

Part I. Lab questions.

- Do not include ANY Minitab code to your assignment. Use spaces left to answer lab questions.

1. (Mean and variance of random variables using Minitab)

Use the following minitab commands to generate 10 values of a random variable and probabilities: (Enable commands)

MTB > set c1

DATA> 1 2 3 4 5 6 7 8 9 10

DATA> end

MTB > set c2

DATA> 0.1 0.12 0.23 0.04 0.15 0.06 0.09 0.08 0.11 0.02

DATA> end

a. [3] Obtain the mean of X , i.e., $\mu = E(X)$ by *Calc* → *Calculator* → *sum(c1 * c2)*. What is μ ? **4.76**

b. [3] By clicking *Calc* → *Calculator* → *sum((c1**2)*c2) - (sum(c1*c2))**2*. obtain $Var(X) = \sigma^2$. What is σ^2 ? **6.9824** What is σ ? **2.6424**

c. [5] Add 2.5 to each number ('Let $c3=c1+2.5$ '). Now compute the mean, variance and standard deviation for $c3$ (repeat part a and b for $c3$: $E(X)=$ **7.26**, $\sigma^2 =$ **6.9824**, $\sigma =$ **2.6424**). How does your answer compare with that in part (a) and (b)?

Mean added by 2.5 but variance and standard deviation didn't change.

d. [5] Multiply the $c1$ by 4 (Let $c4=c1*4$) and compute the mean, variance and standard deviation for $c4$: $E(X)=$ **19.04**, $\sigma^2 =$ **111.718**, $\sigma =$ **10.57**. How does your answer compare with that in part (a) and (b)?

Mean and standard deviation multiplied by 4 but variance multiplied by 16.

2. (Binomial Distribution) Suppose that X has a binomial distribution with $n=40$ and $p=0.3$. Use minitab to simulate 50 values of X .

MTB > random 50 c1;

SUBC > binomial 40 0.5.

a. [3] What proportion of your values are less than 20? **17/50**. (any number between 13 and 23 is acceptable since the numbers were generated randomly.)

b. [3] What is the exact probability that X will be less than 20? **0.4373**.

Hint: To find $P(X \leq k)$, for any $k \geq 0$, use the 'cdf' command; this works by typing

MTB > cdf;

SUBC > binomial 40 0.5.

c. [5] Find $P(X < 24) = \underline{0.8659}$, and $P(12 \leq X \leq 25) = \underline{0.9596 - 0.0032 = 0.9564}$.

d. [3] If you simulate 10000 values of X , what would be the expected number of values (among the 10000) that are less than 24? $\underline{(1000)(0.8659) = 8659}$.

3. (*Poisson Distribution*) Suppose that X has a Poisson distribution with mean $\mu = 12$. Use the 'cdf' command

MTB > cdf;

SUBC > poisson 12.

Find $P(X < 10) = [3] \underline{0.2424}$, and $P(15 \leq X \leq 20) = [3] \underline{0.9884 - 0.772 = 0.2164}$.

4. (*Hypergeometric Distribution*) Suppose that Y has a hypergeometric distribution with parameters $N = 20$, $M = 8$, and $n = 5$.

a. Type 0, 1, 2, 3, 4, 5 in column c10, use the command

MTB > cdf c10;

SUBC > hypergeometric 20 8 5.

to find $P(Y \leq 2) = [3] \underline{0.7038}$, and $P(Y > 2) = [3] \underline{1 - 0.7038 = 0.2962}$.

b. [3] Use the "inverse cdf" command which for a given number a between 0 and 1 yields the value of (?) that satisfies $P(Y \leq ?) = a$, and works by typing

MTB > invcdf a;

SUBC > hypergeometric 20 8 5.

to find the value of c in $P(Y \leq c) = 0.942208$. The value of c is 3.

Part II. Long-answer questions; Give the solutions for the following questions in details

1. In a certain factory, Machines A, Machine B, and Machine C are all producing widgets. Widgets produced by Machine A have a 1% chance of being defective. Likewise, widgets produced by Machine B and Machine C are defective 4% and 2% of the time, respectively. Of the total production of widgets in the factory, Machine A produces 30%, Machine B produces 25%, and Machine C produces 45%. Suppose a widget is selected at random from this factory.

a. [5] What is the probability the widget is defective?

Solution:

D: defective widgets, A: Machine A, B: Machine B, C: Machine C, so $P(D|A) = 0.01$, $P(D|B) = 0.04$, $P(D|C) = 0.02$, $P(A) = 0.3$, $P(B) = 0.25$, $P(C) = 0.45$

$$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) = 0.01(0.3) + 0.04(0.25) + 0.02(0.45) \\ = 0.022$$

b. [5] If the widget is defective, what is the probability it was produced by Machine B?

Solution:

$$P(B|D) = \frac{P(D|B)P(B)}{P(D)} = \frac{0.04(0.25)}{0.22} = 0.45$$

2. A student is preparing for an upcoming exam. The professor for the course has given the class 30 questions to study from and plans to select 10 of the questions for use on the actual exam. Suppose that the student knows how to solve 25 of the 30 questions.

a. [3] What is the probability that the student will get perfect on the test?

Solution:

X = number of correct answers has an hypergeometric distribution where $N = 30$, $n = 10$, $M = 25$, so

$$P(X = 10) = \frac{C_{10}^{25}C_0^5}{C_{10}^{30}} = 0.109$$

b. [5] What is the probability that the student will get at least 8 questions correct on the test?

Solution:

$$\begin{aligned} P(X \geq 8) &= P(X = 8) + P(X = 9) + P(X = 10) = \frac{C_8^{25}C_2^5}{C_{10}^{30}} + \frac{C_9^{25}C_1^5}{C_{10}^{30}} + \frac{C_{10}^{25}C_0^5}{C_{10}^{30}} \\ &= 0.809 \end{aligned}$$

3. The number of flaws on a VHS magnetic tape produced continuously at a factory follows a Poisson distribution with an average of 0.01 flaws per meter. A standard VHS cassette tape contains 250 meters of magnetic tape.

a. [5] What is the probability that there are at least two flaws in a single VHS cassette tape?

Solution:

X = number of flaws is a Poisson random variable with $\mu = 0.01$ per meter, or $\mu = 0.01(250) = 2.5$ per cassette tape

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!} \right] \\ &= 0.7127 \end{aligned}$$

b. [4] What is the probability that there are no flaws in a single VHS cassette tape; that is, a tape is flawless?

Solution:

$$P(X = 0) = \frac{e^{-2.5}(2.5)^0}{0!} = 0.082$$

c. [6] In a random sample of 20 cassettes, what is the probability that at least two of them are flawless?

Solution:

Y = number of flawless cassette has Binomial distribution with $n = 20$, $p = 0.08$, so

$$\begin{aligned} P(Y \geq 2) &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[\binom{20}{0} (0.08)^0 (0.92)^{20} + \binom{20}{1} (0.08)^1 (0.92)^{19} \right] \\ &= 0.483 \end{aligned}$$

4. Consider events A and B such that probability $P(A) = 0.35$, $P(B) = 0.52$, and $P(A \cup B) = 0.57$. Find

a. [2] $P(A \cap B)$ b. [2] $P((A \cup B)')$ c. [2] $P(A' \cup B')$ d. [2] $P(A' \cap B)$.

Solution:

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.35 + 0.52 - 0.57 = 0.3$

(b) $P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.57 = 0.43$

(c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.3 = 0.7$

(d) $P(A' \cap B) = P(B) - P(A \cap B) = 0.52 - 0.3 = 0.22$

e. [2] Are A and B independent? Why? No, since $P(A \cap B) \neq P(A)P(B)$

f. [2] Are A and B mutually exclusive? Why? No, since $P(A \cap B) \neq 0$

5. Suppose that a random variable X follows the distribution given by:

| | | | | | |
|------------|-----|-----|-----|---|-----|
| a | -2 | -1 | 0 | 1 | 2 |
| $P(X = a)$ | 1/8 | 1/4 | 1/4 | ? | 1/8 |

Let $Y = X^2$. Find values of [6] $P(Y \geq 1)$ and [4] $E(Y)$.

Solution:

Since $\sum P(X) = 1 \rightarrow P(X = 1) = 1/4$, the probability distribution for Y is

| | | | | | |
|------------|-----|-----|-----|-----|-----|
| a | 4 | 1 | 0 | 1 | 4 |
| $P(X = a)$ | 1/8 | 1/4 | 1/4 | 1/4 | 1/8 |

then

| | | | |
|------------|-----|-----|-----|
| a | 0 | 1 | 4 |
| $P(X = a)$ | 1/4 | 2/4 | 1/4 |

so $P(Y \geq 1) = 3/4$ and $E(Y) = 0(1/4) + 1(2/4) + 4(1/4) = 6/4$