CVG2140 – Solutions to Assignment No. 5 (Flexural Stresses)

Problem 1. A simply supported wood beam *AB* with span length L = 3.5 m carries a uniform load of intensity q = 6.4 kN/m (see Fig. 1). Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width b = 150 mm and height h = 280 mm.

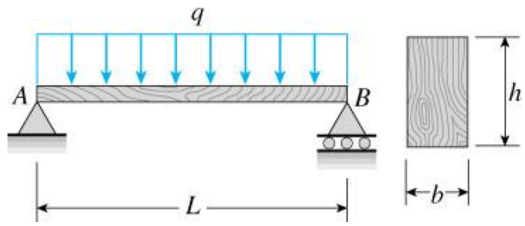


Fig. 1

For a simple-supported beam under the action of uniformly distributed load q, the maximum bending moment occurs at mid-span, and its value is:

$M_{\max} = \dfrac{ql^2}{8}$, where l is the span length

$$\sigma_{\max} = \frac{M_{\max} \times y_{\max}}{I} = \frac{M_{\max}}{S}, \quad S = \frac{bh^2}{6} \quad \Rightarrow \quad \sigma_{\max} = \frac{6M_{\max}}{bh^2} = \frac{3ql^2}{4bh^2}$$

Substituting numerical values results in:

$$\sigma_{\max} = \frac{3 \times 6.4 \times \left(3.5 \times 10^3\right)^2}{4 \times 150 \times 280^2} = \underline{5.0 \text{ MPa}}$$

Problem 2. A seesaw weighing 5 kg/m of length is occupied by two children, each weighing 40 kg (see Fig. 2). The centre of gravity of each child is 2.5 m from the fulcrum. The board is 5.8-m long, 20-cm wide and 4-cm thick. What is the maximum bending stress in the board?

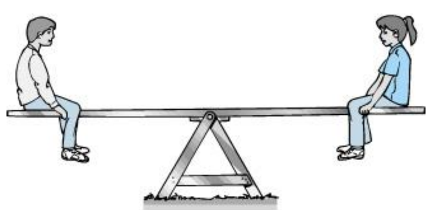
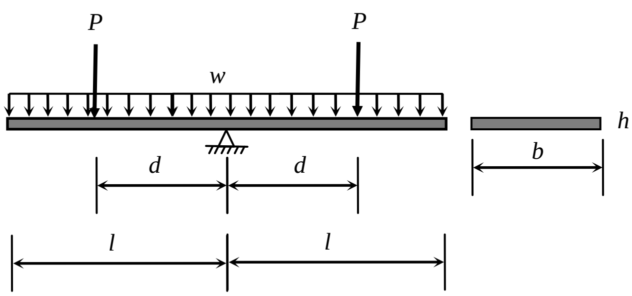


Fig. 2



w = 5 kg/m × 9.8 m/s^2 = 49 N/m, P = 40 kg × 9.8 m/s^2 = 392 N, d = 2.5 m, l = 2.9 m, b = 20 cm, h = 4 cm

The maximum bending moment occurs at mid-span, and its value is:

$$M_{\max} = Pd + \frac{wl^2}{2} = (392 \times 2.5) + \left(\frac{49 \times 2.9^2}{2}\right) = 1{,}186.1 \text{ N·m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{bh^2}{6} \quad \Rightarrow \quad \sigma_{\max} = \frac{6M_{\max}}{bh^2} = \frac{6 \times 1186.1}{0.2 \times 0.04^2} = \underline{22.2 \text{ MPa}}$$

Problem 3. A small dam of height h = 2.4 m is constructed of vertical wood beams AB of thickness t = 160 mm, as shown in Fig. 3. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is γ = 9.81 kN/m^3. Hint: the hydrostatic pressure p in still water is $p = \gamma d$ (Pa), where γ is the weight density of water (N/m^3) and d is the depth from the free surface (m). Do the problem for 1 m of the dam width.

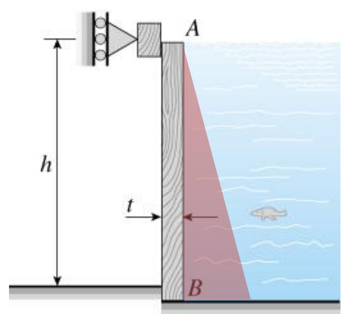
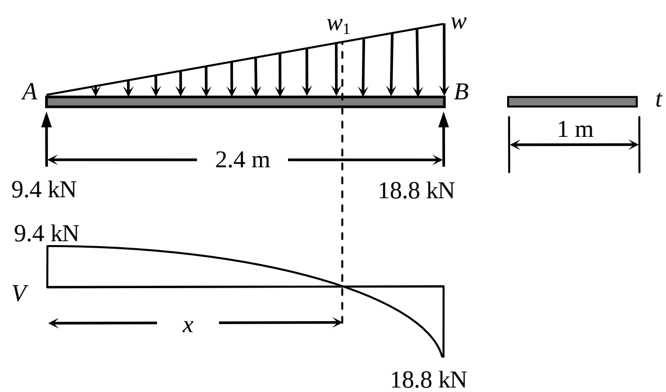
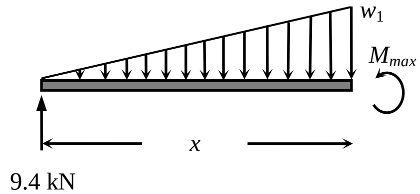


Fig. 3

$$w = \gamma \times h \times width = 9.81 \times 2.4 \times 1 = 23.5 \text{ kN/m}$$



The maximum moment occurs at x, since $V = \frac{dM}{dx} = 0$. By establishing equilibrium at that section:



$$\sum F_y = 9.4 - \left(\frac{1}{2} \times w_1 \times x\right) = 0$$

Since $\frac{w_1}{x} = \frac{w}{2.4}$, $9.4 - \left(\frac{1}{2} \times \frac{23.5}{2.4} \times x^2\right) = 0 \quad \Rightarrow \quad x = 1.39 \text{ m}$

The moment at x = 1.39 m is then calculated by establishing moment equilibrium at that section:

$$\sum M = -(9.4 \times 1.39) + \left(\frac{1}{2} \times 13.6 \times 1.39 \times \frac{1}{3} \times 1.39\right) + M_{\max} = 0 \quad \Rightarrow \quad M_{\max} = 8.69 \text{ kN·m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{bt^2}{6} \quad \Rightarrow \quad \sigma_{\max} = \frac{6M_{\max}}{bt^2} = \frac{6 \times 8.69 \times 10^3}{1 \times 0.16^2} = \underline{2 \text{ MPa}}$$

<u>Problem 4.</u> The beam shown in Fig. 4 has a T cross section with moments of inertia with respect to y and z equal to I_y = 15.63×10^6 mm^4 and I_z = 53.13×10^6 mm^4, respectively. For the beam section where the absolute value of the internal bending moment is maximum, $M = |M_{\max}|$, find the maximum bending stress $|\sigma_{\max}|$ and the bending stress @ H σ_H (is σ_H compressive or tensile?). Considering the entire beam, find the maximum tensile bending stress $\sigma_{\max,\text{Tensile}}$.

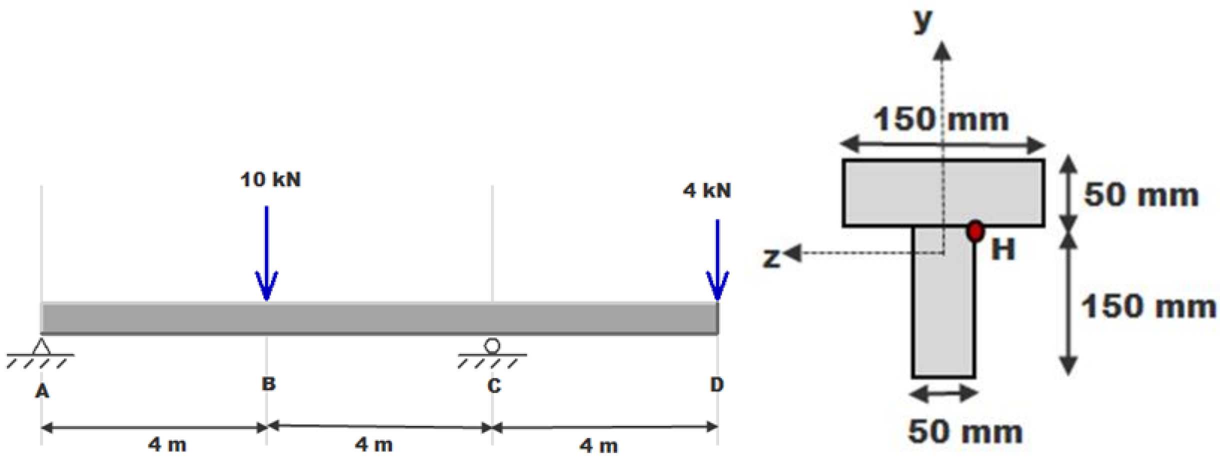


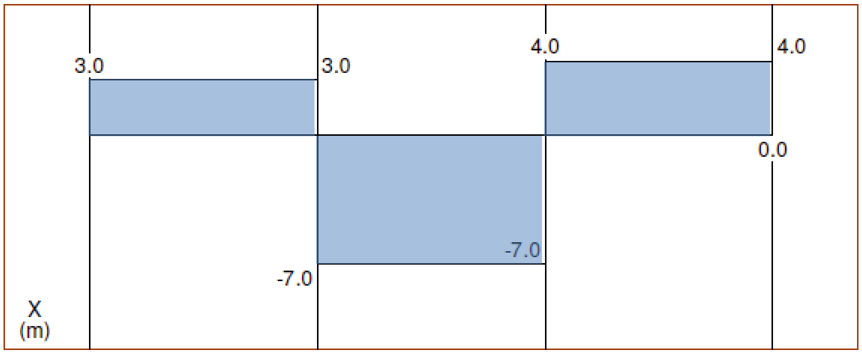
Fig. 4

From equilibrium:

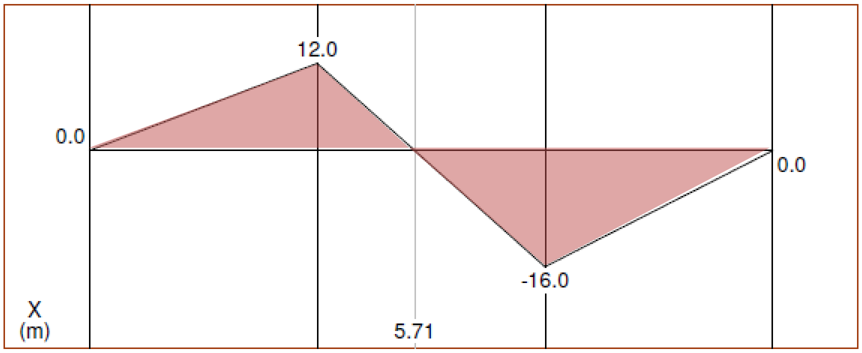
$$\sum F_x = A_x = 0$$

$$\left.\begin{aligned} \sum F_y &= A_y + C_y - 14 = 0 \\ \sum M_A &= -(10)(4) + (C_y)(8) - (4)(12) = 0 \end{aligned}\right\} \Rightarrow \begin{cases} A_y = 3\text{ kN} \\ C_y = 11\text{ kN} \end{cases}$$

The shear and bending moment diagrams of the beam are as follows:



Shear Diagram (kN)



Moment Diagram (kN-m)

a) $\sigma_{\max}$ from the BMD, $|M_{max}|$ = 16 kN·m

(i) The neutral axis (or centroid) of the cross section is located at:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(50\times150\times175)+(50\times150\times75)}{(50\times150)+(50\times150)} = 125\text{ mm from the bottom}$$

(ii) The maximum bending stress $|\sigma_{\max}|$ occurs at the bottom fibre where c = 125 mm.

$$|\sigma_{\max}| = \left|\frac{M_{\max}c}{I_z}\right| = \frac{(16\times10^6)(125)}{(53.125\times10^6)} = \underline{37.6\text{ MPa}}$$

(Note that this is a compressive stress.)

(iii) The bending stress σ_H:

$$\sigma_H = -\frac{M_{\max}y_H}{I_z} = -\frac{(-16\times10^6)(25)}{(53.125\times10^6)} = \underline{7.53\text{ MPa (tensile stress)}}$$

b) The maximum tensile bending stress $\sigma_{\text{max, tensile}}$:

Considering the section where $M = M^{-}_{\max} = -16$ kN·m, $\sigma_{\text{max, tensile}}$ will develop at the top fibre where c = 75 mm.

$$\sigma_{\text{max, tensile}} = -\frac{M_{\max} c}{I_z} = -\frac{\left(-16\times10^{6}\right)\left(75\right)}{\left(53.125\times10^{6}\right)} = 22.6 \text{ MPa}$$

Considering the section where $M = M^{+}_{\max} = 12$ kN·m, $\sigma_{\text{max, tensile}}$ will develop at the bottom fibre where c = -125 mm.

$$\sigma_{\text{max, tensile}} = -\frac{M_{\max} c}{I_z} = -\frac{\left(12\times10^{6}\right)\left(-125\right)}{\left(53.125\times10^{6}\right)} = \underline{28.2 \text{ MPa}}$$

Therefore, the maximum tensile bending stress $\sigma_{\text{max, tensile}}$ develops at the bottom fibre of the section with maximum positive moment, and its value is $\underline{\sigma_{\text{max, tensile}} = 28.2 \text{ MPa}}$.