CVG2140 – Solutions to Assignment No. 4 (Axial Members)

Problem 1. The A-36 steel rod (E = 200 GPa) is subjected to the loading shown in Fig. 1. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of its end D. Neglect the size of the couplings at B, C, and D.

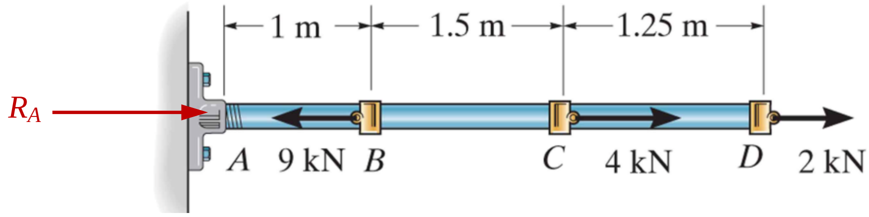
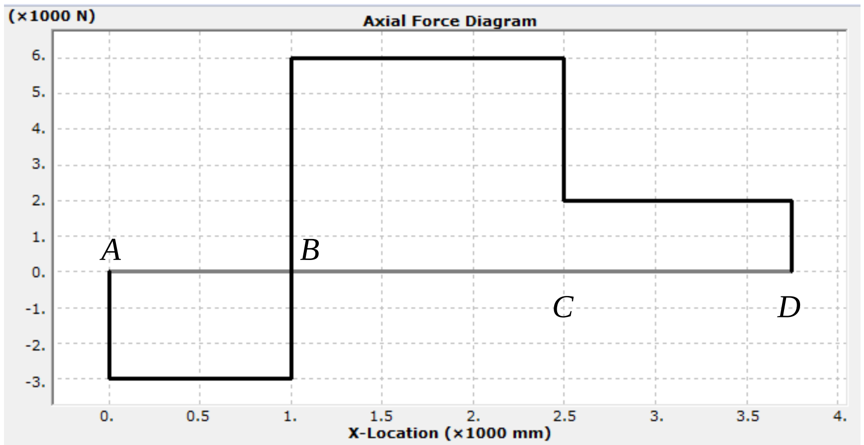


Fig. 1

By establishing equilibrium:

$$\sum F_x = R_A - 9 + 4 + 2 = 0 \quad \Rightarrow \quad R_A = 3 \text{ kN}$$

The distribution of the internal axial load $N(x)$ along the steel rod is shown below:



The displacement of end D is therefore given by:

$$\delta_D = \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = \frac{F_{BA}L_{BA}}{EA} + \frac{F_{CB}L_{CB}}{EA} + \frac{F_{DC}L_{DC}}{EA}$$

$$\delta_D = -\frac{3\times10^3 \times 1{,}000}{200\times10^3 \times 50} + \frac{6\times10^3 \times 1{,}500}{200\times10^3 \times 50} + \frac{2\times10^3 \times 1{,}250}{200\times10^3 \times 50} = \underline{0.85 \text{ mm}}$$

Problem 2. The load of 800 lb is supported by the four 304 stainless steel wires ($E = 28\times10^6$ psi) that are connected to the rigid members *AB* and *DC* (Fig. 2). Determine the vertical displacement of the load if the members were horizontal before the load was applied. Each wire has a cross-sectional area of 0.05 in^2.

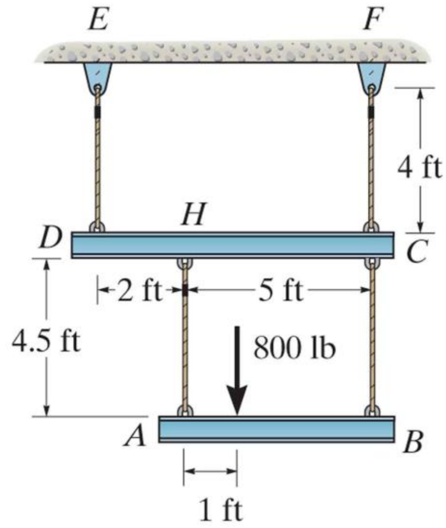
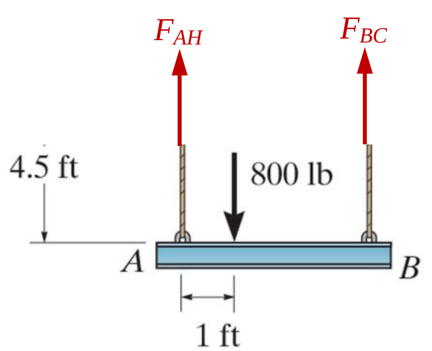
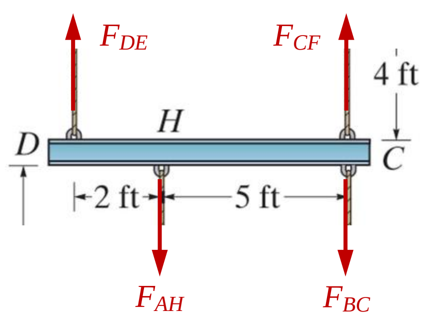


Fig. 2





Establishing equilibrium on bar *AB* by taking moments (assumed positive counterclockwise) about *B* and *A* results in:

$$\sum M_B = 800\times4 - F_{AH}\times5 = 0 \quad\Rightarrow\quad F_{AH} = 640 \text{ lb}$$

$$\sum M_A = -800\times1 + F_{BC}\times5 = 0 \quad\Rightarrow\quad F_{BC} = 160 \text{ lb}$$

Likewise, establishing equilibrium on bar *DC* by taking moments about *C* and *D* results in:

$$\sum M_C = F_{AH}\times5 - F_{DE}\times7 = 0 \quad\Rightarrow\quad F_{DE} = 457.14 \text{ lb}$$

$$\sum M_D = -F_{AH}\times2 - F_{BC}\times7 + F_{CF}\times7 = 0 \quad\Rightarrow\quad F_{CF} = 342.86 \text{ lb}$$

Since *E* and *F* are fixed,

$$\delta_D = \frac{F_{DE}L_{DE}}{EA} = \frac{457.14\times4\times12}{28\times10^6\times0.05} = 0.01567 \text{ in (downwards)}$$

$$\delta_C = \frac{F_{CF} L_{CF}}{EA} = \frac{342.86 \times 4 \times 12}{28 \times 10^6 \times 0.05} = 0.01176 \text{ in (downwards)}$$

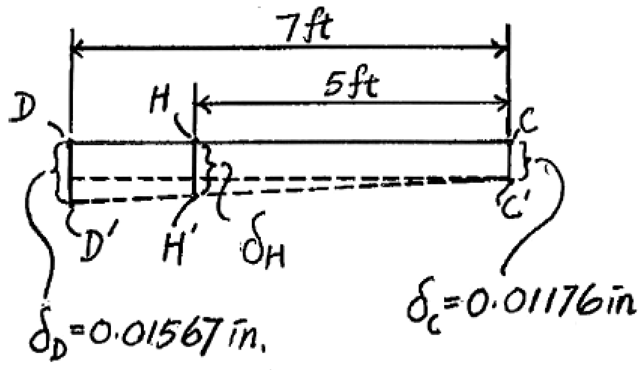


Fig. 2(a)

From the geometry shown in Fig. 2(a),

$$\delta_H = 0.01176 + \frac{5}{7} \; 0.01567 - 0.01176 \; = 0.01455 \text{ in (downwards)}$$

The elongations in the wires *AH* and *BC* are given by:

$$\delta_{A/H} = \frac{F_{AH} L_{AH}}{EA} = \frac{640 \times 4.5 \times 12}{28 \times 10^6 \times 0.05} = 0.02469 \text{ in (downwards)}$$

$$\delta_{B/C} = \frac{F_{BC} L_{BC}}{EA} = \frac{160 \times 4.5 \times 12}{28 \times 10^6 \times 0.05} = 0.006171 \text{ in (downwards)}$$

Thus, the vertical displacements experienced by points *A* and *B* are respectively obtained from:

$$\delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in (downwards)}$$

$$\delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in (downwards)}$$

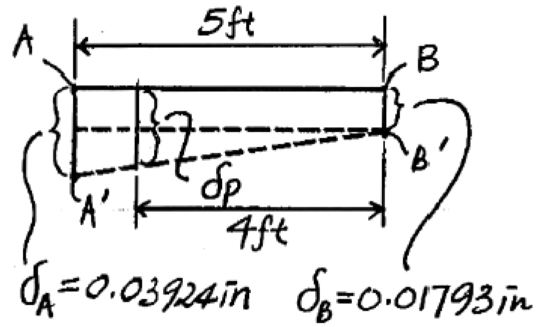
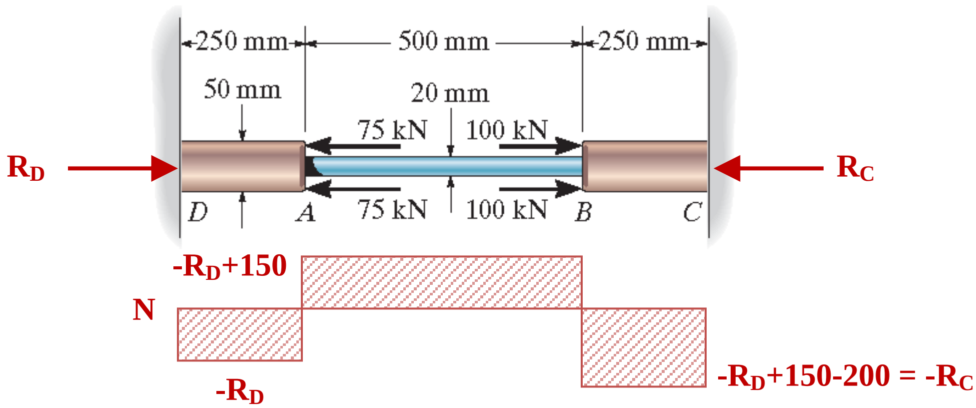


Fig. 2(b)

From the geometry shown in Fig. 2(b),

$$\delta_P = 0.01793 + \frac{4}{5} \; 0.03924 - 0.01793 \; = \underline{0.035 \text{ in (downwards)}}$$

Problem 3. The composite bar in Fig. 3 consists of a 20-mm diameter A-36 steel segment *AB* and 50-mm-diameter red brass C83400 end segments *DA* and *CB*. Determine the displacement of *A* with respect to *B* due to the applied load. E_{st} = 200 GPa, E_{br} = 101 GPa.



Equilibrium: $\sum F_x = R_D - 150 + 200 - R_C = 0 \quad \Rightarrow \quad R_D = R_C - 50$ (1)

Compatibility: $\delta = -\delta_{DA} + \delta_{AB} - \delta_{BC} = 0$ (2)

$$-\frac{N_{DA}L_{DA}}{E_{DA}A_{DA}} + \frac{N_{AB}L_{AB}}{E_{AB}A_{AB}} - \frac{N_{BC}L_{BC}}{E_{BC}A_{BC}} = 0$$

$$-\frac{R_D \times 10^3 \times 250}{101\times10^3 \times \frac{\pi}{4}50^2} + \frac{(-R_D + 150)\times10^3 \times 500}{200\times10^3 \times \frac{\pi}{4}20^2} - \frac{(R_D + 50)\times10^3 \times 250}{101\times10^3 \times \frac{\pi}{4}50^2} = 0$$

From (1) and (2): R_D = 107.89 kN, R_C = 157.89 kN

$$\therefore \quad \delta_{AB} = \frac{N_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{42.11\times10^3 \times 500}{200\times10^3 \times \frac{\pi}{4}20^2} = \underline{0.335 \text{ mm}}$$

Problem 4. The concrete post in Fig. 4 is reinforced using six steel reinforcing rods, each having a diameter of 20 mm. Determine the stress in the concrete and the steel reinforcing bars if the post is subjected to an axial load of 900 kN. E_{st} = 200 GPa and E_c = 25 GPa.

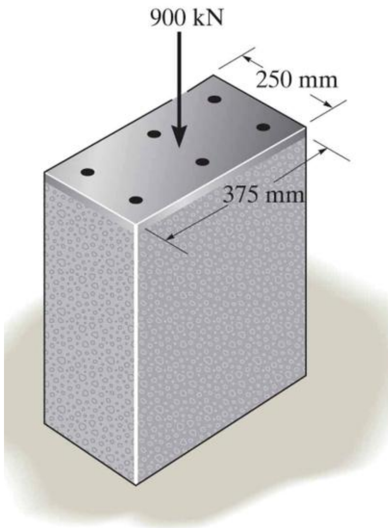


Fig. 4

Establishing equilibrium:

$$\sum F_y = 0 \quad \Rightarrow \quad 900 = F_c + 6F_s \qquad (1)$$

Compatibility of deformation establishes:

$$\delta_c = \delta_s$$

$$\frac{F_c L}{E_c A_c} = \frac{F_s L}{E_s A_s} \qquad (2)$$

The cross-sectional areas of steel and concrete are given by:

$$A_s = \frac{\pi 20^2}{4} = 314.2 \text{ mm}^2$$

$$A_c = \ 250 \times 375 \ - 314.16 = 93,435.8 \text{ mm}^2$$

Substituting (1) into (2) results in:

$$\frac{900 - 6F_s}{E_c A_c} = \frac{F_s}{E_s A_s}$$

$$900 - 6F_s \ \ E_s A_s = F_s E_c A_c$$

$$F_s = \frac{900 E_s A_s}{E_c A_c + 6 E_s A_s} = \frac{900 \times 200 \times 314.2}{25 \times 93,435.8 \ + \ 6 \times 200 \times 314.2} = 20.8 \text{ kN}$$

$$F_c = 900 - 6F_s = 900 - \ \ 6 \times 20.8 \ \ = 775.2 \text{ kN}$$

The axial stresses in the reinforcing steel bars and concrete are therefore given by:

$$\sigma_s = \frac{F_s}{A_s} = \frac{20.8 \times 10^3}{314.2} = \underline{66.2 \text{ MPa}} \qquad \sigma_c = \frac{F_c}{A_c} = \frac{775.2 \times 10^3}{93,435.8} = \underline{8.3 \text{ MPa}}$$