PROBLEM 11.1

The motion of a particle is defined by the relation $x = t^4 - 10t^2 + 8t + 12$, where x and t are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when $t = 1$ s.

SOLUTION

$$x = t^4 - 10t^2 + 8t + 12$$

$$v = \frac{dx}{dt} = 4t^3 - 20t + 8$$

$$a = \frac{dv}{dt} = 12t^2 - 20$$

At $t = 1$ s,

$x = 1 - 10 + 8 + 12 = 11$ $\qquad x = 11.00$ m ◀

$v = 4 - 20 + 8 = -8$ $\qquad v = -8.00$ m/s ◀

$a = 12 - 20 = -8$ $\qquad a = -8.00$ m/s^2 ◀

PROBLEM 11.5

The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when $a = 0$.

SOLUTION

We have $$x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$$

Then $$v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$$

and $$a = \frac{dv}{dt} = 72t^2 - 12t - 24$$

When $a = 0$: $$72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$$

or $$(3t - 2)(2t + 1) = 0$$

or $$t = \frac{2}{3}\text{ s} \quad \text{and} \quad t = -\frac{1}{2}\text{ s} \quad \text{(Reject)}$$

$t = 0.667$ s ◀

At $t = \frac{2}{3}$ s: $$x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3$$

or $x_{2/3} = 0.259$ m ◀

$$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3$$

or $v_{2/3} = -8.56$ m/s ◀

PROBLEM 11.11

The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

SOLUTION

We have $$a = kt^2 \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt^2$$

At $t = 6$ s, $v = 18$ m/s: $$\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$$

or $$v - 18 = \frac{1}{3}k(t^3 - 216)$$

or $$v = 18 + \frac{1}{3}k(t^3 - 216)(\text{m/s})$$

Also $$\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$$

At $t = 0$, $x = 24$ m: $$\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right]dt$$

or $$x - 24 = 18t + \frac{1}{3}k\left(\frac{1}{4}t^4 - 216t\right)$$

Now

At $t = 6$ s, $x = 96$ m: $$96 - 24 = 18(6) + \frac{1}{3}k\left[\frac{1}{4}(6)^4 - 216(6)\right]$$

or $$k = \frac{1}{9}\ \text{m/s}^4$$

Then $$x - 24 = 18t + \frac{1}{3}\left(\frac{1}{9}\right)\left(\frac{1}{4}t^4 - 216t\right)$$

or $$x(t) = \frac{1}{108}t^4 + 10t + 24 \quad ◀$$

and $$v = 18 + \frac{1}{3}\left(\frac{1}{9}\right)(t^3 - 216)$$

or $$v(t) = \frac{1}{27}t^3 + 10 \quad ◀$$

PROBLEM 11.17

The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 5$ m/s when $x = 0.2$ m and that $v = 3$ m/s when $x = 0.4$ m. Determine (*a*) the velocity of the particle when $x = 0.5$ m, (*b*) the position of the particle at which its velocity is zero.

SOLUTION

$$a = \frac{v\,dv}{dx} = \frac{-k}{x}$$

Separate and integrate using $x = 0.2$ m, $v = 5$ m/s.

$$\int_5^v v\,dv = -k\int_{0.2}^{x} \frac{dx}{x}$$

$$\frac{1}{2}v^2\Big|_{0.2}^{v} = -k\ln x\Big|_{0.2}^{x}$$

$$\frac{1}{2}v^2 - \frac{1}{2}(5)^2 = -k\ln\left(\frac{x}{0.2}\right) \qquad (1)$$

When $v = 3$ m/s, $x = 0.4$ m

$$\frac{1}{2}(3)^2 - \frac{1}{2}(5)^2 = -k\ln\left(\frac{0.4}{0.2}\right)$$

Solve for k.

$$k = 11.5416\ \text{m}^2/\text{s}^2$$

(*a*) Substitute $k = 11.5416\ \text{m}^2/\text{s}^2$ and $x = 0.5$ m into (1).

$$\frac{1}{2}v^2 - \frac{1}{2}(5)^2 = -11.5416\ln\left(\frac{0.5}{0.2}\right)$$

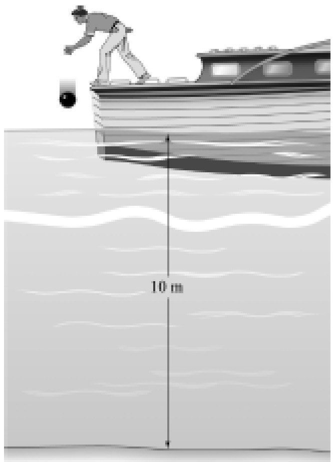
$v = 1.962$ m/s ◀

(*b*) For $v = 0$,

$$0 - \frac{1}{2}(5)^2 = -11.5416\ln\left(\frac{x}{0.2}\right)$$

$$\ln\left(\frac{x}{0.2}\right) = 1.083$$

$x = 0.591$ m ◀



PROBLEM 11.23

A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 8 m/s. Assuming the ball experiences a downward acceleration of $a = 3 - 0.1v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

SOLUTION

$$v_0 = 8 \text{ m/s}, \quad x - x_0 = 10 \text{ m}$$

$$a = 3 - 0.1v^2 = k(c^2 - v^2)$$

where $k = 0.1 \text{ m}^{-1}$ and $c^2 = \dfrac{3}{0.1} = 30 \dfrac{\text{m}^2}{\text{s}^2}$

$$c = 5.4772 \text{ m/s}$$

Since $v_0 > c$, write

$$a = v\frac{dv}{dx} = -k(v^2 - c^2)$$

$$\frac{v\,dv}{v^2 - c^2} = -k\,dx$$

Integrating,

$$\frac{1}{2}\ln(v^2 - c^2)\Big|_{v_0}^{v} = -k(x - x_0)$$

$$\ln\frac{v^2 - c^2}{v_0^2 - c^2} = -2k(x - x_0)$$

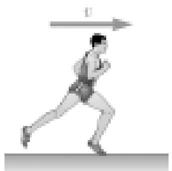
$$\frac{v^2 - c^2}{v_0^2 - c^2} = e^{-2k(x - x_0)}$$

$$v^2 = c^2 + \left(v_0^2 - c^2\right)e^{-2k(x - x_0)}$$

$$= 30 + [(8)^2 - 30]\,e^{-2(0.1)(10)}$$

$$= 34.6014 \text{ m}^2/\text{s}^2$$

$v = 5.88$ m/s ◀



PROBLEM 11.28

Based on observations, the speed of a jogger can be approximated by the relation $v = 12(1-0.06x)^{0.3}$, where v and x are expressed in km/h and km, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in m/s² at $t = 0$, (*c*) the time required for the jogger to run 9 km.

SOLUTION

(*a*) We have $$\frac{dx}{dt} = v = 12(1-0.06x)^{0.3}$$

At $t = 0, x = 0$: $$\int_0^x \frac{dx}{(1-0.06x)^{0.3}} = \int_0^t 12\,dt$$

or $$\frac{1}{0.7}\left(-\frac{1}{0.06}\right)[(1-0.06x)^{0.7}]_0^x = 12t$$

or $$1-(1-0.06x)^{0.7} = 0.504t \qquad (1)$$

or $$x = \frac{1}{0.06}[1-(1-0.504t)^{1/0.7}]$$

At $t = 1$ h: $$x = \frac{1}{0.06}\{1-[1-0.504(1)]^{1/0.7}\}$$

or $x = 10.55$ km ◀

(*b*) We have $$a = v\frac{dv}{dx}$$
$$= 12(1-0.06x)^{0.3}\frac{d}{dx}[12(1-0.06x)^{0.3}]$$
$$= 12^2(1-0.06x)^{0.3}[(0.3)(-0.06)(1-0.06x)^{-0.7}]$$
$$= -2.592(1-0.06x)^{-0.4}$$

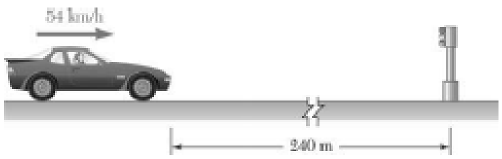
At $t = 0, x = 0$: $$a_0 = \frac{-2.592 \text{ km}}{\text{h}^2}\times\frac{1000 \text{ m}}{1 \text{ km}}\times\left(\frac{1 \text{ h}}{3600}\right)^2$$

or $a_0 = -2\times 10^{-4}$ m/s² ◀

(*c*) From Eq. (1) $$t = \frac{1}{0.504}[1-(1-0.06x)^{0.7}]$$

When $x = 9$ km: $$t = \frac{1}{0.504}\{1-[1-0.06(9)]^{0.7}\}$$
$$= 0.832 \text{ hrs}$$

or $t = 49.9$ min ◀



PROBLEM 11.34

A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (*a*) the required uniform deceleration of the car, (*b*) the speed of the car as it passes the light.

SOLUTION

Uniformly accelerated motion:

$$x_0 = 0 \quad v_0 = 54 \text{ km/h} = 15 \text{ m/s}$$

(a) $x = x_0 + v_0 t + \frac{1}{2} a t^2$

when $t = 24\text{s}$, $x = 240$ m:

$$240 \text{ m} = 0 + (15 \text{ m/s})(24 \text{ s}) + \frac{1}{2} a (24 \text{ s})^2$$

$$a = -0.4167 \text{ m/s}^2$$

$a = -0.417 \text{ m/s}^2$ ◀

(*b*) $v = v_0 + a t$

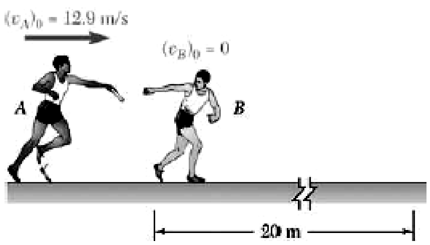
when $t = 24\text{s}$:

$$v = (15 \text{ m/s}) + (-0.4167 \text{ m/s})(24 \text{ s})$$

$$v = 5.00 \text{ m/s}$$

$$v = 18.00 \text{ km/h}$$

$v = 18.00$ km/h ◀



PROBLEM 11.39

As relay runner A enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 s later as they leave the exchange zone with the same velocity. Determine (*a*) the uniform acceleration of each of the runners, (*b*) when runner B should begin to run.

SOLUTION

(*a*) For runner A: $x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2$

At $t = 1.82$ s: $20 \text{ m} = (12.9 \text{ m/s})(1.82 \text{ s}) + \frac{1}{2} a_A (1.82 \text{ s})^2$

or $a_A = -2.10 \text{ m/s}^2$ ◀

Also $v_A = (v_A)_0 + a_A t$

At $t = 1.82$ s: $(v_A)_{1.82} = (12.9 \text{ m/s}) + (-2.10 \text{ m/s}^2)(1.82 \text{ s})$
$= 9.078$ m/s

For runner B: $v_B^2 = 0 + 2a_B [x_B - 0]$

When $x_B = 20$ m, $v_B = v_A$: $(9.078 \text{ m/s})^2 = 2a_B (20 \text{ m})$

or $a_B = 2.0603 \text{ m/s}^2$

$a_B = 2.06 \text{ m/s}^2$ ◀

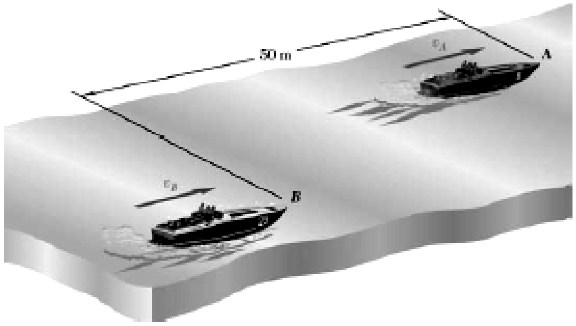
(*b*) For runner B: $v_B = 0 + a_B (t - t_B)$

where t_B is the time at which he begins to run.

At $t = 1.82$ s: $9.078 \text{ m/s} = (2.0603 \text{ m/s}^2)(1.82 - t_B)\text{s}$

or $t_B = -2.59$ s

Runner B should start to run 2.59 s before A reaches the exchange zone. ◀



PROBLEM 11.40

In a boat race, boat A is leading boat B by 50 m and both boats are traveling at a constant speed of 180 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when B passes A, $t = 8$ s and $v_A = 225$ km/h, determine (*a*) the acceleration of A, (*b*) the acceleration of B.

SOLUTION

(*a*) We have $v_A = (v_A)_0 + a_A t$

$$(v_A)_0 = 180 \text{ km/h} = 50 \text{ m/s}$$

At $t = 8$ s: $v_A = 225 \text{ km/h} = 62.5 \text{ m/s}$

Then $62.5 \text{ m/s} = 50 \text{ m/s} + a_A(8 \text{ s})$

or $a_A = 1.563 \text{ m/s}^2$ ◀

(*b*) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 50 \text{ m} + (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(1.5625 \text{ m/s}^2)(8 \text{ s})^2 = 500 \text{ m}$$

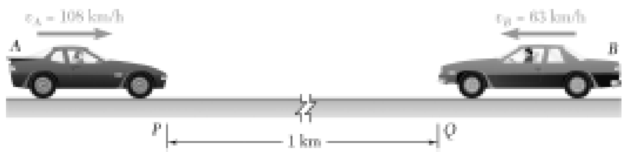
and $x_B = 0 + (v_B)_0 t + \frac{1}{2}a_B t^2$ $(v_B)_0 = 50$ m/s

At $t = 8$ s: $x_A = x_B$

$$500 \text{ m} = (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}a_B(8 \text{ s})^2$$

or $a_B = 3.13 \text{ m/s}^2$ ◀

PROBLEM 11.43



Two automobiles *A* and *B* are approaching each other in adjacent highway lanes. At $t = 0$, *A* and *B* are 1 km apart, their speeds are v_A = 108 km/h and v_B = 63 km/h, and they are at Points *P* and *Q*, respectively. Knowing that *A* passes Point *Q* 40 s after *B* was there and that *B* passes Point *P* 42 s after *A* was there, determine (*a*) the uniform accelerations of *A* and *B*, (*b*) when the vehicles pass each other, (*c*) the speed of *B* at that time.

SOLUTION

(*a*) We have $x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2 \quad (v_A)_0 = 108 \text{ km/h} = 30 \text{ m/s}$

At $t = 40$ s: $1000 \text{ m} = (30 \text{ m/s})(40 \text{ s}) + \frac{1}{2} a_A (40 \text{ s})^2$

or $a_A = -0.250 \text{ m/s}^2$ ◀

Also, $x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \quad (v_B)_0 = 63 \text{ km/h} = 17.5 \text{ m/s}$

At $t = 42$ s: $1000 \text{ m} = (17.5 \text{ m/s})(42 \text{ s}) + \frac{1}{2} a_B (42 \text{ s})^2$

or $a_B = 0.30045 \text{ m/s}^2$ $a_B = 0.300 \text{ m/s}^2$ ◀

(*b*) When the cars pass each other

$$x_A + x_B = 1000 \text{ m}$$

Then
$$(30 \text{ m/s})t_{AB} + \frac{1}{2}(-0.250 \text{ m/s})t_{AB}^2 + (17.5 \text{ m/s})t_{AB} + \frac{1}{2}(0.30045 \text{ m/s}^2)t_{AB}^2 = 1000 \text{ m}$$

or $0.05045 t_{AB}^2 + 95 t_{AB}^2 - 2000 = 0$

Solving $t = 20.822$ s and $t = -1904$ s

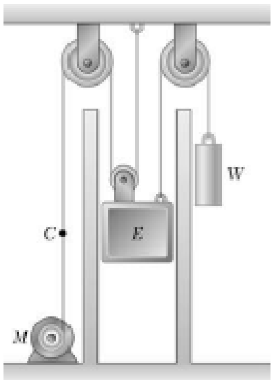
$t > 0 \Rightarrow t_{AB} = 20.8$ s ◀

PROBLEM 11.43 (Continued)

(*c*) We have $v_B = (v_B)_0 + a_B t$

At $t = t_{AB}$: $v_B = 17.5 \text{ m/s} + (0.30045 \text{ m/s}^2)(20.822 \text{ s})$
$= 23.756 \text{ m/s}$

or $v_B = 85.5 \text{ km/h}$ ◀

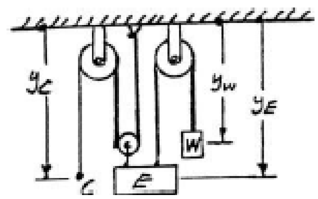


PROBLEM 11.47

The elevator shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable *C*, (*b*) the velocity of the counterweight *W*, (*c*) the relative velocity of the cable *C* with respect to the elevator, (*d*) the relative velocity of the counterweight *W* with respect to the elevator.

SOLUTION

Choose the positive direction downward.



(*a*) Velocity of cable *C*.

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $v_E = 4$ m/s

or $v_C = -2v_E = -8$ m/s $\quad \mathbf{v}_C = 8.00 \text{ m/s} \uparrow$ ◀

(*b*) Velocity of counterweight *W*.

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -4 \text{ m/s}$$

$\mathbf{v}_W = 4.00 \text{ m/s} \uparrow$ ◀

(*c*) Relative velocity of *C* with respect to *E*.

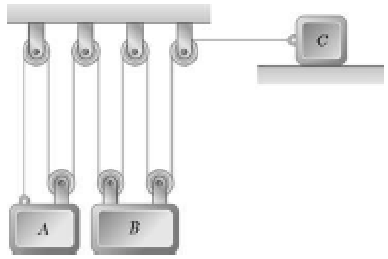
$$v_{C/E} = v_C - v_E = (-8 \text{ m/s}) - (+4 \text{ m/s}) = -12 \text{ m/s}$$

$\mathbf{v}_{C/E} = 12.00 \text{ m/s} \uparrow$ ◀

(*d*) Relative velocity of *W* with respect to *E*.

$$v_{W/E} = v_W - v_E = (-4 \text{ m/s}) - (4 \text{ m/s}) = -8 \text{ m/s}$$

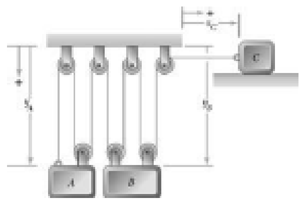
$\mathbf{v}_{W/E} = 8.00 \text{ m/s} \uparrow$ ◀



PROBLEM 11.57

Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \qquad (1)$$

and $$3a_A + 4a_B + a_C = 0 \qquad (2)$$

<u>Given</u>: $(v_B) = 0,$

$a_A = \text{constent}$

$(\mathbf{a}_C) = 75 \text{ mm/s}^2 \longrightarrow$

At $t = 2$ s, $\mathbf{v}_B = 480 \text{ mm/s}\downarrow$

$\mathbf{v}_C = 280 \text{ mm/s} \longrightarrow$

(*a*) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then $v_B = 0 + a_B t$

At $t = 2$ s: $480 \text{ mm/s} = a_B (2 \text{ s})$

$a_B = 240 \text{ mm/s}^2$ or $\mathbf{a}_B = 240 \text{ mm/s}^2\downarrow$ ◀

Substituting into Eq. (2)

$$3a_A + 4(240 \text{ mm/s}^2) + (75 \text{ mm/s}^2) = 0$$

$a_A = -345 \text{ mm/s}$ or $\mathbf{a}_A = 345 \text{ mm/s}^2\uparrow$ ◀

PROBLEM 11.57 (Continued)

(*b*) We have $v_C = (v_C)_0 + a_C t$

At $t = 2$ s: $280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s})$

$v_C = 130 \text{ mm/s}$ or $(\mathbf{v}_C)_0 = 130.0 \text{ mm/s} \longrightarrow$ ◀

Then, substituting into Eq. (1) at $t = 0$

$$3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0$$

$v_A = -43.3 \text{ mm/s}$ or $(\mathbf{v}_A)_0 = 43.3 \text{ mm/s} \uparrow$ ◀

(*c*) We have $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$

At $t = 3$ s: $x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2}(75 \text{ mm/s}^2)(3 \text{ s})^2$

$= 728 \text{ mm}$ or $\mathbf{x}_C - (\mathbf{x}_C)_0 = 728 \text{ mm} \longrightarrow$ ◀