UNIVERSITY OF VICTORIA
SAMPLE EXAMINATION
MATHEMATICS 102

Duration: **3 hours**

THIS QUESTION PAPER HAS 6 PAGES plus COVER.

Instructions:

Please PRINT your name (with your last name first) on this examination paper. Also fill in your student number and section. With a pencil, put your name (with your last name first) and student number on the computer marking sheet (N.C.S.), and fill in the corresponding bubbles.

This examination is multiple choice. Each question is worth 2 marks, and there are 36 questions. The maximum number of possible marks is 70 but you may answer all questions. For each question, select the correct choice from the possibilities given, and carefully mark, in pencil, the corresponding letter on the computer marking sheet. For numerical answers, pick the number **closest** to your answer. If your answer is **exactly** in the middle of the two possibilities, then pick the larger value. Sometimes (J) none or does not exist is an option. For symbolic (algebraic) answers, pick the answer from (A)-(I) that is exactly correct. If none is correct, or an answer does not exist, then pick (J).

You may use only a basic scientific calculator, *i.e.*, not programmable and not graphical.

Before you leave the examination room, put your computer marking sheet inside your examination paper and place them in the box for your section. It is your responsibility to make sure that these are handed in and are in the correct place. You will be required to show ID and sign the list during the exam. Good Luck!

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1. Evaluate

$$\lim_{x\to 2}\left(\frac{-2x^2+4x}{x-2}\right).$$

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (J) does not exist

2. Evaluate

$$\lim_{h\to 0}\left(\frac{(-2+h)^2-4}{h}\right).$$

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (J) does not exist

3. Let

$$P(x)=\frac{60x^2+5x}{3x^2+2}.$$

Find $\lim_{x\to\infty} P(x)$.

(A) 0 (B) 5 (C) 10 (D) 15 (E) 20 (F) 25 (G) 30 (H) 35 (I) 40 (J) does not exist

4. Given $f(x)=\frac{1}{x}$ for $x>0$, calculate and simplify the difference quotient

$$\frac{f(x+h)-f(x)}{h}, \qquad \text{where } h\neq 0.$$

(A) 0 (B) $\frac{1}{x(x+h)}$ (C) $\frac{1}{x}$ (D) $\frac{1}{x+h}$ (E) $\frac{-h}{x(x+h)}$

(F) $\frac{2x+h}{x(x+h)h}$ (G) $\frac{-1}{x^2}$ (H) $\frac{1}{x^2}$ (I) $\frac{-1}{x(x+h)}$ (J) none of the above

5. Find an equation of the tangent line to the curve

$$y=\left(x^2-15\right)^6$$

at the point $x=4$.

(A) $y=8x-31$ (B) $y=6x-25$ (C) $y=48x$ (D) $6y=x$

(E) $y=48x-191$ (F) $y=6x-23$ (G) $y=12\left(x^2-15\right)^5 x$ (H) $y=8x-33$

(I) $y=48x+193$ (J) none of the above

6. Let $f(x) = \dfrac{x^2 - x - 6}{x - 3}$ for $x \neq 3$. Determine the value of $f(x)$ at the point $x = 3$ (i.e., $f(3)$) so that $f(x)$ is continuous at $x = 3$.

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1 (F) 2 (G) 3 (H) 4 (I) 5

(J) no such value exists

7. Find the *positive* value of x for which the graph of $f(x) = 2x^3 - 3x^2 - 12x + 5$ has a tangent line that is horizontal.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 7 (H) 8 (I) 9

(J) no such value exists

8. Given that the distance $s(t)$ at any time $t > 0$ is given by $s(t) = t\ln(t) - t$, find the acceleration $s''(t)$.

(A) $\ln(\ln(t))$ (B) $\ln(t)$ (C) $1/t$ (D) $-1/t^2$ (E) $2/t^3$

(F) $-6/t^4$ (G) $-\ln(t)$ (H) $-\ln(\ln(t))$ (I) $-1/t$ (J) none of the above

9. Given $f(x) = x^3 - x^2 - x$, $f'(x) = (3x+1)(x-1)$, and $f''(x) = 6x - 2$. Find the value of x where $f(x)$ has a relative maximum.

(A) -1 (B) $\frac{-1}{2}$ (C) $\frac{-1}{3}$ (D) $\frac{-1}{6}$ (E) 0 (F) $\frac{1}{6}$ (G) $\frac{1}{3}$ (H) $\frac{1}{2}$ (I) $\frac{2}{3}$

(J) no such value exists

10. Find the absolute maximum value of $f(x)$ on the interval $-3 \leq x \leq 3$ where $f(x) = x^3 - x^2 - x$, $f'(x) = (3x+1)(x-1)$, and $f''(x) = 6x - 2$.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12 (F) 13 (G) 14 (H) 15 (I) 16

(J) The maximum does not exist.

11. Given $f(x) = x^3 - x^2 - x$, $f'(x) = (3x+1)(x-1)$, and $f''(x) = 6x - 2$. Find the x coordinate of an inflection point, if any, on the graph of $y = f(x)$.

(A) -1 (B) $\frac{-1}{2}$ (C) $\frac{-1}{3}$ (D) $\frac{-1}{4}$ (E) $\frac{-1}{6}$ (F) $\frac{1}{3}$ (G) $\frac{1}{2}$ (H) $\frac{2}{3}$ (I) 1

(J) There are no inflection points.

12. Find the exact interval on which the graph of $f(x) = x^4 - 6x^2 + 1$ is concave down for every x in the interval.

(A) $-5 < x < \frac{1}{5}$ (B) $-4 < x < \frac{1}{4}$ (C) $-3 < x < \frac{1}{3}$ (D) $-2 < x < \frac{1}{2}$ (E) $-1 < x < 1$
(F) $-\frac{1}{2} < x < 2$ (G) $-\frac{1}{3} < x < 3$ (H) $-\frac{1}{4} < x < 4$ (I) $-\frac{1}{5} < x < 5$ (J) $-\infty < x < \infty$

13. If $y = k$ is a horizontal asymptote for the curve $y = \dfrac{5}{x} + 2$, find the value of k.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) does not exist

14. A rectangular garden of area 75 square meters is to be surrounded by a brick wall on 3 sides costing \$10 per meter and a wooden fence on the other side costing \$5 per meter. Use calculus to find the minimum **cost** of materials.

(A) 5 (B) 7.5 (C) 10 (D) 12.5 (E) 100 (F) 125 (G) 200 (H) 225 (I) 300 (J) 325

15. The demand equation for x units is $p(x) = 1000 - 2x$ (where p is the price per unit) and the cost function is $C(x) = 8000 + 500x - x^2$. Determine the value of x that maximizes the profit.

(A) 0 (B) 20 (C) 50 (D) 100 (E) 150 (F) 200 (G) 250 (H) 300 (I) 350 (J) 400

16. If

$$f(x) = \frac{\sqrt{x} - 1}{\sqrt{x} + 1},$$

find $f'(x)$ and then evaluate the derivative at $x = 4$, i.e., find $f'(4)$.

(A) $\frac{1}{9}$ (B) $\frac{1}{12}$ (C) $\frac{1}{15}$ (D) $\frac{1}{18}$ (E) $\frac{1}{21}$ (F) $\frac{1}{24}$ (G) $\frac{1}{27}$ (H) $\frac{1}{30}$ (I) $\frac{1}{33}$ (J) $\frac{1}{36}$

17. If

$$f(x) = x\left(1 + x^4\right)^5,$$

find $f'(x)$ and then evaluate the derivative at $x = 1$, i.e., evaluate $f'(1)$.

(A) 260 (B) 270 (C) 280 (D) 290 (E) 300 (F) 310 (G) 320 (H) 330 (I) 340 (J) 350

18. Given $yx^2 + xy^2 = 7$, use implicit differentiation to find $\dfrac{dy}{dx}$.

(A) $\dfrac{y^2+2xy}{x^2+2xy}$ (B) $\dfrac{x^2+2xy}{y^2+2xy}$ (C) $\dfrac{2xy}{x^2+2xy}$ (D) $\dfrac{y^2+2xy}{x^2+xy^2}$ (E) $\dfrac{y+2x}{x+2y}$
(F) $-\dfrac{y^2+2xy}{x^2+2xy}$ (G) $-\dfrac{x^2+2xy}{y^2+2xy}$ (H) $-\dfrac{2xy}{x^2+2xy}$ (I) $-\dfrac{y^2+2xy}{x^2+xy^2}$ (J) none of the above

19. A spherical balloon, with volume $V = \frac{4}{3}\pi r^3$ and radius r is being inflated so that its volume increases at the rate of 480 cm^3/sec. Find the rate, in cm/sec, at which the balloon's radius r is growing when $r = 5$ cm.

(A) 0.38 (B) 0.61 (C) 0.84 (D) 1.07 (E) 1.30
(F) 1.53 (G) 1.76 (H) 1.99 (I) 2.22 (J) 2.45

20. The height of a certain weed after t weeks is $f(t) = 6\left(1 - 5e^{-t/2}\right)$ cm. Find how fast the weed is growing at the end of two weeks.

(A) 2 (B) 2.5 (C) 3 (D) 3.5 (E) 4 (F) 4.5 (G) 5 (H) 5.5 (I) 6 (J) 6.5

21. Find the slope of the graph of $y = 4\ln(x^3 + 2x + 1)$ at the point $x = 1$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6 (H) 7 (I) 8 (J) 9

22. Find the exact interval on which the graph of $f(x) = x^2e^{-3x}$ is *increasing* for every x in the interval.

(A) $-\infty < x < 0$ (B) $-2 < x < \frac{1}{4}$ (C) $-1 < x < \frac{1}{3}$ (D) $0 < x < \frac{2}{3}$ (E) $\frac{1}{3} < x < 1$
(F) $\frac{1}{2} < x < 2$ (G) $\frac{2}{3} < x < 3$ (H) $\frac{3}{4} < x < 4$ (I) $1 < x < \infty$ (J) $-\infty < x < \infty$

23. Use logarithmic differentiation to find the derivative of $y = x^{1+x}$ at $x = 1$.

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3 (F) 4 (G) 5 (H) 6 (I) 7 (J) 8

24. Differentiate $y = 2\sin(2t)$ and evaluate your answer at $t = 0$, i.e., find $y'(0)$.

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 3 (I) 4 (J) 5

25. If

$$f(x) = \cos\left(1 - x^4\right),$$

find $f'(x)$.

(A) $-\sin(1 - x^4)$ (B) $4x^3\sin(1 - x^4)$ (C) $\sin(4x^3)$ (D) $-4x^3\sin(1 - x^4)$
(E) $\sin(1 - x^4)$ (F) $4x^3\cos(1 - x^4)$ (G) $-4x^3\cos(1 - x^4)$ (H) $\cos(4x^3)$
(I) $\cos(4x^3)\sin(4x^3)$ (J) none of the above

26. The population of Canada was 28 million in January 1995 with a growth rate of 0.8% per year. Assuming uninhibited exponential growth, predict Canada's population in January 2004.

(A) $29,600,000$ (B) $29,700,000$ (C) $29,800,000$ (D) $29,900,000$ (E) $30,000,000$
(F) $30,100,000$ (G) $30,200,000$ (H) $30,300,000$ (I) $30,400,000$ (J) $30,500,000$

27. The decay constant for a radioactive material is 0.0244 with time measured in years. Assuming exponential decay, how long (in years) will it take for a quantity P_0 to decay to $\frac{1}{2}P_0$?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 22 (F) 24 (G) 26 (H) 28 (I) 30 (J) 32

28. Evaluate

$$\int_1^2 \left(x^3 - 4x + 1\right) dx$$

(A) -1.5 (B) -1.25 (C) -1.00 (D) -0.75 (E) -0.50
(F) -0.25 (G) 0 (H) 0.25 (I) 0.5 (J) 0.75

29. Find the area under the curve $y = \frac{4}{x}$ and above the x-axis, and bounded by the vertical lines at $x = e$ and $x = e^3$.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8 (F) 9 (G) 10 (H) 11 (I) 12 (J) 13

30. Find the area of the region enclosed by the curves $y = 8 - x^2$ and $y = x^2$.
[A sketch of the region may help you].

(A) 7 (B) 9 (C) 11 (D) 13 (E) 15 (F) 17 (G) 19 (H) 21 (I) 23 (J) 25

31. Use integration by substitution to evaluate

$$\int \frac{3x^2}{\left(1+x^3\right)^5} dx.$$

(A) $\dfrac{-x}{12\left(1+x^3\right)^4} + C$ (B) $\dfrac{-x}{4\left(1+x^3\right)^4} + C$ (C) $\dfrac{-x}{3\left(1+x^3\right)^4} + C$ (D) $\dfrac{-1}{6\left(1+x^3\right)^4} + C$

(E) $\dfrac{1}{4\left(1+x^3\right)^4} + C$ (F) $\dfrac{-1}{4\left(1+x^3\right)^4} + C$ (G) $\dfrac{x}{12\left(1+x^3\right)^4} + C$ (H) $\dfrac{x}{6\left(1+x^3\right)^4} + C$

(I) $-\dfrac{1}{15}\ln\left(1+x^3\right)^5 + C$ (J) none of the above

32. Evaluate

$$\int \frac{x+1}{x^2+2x+5}dx.$$

(A) $(x+1)+\ln|x^2+2x+5|+C$ (B) $(x^2+2x+5)^{-1}+C$ (C) $2(x^2+2x+5)^{-1}+C$
(D) $4(x^2+2x+5)^{-2}+C$ (E) $(x+1)(x^2+2x+5)^{-1}+C$ (F) $2(x^2+2x+5)^{-2}+C$
(G) $\frac{1}{2}\ln|x^2+2x+5|+C$ (H) $2\ln|x^2+2x+5|+C$ (I) $\ln|x^2+2x+5|+C$
(J) none of the above

33. Use integration by parts to evaluate the following integral for $x>0$

$$\int \sqrt{x}\ln x\ dx.$$

(A) $\frac{2}{3}x^{3/2}\left(\ln x+\frac{1}{2}\right)+C$ (B) $\frac{2}{3}x^{3/2}\left(\ln x-\frac{1}{2}\right)+C$ (C) $\frac{2}{3}x^{3/2}(\ln x-1)-\sqrt{x}\ln x+C$
(D) $\frac{2}{3}x^{3/2}\left(\ln x+\frac{2}{3}\right)+C$ (E) $\frac{2}{3}x^{3/2}\left(\ln x-\frac{2}{3}\right)+C$ (F) $\frac{2}{3}x^{3/2}(\ln x-1)+\sqrt{x}\ln x+C$
(G) $\frac{2}{3}x^{3/2}\left(\ln x+\frac{4}{9}\right)+C$ (H) $\frac{2}{3}x^{3/2}\left(\ln x-\frac{4}{9}\right)+C$ (I) $\frac{2}{3}x^{3/2}(\ln x+1)-\sqrt{x}\ln x+C$
(J) none of the above

34. During a certain 12-hour period the temperature at time t (measured in hours from the start of the period) was $10+4t-\dfrac{1}{3}t^2$ degrees Celsius. What was the *average* temperature during that period?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10 (F) 12 (G) 14 (H) 16 (I) 18 (J) 20

35. Solve the differential equation $y'=5y$ given that $y(0)=3$.

(A) $y=5x+3$ (B) $y=3e^{5x}$ (C) $y=3e^{5x}+C$ (D) $y=5e^{3x}$ (E) $y=e^{5x}+3$
(F) $y=5x^2+3$ (G) $y=e^3e^{5x}$ (H) $y=e^{5x}+2$ (I) $y=\dfrac{e^{5x}}{3}$ (J) none of the above

36. The marginal cost in dollars for x units of a certain product is

$$C'(x)=3x^2-8x^3.$$

Find $C(x)$ given that $C(0)=50$.

(A) $6x-24x^2$ (B) $6x-24x^2+100$ (C) x^3-2x^4+50 (D) $50(6x-24x^2)$
(E) $50(x^3-2x^4)$ (F) $6x-2x^4$ (G) x^3-24x^2+100 (H) 50
(I) $6-48x$ (J) none of the above

-END-