

CVG2140 – Solutions to Assignment No. 8 (Combined Stresses)

Problem 1. Determine the principal stresses and their directions at the surface point C of the solid cylinder shown in Fig. 1 (diameter = 50 mm, length = 250 mm) subjected to an axial force P and torque T .

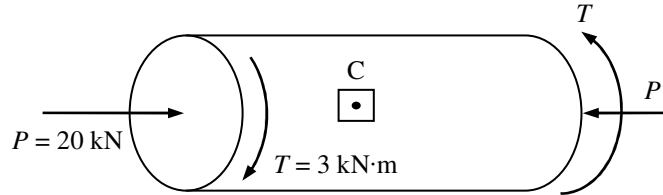


Fig. 1

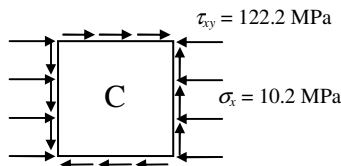
At C:

$$\sigma_x = \frac{P}{A} = \frac{-20 \times 10^3}{\pi \times 25^2} = -10.2 \text{ MPa}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16 \times 3 \times 10^6}{\pi \times 50^3} = 122.2 \text{ MPa}$$

The state of stress at point C is therefore:



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 122.2}{-10.2} = -24 \Rightarrow 2\theta_p = -87.6^\circ \Rightarrow \theta_p = \begin{cases} -43.8^\circ \\ -43.2^\circ + 90^\circ = 46.2^\circ \end{cases}$$

Substituting θ_p and $\theta_p + 90^\circ$ into the transformation equations:

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-10.2}{2} + \frac{-10.2}{2} \cos(-87.6) + 122.2 \sin(-87.6) = -127.4 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-10.2}{2} + \frac{-10.2}{2} \cos(2 \times 46.2) + 122.2 \sin(2 \times 46.2) = 117.2 \text{ MPa} \end{aligned}$$

Therefore, $\underline{\sigma_1 = \sigma_{\max} = 117.2 \text{ MPa at } 46.2^\circ}$ and $\underline{\sigma_2 = \sigma_{\min} = -127.4 \text{ MPa at } -43.8^\circ}$

Problem 2. Determine the principal stresses and their directions at three surface locations, A, B, and C, of the brass cantilever beam with rectangular cross-section shown in Fig. 2. The beam is subjected to its own weight as well as a concentrated load of 12 kN at its free end. The material properties of brass are as follows: density $\rho = 8500 \text{ kg/m}^3$, Young's modulus $E = 100 \text{ GPa}$, and yield stress $\sigma_y = 450 \text{ MPa}$.

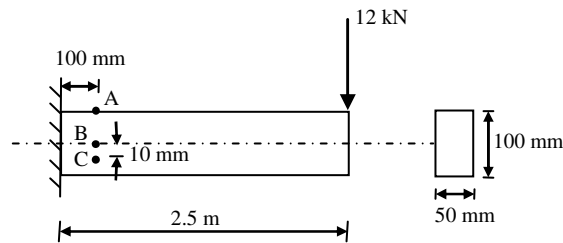
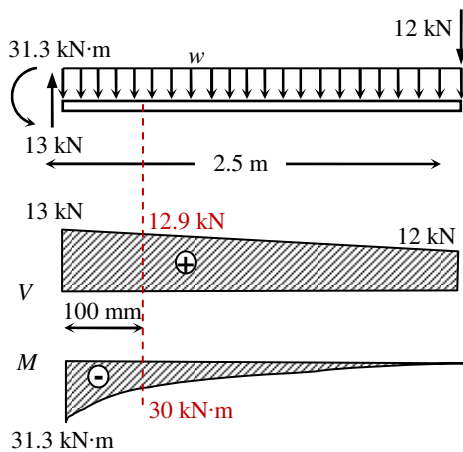


Fig. 2

The weight per linear meter of the cantilever beam is:

$$w = \rho g A = 8500 \times 9.81 \times 0.1 \times 0.05 = 0.417 \text{ kN/m}$$



At the three locations:

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{VQ}{Ib} = \frac{12VQ}{b^2h^3}$$

	Q (mm^3)	σ_x (MPa)	τ_{xy} (MPa)	σ_1 (MPa)	θ_{p1}	σ_2 (MPa)	θ_{p2}
A	0	360	0	360	0°	0	90°
B	62.5×10^3	0	-3.87	3.87	45°	-3.87	135°
C	60×10^3	-72	-3.71	0.19	92.9°	-72.19	2.9°

Problem 3. The state of stress at a point is shown in Fig. 3. By using Mohr's circle, determine:

- The principal stresses and their orientation;
- The maximum in-plane shear stress and average normal stress, specifying the corresponding orientation; and,
- The equivalent state of stress if the coordinate system is oriented 30° clockwise from the element shown in Fig. 3.

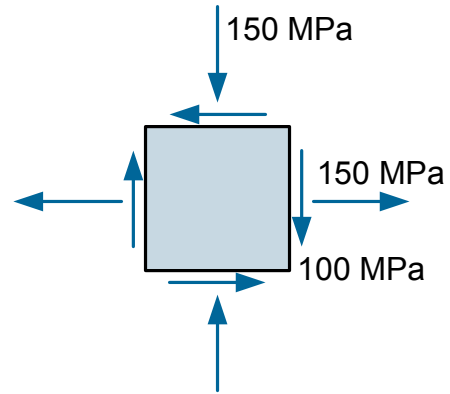


Fig. 3

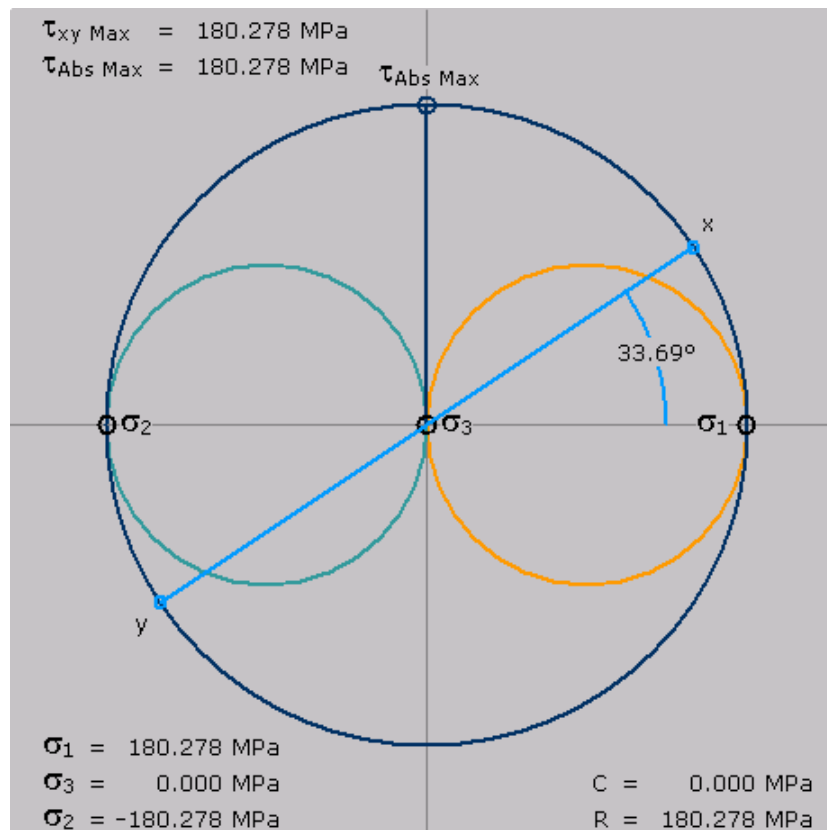
From Fig. 3:

$$\sigma_x = 150 \text{ MPa}, \sigma_y = -150 \text{ MPa}, \tau_{xy} = -100 \text{ MPa}$$

Therefore,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 - 150}{2} = 0 \text{ MPa}$$

$$R = \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{150 + 150}{2}\right)^2 + 100^2} = 180.28 \text{ MPa}$$

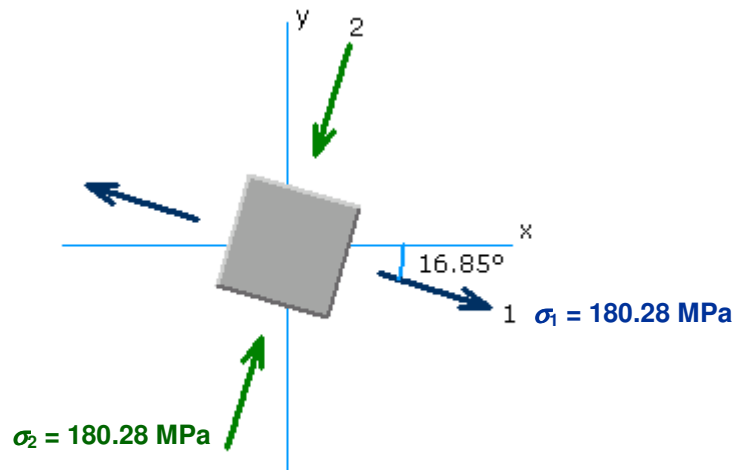


$$a) \quad 2\theta_p = \tan^{-1}\left(\frac{100}{150}\right) = 33.69^\circ$$

$\theta_p = -33.69^\circ/2 = -16.85^\circ$ (negative, because the rotation from x to σ_1 is clockwise)

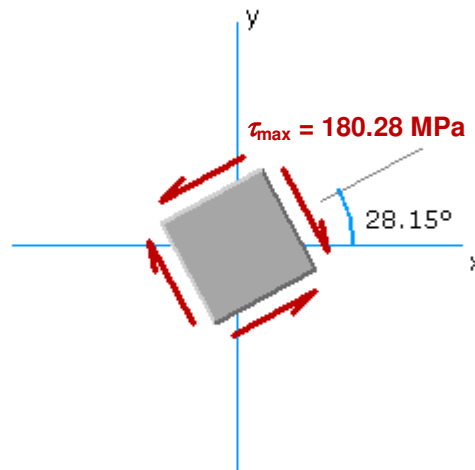
$\sigma_1 = 0 + 180.28 = 180.28 \text{ MPa}$ acting at -16.85°

$\sigma_2 = 0 - 180.28 = -180.28 \text{ MPa}$ acting at $-16.85^\circ + 90^\circ = 73.15^\circ$

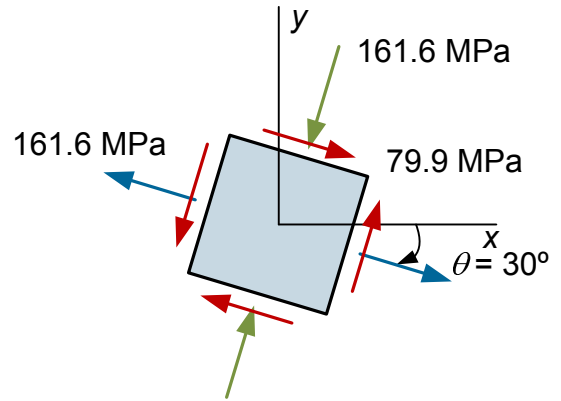
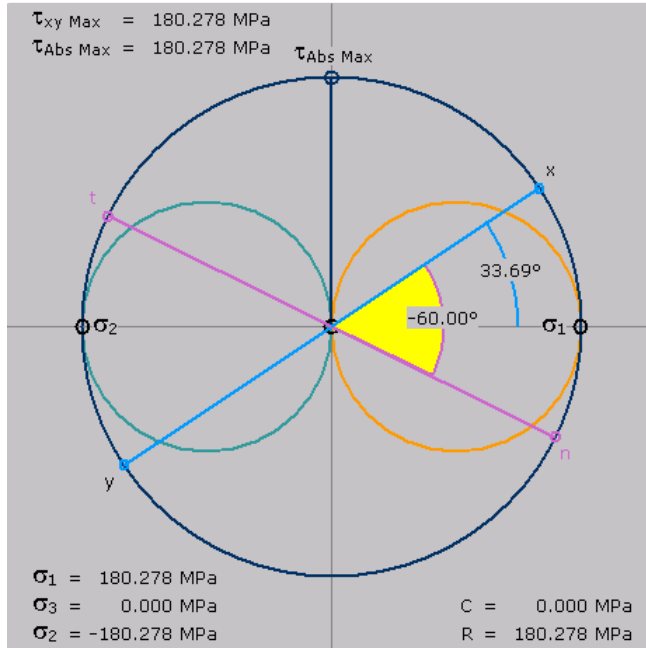


$$b) \quad \tau_{\max} = R = 180.28 \text{ MPa acting at } -16.85^\circ + 45^\circ = 28.15^\circ$$

$$\sigma_{\text{avg}} = C = 0 \text{ MPa}$$



c)



$$\sigma_{x'} = R \times \cos(60^\circ - 33.69^\circ) = 180.28 \times \cos(26.31^\circ) = 161.6 \text{ MPa}$$

$$\sigma_{y'} = -R \times \cos(60^\circ - 33.69^\circ) = -161.6 \text{ MPa}$$

$$\tau_{x'y'} = R \times \sin(60^\circ - 33.69^\circ) = 180.28 \times \sin(26.31^\circ) = 79.9 \text{ MPa}$$