}}{2}}$

(*For question #7 you may use a graphical technique or one involving identities*)

8. Let $f(x) = \ln(5-x)$ and $g(x) = \sqrt{x+4}$

a. [1] What is the domain of g? (*answer only ok*)

$x+4 \geq 0 \Rightarrow x \geq -4 \Rightarrow \text{dom}(g) = [-4, \infty) = \{x \in \mathbb{R} \mid x \geq -4\}$

b. [2.5] What is the domain of $f+g$? (*answer only ok*)

$f+g(x) = \ln(5-x) + \sqrt{x+4} \leftarrow$

$5-x>0 \Rightarrow x<5$ and $x \geq -4$

$\}$ $\text{dom}(f+g) = [-4, 5)$

c. [2] What is the rule of $f \circ g(x) = f(g(x))$? Do not try to simplify.

$f \circ g(x) = f(g(x)) = \ln(5 - g(x)) = \ln(5 - \sqrt{x+4})$

d. [3] What is the domain of $f \circ g(x)$?

$f \circ g(x) = \ln(5 - \sqrt{x+4})$

- presence of $\sqrt{x+4} \Rightarrow x+4 \geq 0 \Rightarrow x \geq -4$

and

- " " $\ln(\quad) \Rightarrow 5 - \sqrt{x+4} > 0$

$\ldots \Rightarrow \sqrt{x+4} < 5$

$x+4 < 25$

$x < 21$

$\boxed{\therefore \text{dom}(f \circ g) = [-4, 21)}$