MAT1341D-TS- Version A - Solution

1. Parametric equations of the line containing $(-5,\ 0,\ 1)$ and which is parallel to the two planes $2x - 4y + z = 0$ and $x - 3y - 2z = 1$ are:

A. $x = 5 + 11t,\ y = 3t,\ z = 1 + 2t,\ t \in \mathbf{R}$

B. $x = -5 + 11t,\ y = -3t,\ z = 1 + 2t,\ t \in \mathbf{R}$

C. $x = 5t,\ y = 0,\ z = t,\ t \in \mathbf{R}$

D. $x = -5 + 5t,\ y = -5t,\ z = 1 - 10t,\ t \in \mathbf{R}$

E. $x = -5 + 11t,\ y = 5t,\ z = 1 - 2t,\ t \in \mathbf{R}$

F. $x = -5t,\ y = 0,\ z = t,\ t \in \mathbf{R}$

$$\vec{V} = (2,-4,1) \times (1,-3,-2) = (11,5,-2)$$

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2. Which two of the following are vector parametric descriptions for the plane with equation $x + y - 2z = 4$?

I. $v = (0,0,0) + s\ (0,2,1) + t\ (2,0,1); s,t \in \mathbf{R}$.

II. $v = (4,0,0) + s\ (1,-1,0) + t\ (0,2,1); s,t \in \mathbf{R}$.

III. $v = (4,0,0) + s\ (1,1,1) + t\ (1,1,0); s,t \in \mathbf{R}$.

IV. $v = (0,0,-2) + s\ (1,-1,0) + t\ (2,0,1); s,t \in \mathbf{R}$.

$V = V_0 + tV_1 + sV_2$

A. I & II B. I & III C. I & IV D. II & III E. II & IV F. III & V

$\vec{n} = (1,1,-2)$

$V_1 \cdot \vec{n} = 0,\ V_2 \cdot \vec{n} = 0$

$(1,-1,0)\cdot(1,1,-2) = 0, \quad (0,2,1)\cdot(1,1,-2) = 0$

$(2,0,1)\cdot(1,1,-2) = 0$

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3. An equation for the plane parallel to the x–axis and passing through the points $(2, 1, -1)$ and $(3, 2, 1)$ is:

A. $-3x + 7y - 2z = 3$

B. $x - y = 1$

C. $2y - z = 3$

D. $2x - z = 5$

E. $x + y - z = 4$

F. $x + y + z = 2$

Let $by + cz = d$, sub $(2,1,-1)$ $\Rightarrow b(1) + c(-1) = d$

sub $(3,2,1) \Rightarrow b(2) + c(1) = d$

$\Rightarrow b = \frac{2}{3}d,\ c = -\frac{1}{3}d \Rightarrow \frac{2}{3}dy - \frac{1}{3}dz = d$

$\Rightarrow 2y - z = 3$

4. Find an equation of the plane which passes through the point $(1,\ -7,\ 8)$ and which is perpendicular to the line whose (scalar) parametric equations are:

$$x = 2 + 2t, \quad y = 7 - 4t, \quad z = -3 + t; \quad t \in \mathbf{R}.$$

A. $2x - 4y + z = -38$

B. $-4x + 2y + z = -10$

C. $-4x + 2y + z = 10$

D. $2x - 4y + z = 38$

E. $2x + 7y - 3z = -71$

F. $2x - 4y + z = -28$

$\vec{v} = (2, -4, 1)$

Let Π: $2x - 4y + z = d$

sub $(1, -7, 8)$: $d = 38$

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5. One of the following is an equation for the plane with vector parametric description

$$v = (2, 0, 3) + s(1, 0, 1) + t(0, -1, 0); s, t \in \mathbf{R}.$$

Which is it?

A. $4x - 9y + z = 18$

B. $x + y - 2z = 14$

C. $x - 2y + 2z = 0$

D. $x + 2y - z = 0$

E. $x - z = -1$

F. $9x - 11y + 18z = -40$

$(1,0,1) \times (0,-1,0) = (1,0,-1) = \vec{n}$

Let $x - z = d$

Sub $(2,0,3)$: $d = -1$

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6. Find the polar form of

$$\frac{1 - \sqrt{3}i}{-1 + i}$$

A. $\sqrt{2}(\cos(-7\pi/12) + i\sin(-7\pi/12))$

B. $\sqrt{2}(\cos(5\pi/12) + i\sin(5\pi/12))$

C. $\sqrt{2}(\cos(-\pi/12) + i\sin(-\pi/12))$

D. $\sqrt{2}(\cos(\pi/12) + i\sin(\pi/12))$

E. $\sqrt{2}(\cos(-5\pi/12) + i\sin(-5\pi/12))$

F. $\sqrt{2}(\cos(11\pi/12) + i\sin(11\pi/12))$

$1 - \sqrt{3}i = 2e^{i\frac{5\pi}{3}}$

$-1 + i = \sqrt{2}e^{i\frac{3\pi}{4}}$

$\frac{1-\sqrt{3}i}{-1+i} = \frac{2}{\sqrt{2}} e^{i\left(\frac{5\pi}{3} - \frac{3\pi}{4}\right)}$

$= \sqrt{2} e^{i\frac{11\pi}{12}}$

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7. What is the area of the triangle with vertices $(3,\ 0,\ -2)$, $(5,\ 2,\ -1)$ and $(5,\ 9,\ 0)$?

A. 13/2

B. 15/2

C. 17/2

D. 10

E. 13

F. 15

$\overrightarrow{AB} = (5,2,-1)-(3,0,-2) = (2,2,1)$

$\overrightarrow{AC} = (5,9,0)-(3,0,-2) = (2,9,2)$

$\text{Area} = \frac{1}{2}\left|(2,2,1)\times(2,9,2)\right|$

$= \frac{1}{2}\left|(-5,-2,14)\right|$

$= \frac{1}{2}\sqrt{225} = 15/2$

8. Let L be the line passing through $(1,\ 1,\ 0)$ and $(2,\ 3,\ 1)$. The point of intersection of L with the plane $x+y-z=1$ is:

A. $(1/2,\ 1/2,\ 0)$

B. $(0,\ 1/2,\ -1/2)$

C. $(0,\ 1,\ 0)$

D. $(1/2,\ 0,\ -1/2)$

E. $(1,\ 0,\ 0)$

F. $(-1,\ 0,\ -1)$

L: $x=1+t$, $y=1+2t$, $z=t$

Sub into Π:

$1+t+1+2t-t=1$

$2t=-1$. $t=-\frac{1}{2}$

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9. Express the following complex numbers in the form $a + b\,i$:

$$z_1 = \frac{i}{-1+i}$$

$$z_2 = (2+i)(1+i)$$

A. $z_1 = \frac{1}{2} + \frac{1}{2}i$; $z_2 = 1 - 3i$

B. $z_1 = \frac{1}{2} - \frac{1}{2}i$; $z_2 = 1 + 3i$

C. $z_1 = 1 - i$; $z_2 = 2 + 2i$

D. $z_1 = -1 + i$; $z_2 = 1 + 2i$

E. $z_1 = 2 - \frac{1}{4}i$; $z_2 = 3 - i$

F. $z_1 = 1 - i$; $z_2 = 2$

$$z_1 = \frac{i(-1-i)}{(-1+i)(-1-i)} = \frac{-i-i^2}{2}$$

$$= \frac{1}{2} - \frac{1}{2}i$$

$$z_2 = 2 + 3i + i^2 = 1 + 3i$$

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10. If $v = (2,\ 2,\ 2)$ and $u = (-1, 0, -1)$ then $\operatorname{proj}_u v =$

A. $\frac{4}{9}(2, 1, 2)$

B. $\frac{12}{7}(3,\ 3,\ 3)$

C. $\frac{4}{3}(2, 1, 2)$

D. $(2, 0, 2)$

E. $\frac{\sqrt{2}}{2}(1, 0, 1)$

F. $\frac{11}{7}(3,\ 3,\ 3)$

$$\operatorname{proj}_u V = \frac{V \cdot u}{u \cdot u} u = \frac{-4}{2}(-1, 0, -1)$$

$$= (2, 0, 2)$$

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11. Find the volume of the parallelepiped determined by the vectors $u = (1,\ 1,\ -1)$, $v = (2,\ 0,\ 1)$ and $w = (1,\ -1,\ 3)$.

A. 6

B. 8

C. 16

D. 2

E. 4

F. -2

$$V = |u \cdot (v \times w)|$$

$$= |(1,1,-1) \cdot (1,-5,-2)|$$

$$= |1-5+2| = 2$$

12. If $A = (1,2,1)$, $B = (2,2,1)$ and $C = (2,2,2)$, find the angle $\angle ACB$.

A. $\pi/4$

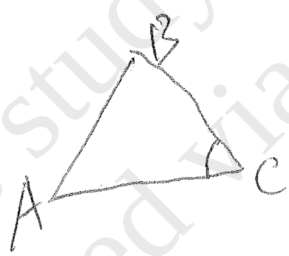
B. $\pi/6$

C. $3\pi/4$

D. $4\pi/3$

E. $\pi/2$

F. $\pi/3$



$$\cos\angle ACB = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}|\,|\vec{CB}|}$$

$$= \frac{(-1,0,-1)\cdot(0,0,-1)}{|(-1,0,-1)|\,|(0,0,-1)|} = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

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