CVG2140 – Solutions to Assignment No. 2 (Shear Force & Bending Moment Diagrams)

Problem 1. Determine the forces in all members of the truss shown in Fig. 1.

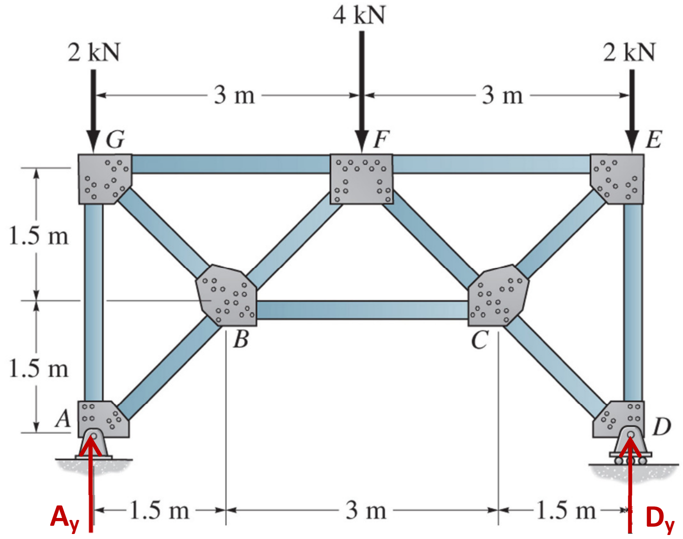


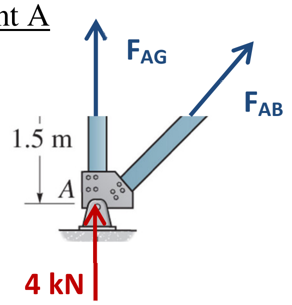
Fig. 1

Since the truss has symmetry in geometry, boundary conditions (supports) and loading, only half of it will be solved; the other half will be the same. By establishing equilibrium (moments are assumed positive counterclockwise), the external reactions at the supports are:

$$\left.\begin{aligned}\sum F_y &= A_y - 2 - 4 - 2 + D_y = 0\\ \sum M_A &= -(4\times 3) - (2\times 6) + (D_y \times 6) = 0\end{aligned}\right\} \Rightarrow \begin{aligned}A_y &= 4\text{ kN}\\ D_y &= 4\text{ kN}\end{aligned}$$

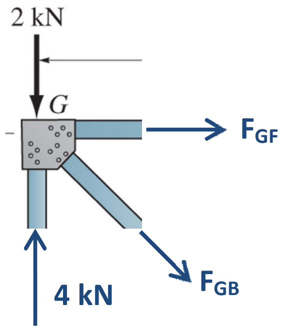
The internal forces along each truss member are solved using the method of joints.

Joint A



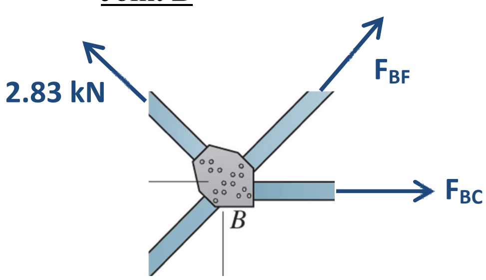
$$\left.\begin{aligned}\sum F_x &= \frac{1.5}{\sqrt{4.5}} F_{AB} = 0\\ \sum F_y &= 4 + F_{AG} + \frac{1.5}{\sqrt{4.5}} F_{AB} = 0\end{aligned}\right\} \Rightarrow \begin{aligned}F_{AB} &= 0\text{ kN}\\ F_{AG} &= -4\text{ kN}(\text{C})\end{aligned}$$

Joint G



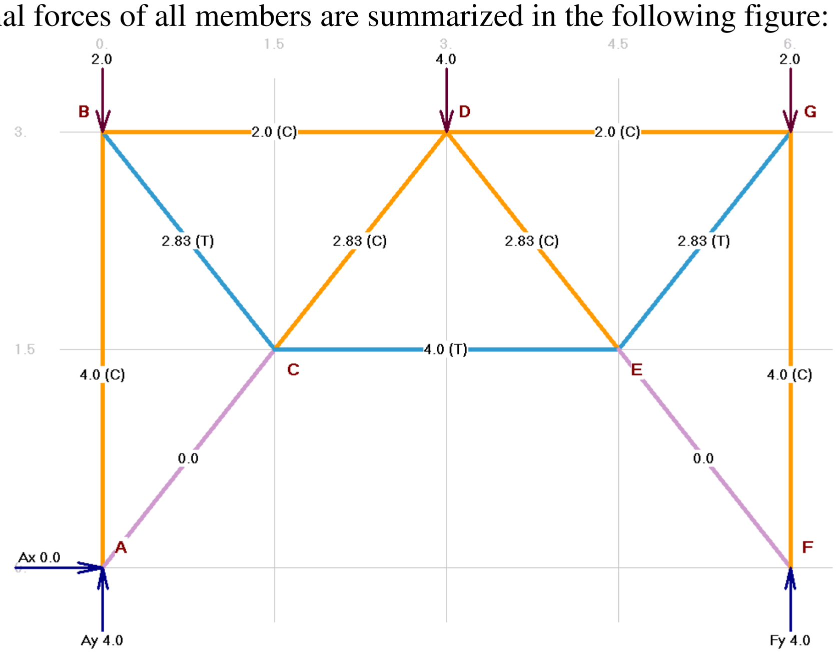
$$\left.\begin{aligned}\sum F_x &= F_{GF} + \frac{1.5}{\sqrt{4.5}} F_{GB} = 0 \\ \sum F_y &= 4 - 2 - \frac{1.5}{\sqrt{4.5}} F_{GB} = 0\end{aligned}\right\} \Rightarrow \begin{aligned} F_{GF} &= -2.0\ \text{kN}(\text{C}) \\ F_{GB} &= 2.83\ \text{kN}(\text{T})\end{aligned}$$

Joint B



$$\left.\begin{aligned}\sum F_x &= F_{BC} + \frac{1.5}{\sqrt{4.5}} F_{BF} - \left(\frac{1.5}{\sqrt{4.5}} \times 2.83\right) = 0 \\ \sum F_y &= \frac{1.5}{\sqrt{4.5}} F_{BF} + \left(\frac{1.5}{\sqrt{4.5}} \times 2.83\right) = 0\end{aligned}\right\} \Rightarrow \begin{aligned} F_{BF} &= -2.83\ \text{kN}(\text{C}) \\ F_{BF} &= 4\ \text{kN}(\text{T})\end{aligned}$$

The internal forces of all members are summarized in the following figure:



<u>Problem 2.</u> For the cantilever beam shown in Fig. 2, express the shear force and bending moment as functions of *x*, and then draw the shear force and bending moment diagrams.

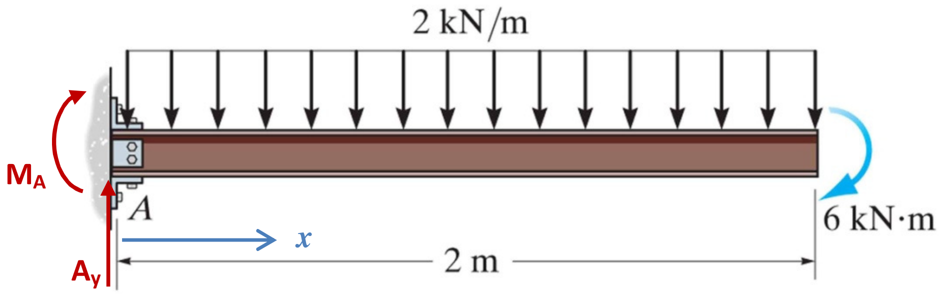


Fig. 2

By establishing equilibrium (moments are assumed positive counterclockwise):

$$\left.\begin{aligned}\sum F_y &= A_y-(2\times 2)=0\\ \sum M_A &= M_A-(2\times 2\times 1)-6=0\end{aligned}\right\}\Rightarrow \begin{aligned}A_y &= 4\text{ kN}\\ M_A &= 10\text{ kN}\cdot\text{m}\end{aligned}$$

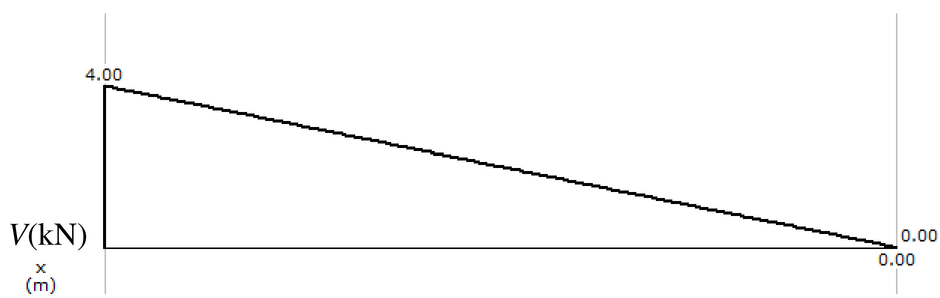
At any distance *x* from the left support *A*:

$$\sum F_y = A_y-(2\times x)-V(x)=0 \quad\Rightarrow\quad V(x)=A_y-2x=4-2x$$

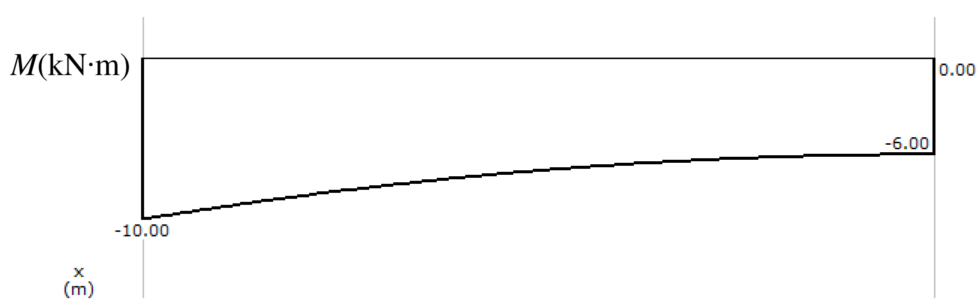
$$\sum M_x = M_A-(A_y\times x)+\left(2\times x\times\frac{x}{2}\right)+M(x)=0 \quad\Rightarrow$$

$$M(x)=-M_A+(A_y\times x)-\left(2\times x\times\frac{x}{2}\right)=-10+4x-x^2$$

The shear force diagram for the cantilever beam in Fig. 2 is shown below:



The bending moment diagram for the cantilever beam in Fig. 2 is shown below:



Problem 3. Draw the shear force and bending moment diagrams for the beam shown in Fig. 3. Determine the functions of the shear force and bending moment in terms of *x*.

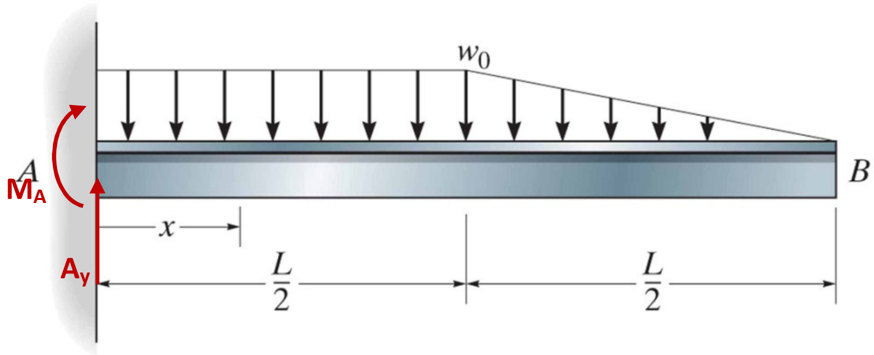
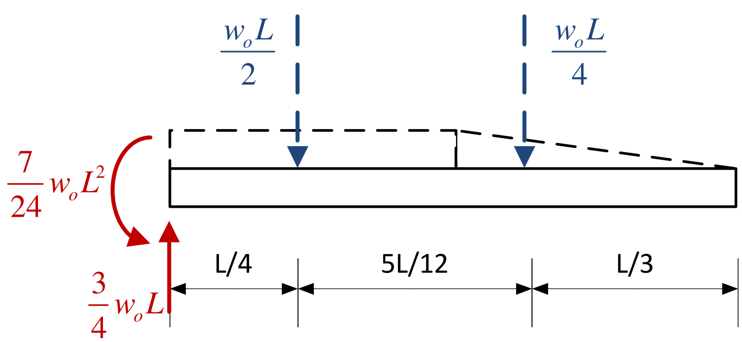
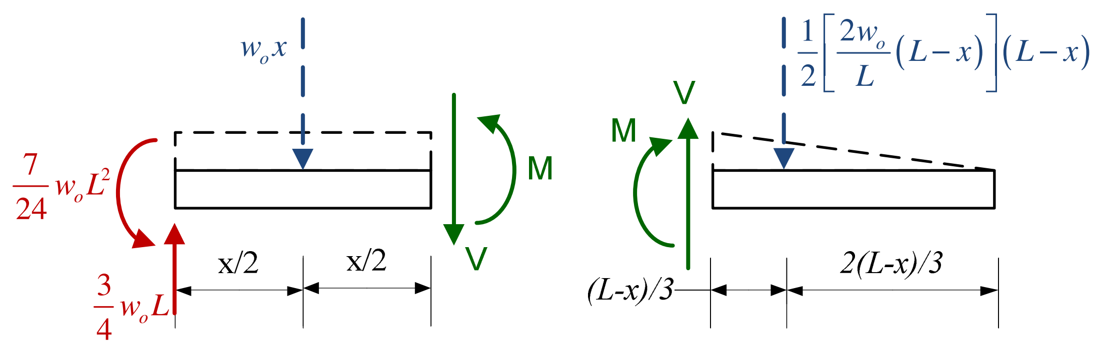


Fig. 3

By establishing equilibrium (moments are assumed positive counterclockwise):

$$\left.\begin{aligned}\sum F_y &= A_y - \left(w_0 \times \frac{L}{2}\right) - \left(\frac{1}{2} \times w_0 \times \frac{L}{2}\right) = 0 \\ \sum M_A &= M_A - \left(w_0 \times \frac{L}{2} \times \frac{L}{4}\right) - \left[\frac{1}{2} \times w_0 \times \frac{L}{2} \times \left(\frac{L}{2} + \frac{1}{3} \cdot \frac{L}{2}\right)\right] = 0\end{aligned}\right\} \Rightarrow \quad \begin{aligned} A_y &= \frac{3}{4} w_0 L \\ M_A &= \frac{7}{24} w_0 L^2 \end{aligned}$$





(a) (b)

At any distance $0 \le x \le L/2$ from the left support *A* (diagram (a) above):

$$\sum F_y = A_y - \left(w_0 \times \frac{L}{2}\right) - V(x) = 0 \quad \Rightarrow \quad V(x) = A_y - w_0 x = w_0 \left(\frac{3}{4} L - x\right)$$

$$\sum M_x = M_A - (A_y \times x) + \left(w_0 \times x \times \frac{x}{2} \right) + M(x) = 0 \quad \Rightarrow$$

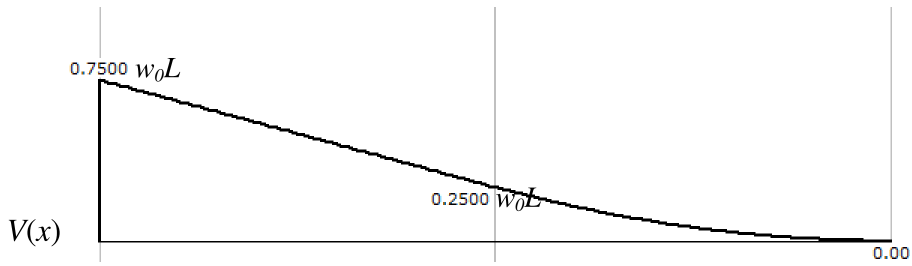
$$M(x) = -\frac{7}{24} w_0 L^2 + \frac{3}{4} w_0 L x - \frac{1}{2} w_0 x^2$$

At any distance $L/2 \le x \le L$ from the left support A (diagram (b) above):

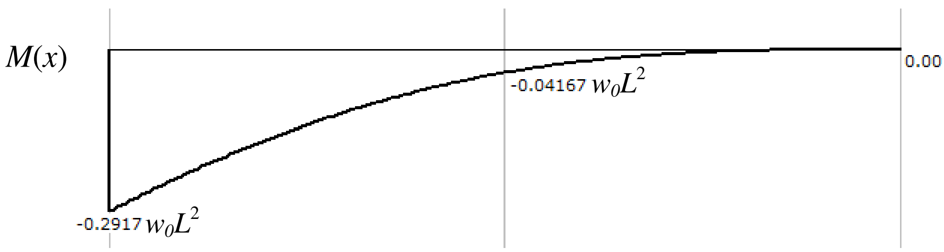
$$\sum F_y = V(x) - \frac{1}{2}\left[\frac{2w_0}{L}(L-x) \right](L-x) = 0 \quad \Rightarrow \quad V(x) = \frac{w_0}{L}(L-x)^2$$

$$\sum M_x = -M(x) - \frac{1}{2}\left[\frac{2w_0}{L}(L-x) \right](L-x)\left(\frac{L-x}{3} \right) = 0 \quad \Rightarrow \quad M(x) = -\frac{w_0}{3L}(L-x)^3$$

The shear force diagram for the cantilever beam in Fig. 3 is shown below:



The bending moment diagram for the cantilever beam in Fig. 3 is shown below:



Problem 4. Draw the shear force and bending moment diagrams for the compound beam illustrated in Fig. 4.

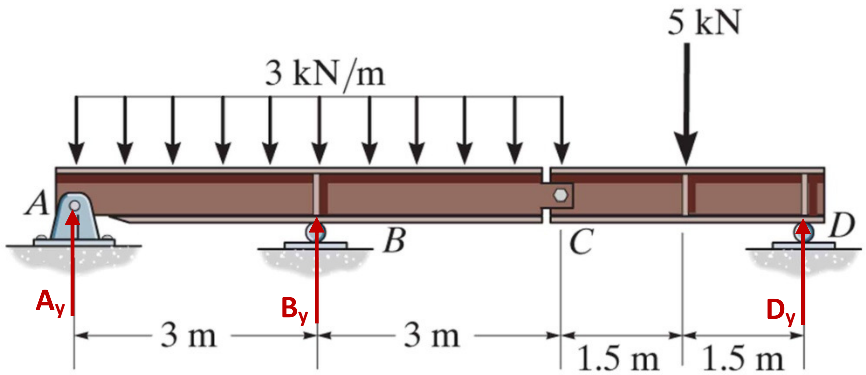


Fig. 4

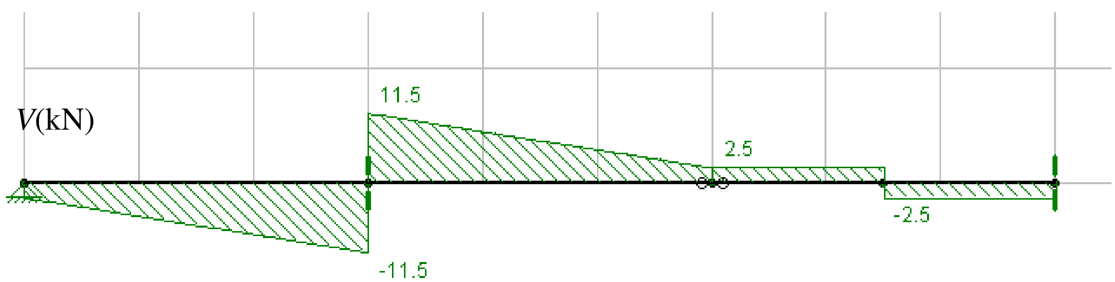
By establishing equilibrium on segment *CD* (moments are assumed positive counterclockwise):

$$\left.\begin{aligned}\sum F_y &= C_y + D_y - 5 = 0\\ \sum M_C &= -(5\times1.5)+(D_y\times3)=0\end{aligned}\right\}\Rightarrow \quad \begin{aligned}C_y &= 2.5\text{ kN}\\ D_y &= 2.5\text{ kN}\end{aligned}$$

Likewise, by establishing equilibrium on segment *ABC*:

$$\left.\begin{aligned}\sum F_y &= A_y + B_y - (3\times6) - C_y = 0\\ \sum M_A &= -(3\times6\times3)+(B_y\times3)-(C_y\times6)=0\end{aligned}\right\}\Rightarrow \quad \begin{aligned}A_y &= -2.5\text{ kN}\\ B_y &= 23\text{ kN}\end{aligned}$$

The shear force diagram for the compound beam in Fig. 4 is shown below:



The bending moment diagram for the compound beam in Fig. 4 is shown below: