MAT2384 C-Ordinary differential Equations and Numerical Methods

Practice Midterm II (Winter 2016)

Professor: Joseph Khoury. **Duration: 80 minutes**

Last Name: _____

First Name: _____

Student Number: _____

(1) This is a closed book exam.

(2) Only basic scientific calculators are allowed. Graphing or programmable calculators are not permitted.

(3) The exam has 4 questions worth a total of 24 points.

(4) The exam has 6 pages.

(5) Please write your answers in a complete and clear way. You may use the back of the pages or the additional pages at the end if you need more space for your work.

(6) You must answer all the questions.

1. [**6 points**] Use the **undetermined coefficients method** to find the general solution of the ODE:

$$y''' - 6y'' + 9y' = 12e^{3x} - 10\cos x + 9.$$

2. [**6 points**] Consider the following definite intégral:

$$I = \int_0^{0.5} x^2 \cos(x) dx.$$

(1) Use **Simpson rule** to approximate the value of I with a maximal absolute error of 0.0001. Round your answer to six decimal places. *Reminder*: For Simpson rule,

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(b)\right]$$

and the error satisfies $|E_S| \leq \frac{M(b-a)^5}{180n^4}$ where M is an upper bound for $|f^{(4)}(x)|$ on the interval $[a, b]$.

(2) Use Gaussian Quadrature of order 3 pour to approximate the value of de I (round the values of the coefficients and the nodes to 6 decimal places). *Reminder*: For the Gaussian Quadrature of order 3, the coefficients are $w_1 = 0.555555556$, $w_2 = 0.888888889$ and $w_3 = 0.555555556$ and the nodes are $t_1 = -0.7745966692$, $t_2 = 0.0$ and $t_3 = 0.7745966692$.

3. [**6 points**] Use the **method of variation of parameters** to solve the IVP:

$$y'' - 8y' + 16y = x^{\frac{1}{3}}\, e^{4x}, \quad y(0) = 1, \quad y'(0) = 1.$$

4. [**6 points**] Give the general solution to the following system of first order ODEs:

$$\vec{y'} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \vec{y} + \begin{bmatrix} -x+1 \\ e^{2x} \end{bmatrix}.$$