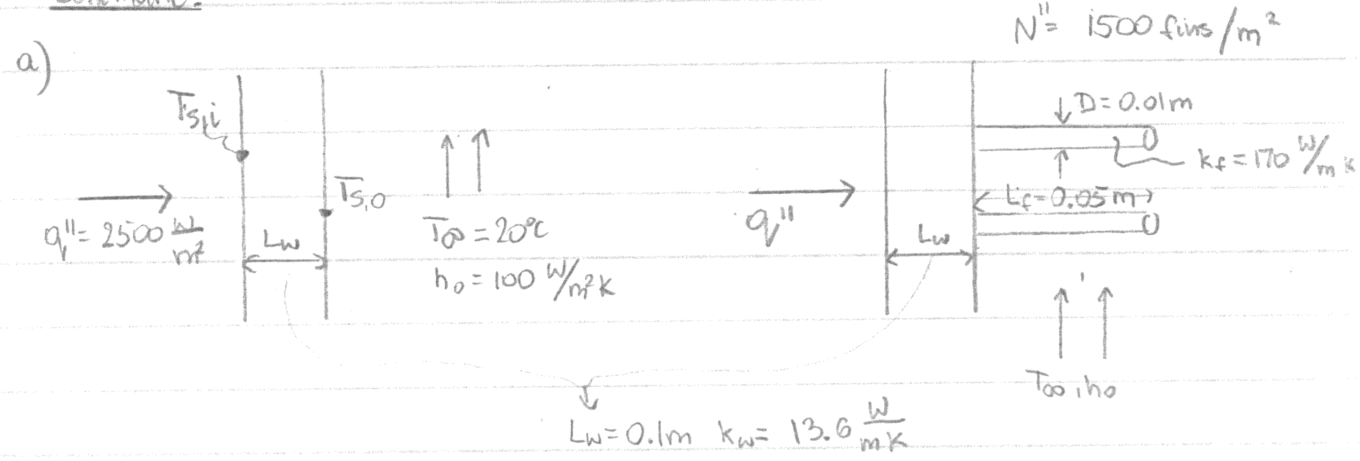
CHG 2314 - Midterm Solutions
Winter 2016

Problem 1

Known: Wall inside surface receives constant heat flux (q''). The outside surface is convectively cooled as a) flat surface, or b) with addition of fins

Find: Inner surface $(T_{s,i})$ and outer surface $(T_{s,o})$ temperatures of the wall for cases a) and b)

Schematic:



Assumptions:
1) Steady-state 1-D conduction in wall & fins
2) h_o not affected by the presence of fins
3) Negligible radiation effects
4) constant properties

Analysis: a) Outside surface is flat

- Thermal circuit $\quad q'' \longrightarrow T_{s,i} \text{---}\underset{R''_{cond}}{\text{WW}}\text{---} T_{s,o} \text{---}\underset{R''_{conv}}{\text{WW}}\text{---} T_\infty$

where:
$$R''_{cond} = \frac{L_w}{k} = \frac{0.1\,m}{13.6\,K/W\cdot m} = 0.00735\ \frac{Km^2}{W}$$
$$R''_{conv} = \frac{1}{h_o} = \frac{1}{100\ W/m^2K} = 0.01\ \frac{Km^2}{W}$$

$$q'' = \frac{T_{s,o} - T_\infty}{R''_{conv}} = h_o(T_{s,o} - T_\infty) \Rightarrow T_{s,o} = \frac{q''}{h_o} + T_\infty = \frac{2500\ \frac{W}{m^2}}{100\ \frac{W}{m^2K}} + 2($$

$$\therefore T_{s,o} = 25 + 20 = 45°C$$

Problem 1 - continued

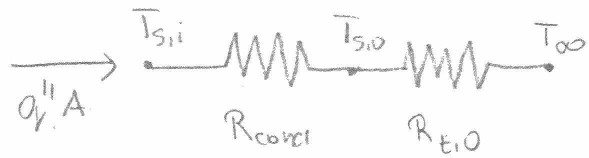
Similarly : $q'' = \dfrac{T_{s,i} - T_{s,o}}{R''_{cond}} \Rightarrow T_{s,i} = q'' R''_{cond} + T_{s,o} = 2500 \cdot \frac{W}{m^2} \cdot 0.00735 \frac{K m^2}{W} + 45°C$

$\therefore T_{s,i} = 63.4°C$

b) Outside surface is finned

we know number of fins per m^2; thus we will use $A = A_i = A_o = 1 m^2$

Thermal circuit



$R_{cond} = R''_{cond} / A = 0.00735 \ K/W$

$R_{t,o} = \dfrac{1}{h A_t \eta_o}$ where η_o = efficiency of finned surface

$\cdot \ \eta_o = 1 - \dfrac{N A_f}{A_t}(1 - \eta_f)$

where : $A_t = N A_f + A_b = N A_f + (1 - N A_c)$

• we will use eqn for adiabatic fins, thus to take into consideration convection from the tips, we will use $L_c = L_f + \frac{D}{4} = 0.05 + \frac{0.01}{4} = 0.0525 m$

$A_c = \pi \frac{D^2}{4} = \pi \cdot \frac{(0.01)^2}{4} = 7.85 \cdot 10^{-5} m^2$

$A_f = \pi D L_c = \pi \cdot 0.01 \cdot 0.0525 = 1.65 \cdot 10^{-3} m^2$

$\therefore A_b = 1 - 2500 \cdot 7.85 \cdot 10^{-5} = 0.80375 m^2$

$\therefore A_t = 2500 \cdot 1.65 \cdot 10^{-3} + 0.80375 = 4.925 m^2$

$\eta_f = \dfrac{\tanh(m L_c)}{m L_c}$ where $m = \left(\dfrac{hP}{k_f A_c}\right)^{1/2} = \left(\dfrac{4h_o}{k_f D}\right)^{1/2} = \left(\dfrac{4 \cdot 100}{170 \cdot 0.01}\right)^{1/2}$

$\Rightarrow m = 15.34 \ m^{-1} \Rightarrow m L_c = 15.34 \cdot 0.0525 =$

$\therefore m L_c = 0.805$

Sub-in numerical values : $\eta_f = \dfrac{\tanh(0.805)}{0.805} = 0.828$

$\eta_o = 1 - \dfrac{2500 \cdot 1.65 \cdot 10^{-3}}{4.925}(1 - 0.828) = 0.856 \Rightarrow R_{t,o} = \dfrac{1}{100 \cdot 4.925 \cdot 0.856} = 0.00237 \frac{K}{W}$

$T_{s,o} = A q'' R_{t,o} + T_\infty = 2500 W \cdot 0.00237 \frac{K}{W} + 20 = \underline{25.9°C}$; $T_{s,i} = 2500 \cdot 0.00735 + 25.9 = \underline{44.3°C}$

Problem 2

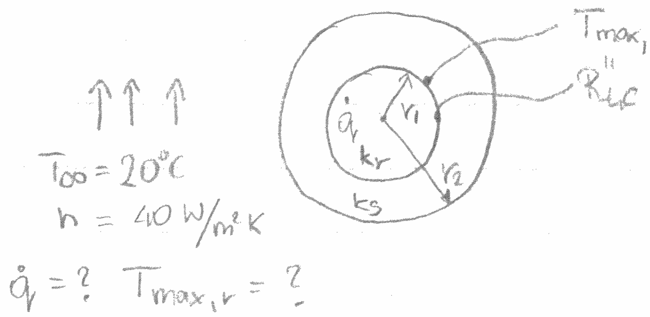
Known: long cylindrical rod experiencing uniform volumetric heat generation, which is encapsulated by a circular sleeve exposed to convective cooling; contact resistance between the sleeve and the rod

Find: a) thermal circuit

b) volumetric heat generation so the sleeve does not exceed 100°C

c) corresponding maximum temperature of the rod

Schematic:



rod: $r_1 = \frac{150}{2} = 75mm = 0.075m$

$k_r = 0.5\ W/mK$

sleeve: $r_2 = \frac{250}{2} = 125mm = 0.125mm$

$R''_{t,c} = 2\times10^{-4}\ \frac{m^2K}{W}$

Assumptions: 1) 1-D conduction

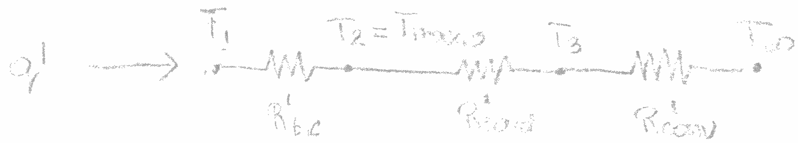
2) Steady state

3) uniform heat generation

4) constant properties

5) negligible radiation

Analysis: a) Equivalent thermal circuit. Length of the rod is unknown, thus heat flow and resistances will be per unit length (q' and R')



where $R'_{t,c} = \frac{R''_{t,c}}{P} = \frac{R''_{t,c}}{2\pi r_1}$, $R'_{cond} = \frac{\ln(r_2/r_1)}{2\pi k_s}$, $R'_{conv} = \frac{1}{2\pi r_2 h}$

$q' = \dot{q}\pi r_1^2$

b) Determination of $\dot{q}$ if $T_2 = 100°C$

From thermal circuit:

$$q' = \frac{T_2 - T_\infty}{R'_{cond} + R'_{conv}} = \dot{q}\pi r_1^2 \Rightarrow \dot{q} = \frac{T_2 - T_\infty}{(R'_{cond} + R'_{conv})\pi r_1^2}$$

Once R'_{cond} and R'_{conv} are evaluated, $\dot{q}$ is the only unknown

Problem 2 - continued

$$R'_{cond} = \frac{\ln(0.125/0.075)}{2\pi \cdot 8} = 1.017 \cdot 10^{-2} \ \frac{m \cdot K}{W}$$

$$R'_{conv} = \frac{1}{2\pi \cdot 0.125 \cdot 40} = 3.184 \cdot 10^{-2} \ \frac{m \cdot K}{W}$$

Sub-in numerical values:

$$\dot{q}_r = \frac{100 - 20}{\left(1.017 \cdot 10^{-2} + 3.184 \cdot 10^{-2}\right) \pi \cdot (0.075)^2} = 1.065 \cdot 10^{5} \ \frac{W}{m^3}$$

c) Determination of $T_{max,r}$

$T_{max,r}$ occurs at the centre of the rod

For the solid circular rod, the temperature distribution is given by:

$$T(r) = \frac{\dot{q} r_0^2}{4k}\left(1 - \frac{r^2}{r_0^2}\right) + T_s \qquad \text{where } r = 0 \quad r_0 = r_1, \ k = k_r$$

$$T_s = T_1 = ?$$

From thermal circuit:

$$q_r' = \frac{T_1 - T_2}{R'_{t,c}} \Rightarrow T_1 = q_r' R'_{t,c} + T_2$$

where: $q_r' = 1.065 \cdot 10^{5} \cdot \pi (0.075)^2 = 1882 \ \frac{W}{m}$

$$R'_{t,c} = \frac{R''_{t,c}}{2\pi r_1} = \frac{2 \cdot 10^{-4}}{2 \cdot \pi \cdot 0.075} = 4.25 \cdot 10^{-4} \ \frac{m \cdot K}{W}$$

Sub-in numerical values: $T_1 = 1882 \cdot 4.25 \cdot 10^{-4} + 100 = 100.8°C$

Sub-in into temp distribution: $T_{max,r} = T(0) = \frac{\dot{q}_r r_1^2}{4 k_r} + T_1$

$$\therefore T_{max,r} = \frac{1.065 \cdot 10^{5} \cdot 0.075^2}{4 \cdot 0.5} + 100.8 = 400.3°C$$

Problem 3

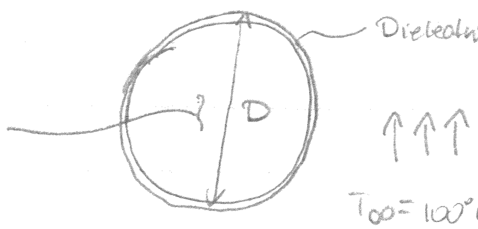
Known: Solid steel sphere, coated with dielectric material of known thermal resistance. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath

Find: a) applicability of LTCM

b) time required for sphere centre to reach 200°C

c) corresponding temperature of dielectric coating

Shematic:



Dielectric coating $R''_{t,c} = 0.04 \frac{m^2 K}{W}$

Sphere

$D = 300$ mm

$T_\infty = 100°C \quad h = 40 \, W/m^2K$

$\rho = 7832 \frac{kg}{m^3}$, $c = 559 \, J/kgK$

$k = 48.8 \, W/mK$, $T_i = 500°C$

Assumptions: 1) Dielectric coating has negligible thermal capacitance compared to sphere

2) Layer is very thin

3) Constant properties

Analysis

a) Applicability of LTCM

Determination of Biot number $Bi = \frac{R_{int}}{R_{ext}}$

Note that external resistance includes resistance to convection and thermal resistance of coating, i.e. $R_{ext} = R''_{t,c} + R''_{conv} \Rightarrow U = \frac{1}{R''_{ext}}$

$$R_{ext} = 0.04 + \frac{1}{40} = 0.065 \frac{K m^2}{W} \Rightarrow U = 15.4 \, W/m^2K$$

$$\therefore Bi = \frac{U \cdot R}{k} = \frac{15.4 \cdot \frac{0.3}{2}}{48.8} = 0.047 < 0.1 \Rightarrow \text{yes LTCM is applicable}$$

b) time for the center sphere temp to reach 200°C

Since there is no heat generation

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{UA}{\rho c V} t\right)$$

note: we use U instead of h because convection is not the only resistance

Problem 3- continued

or: $\frac{T-T_\infty}{T_i-T_\infty} = \exp(-t/\tau)$ where $\tau = \frac{\rho c V}{AU} = \frac{\rho c \frac{\pi}{6} D^3}{\cancel{\pi D^2} U} = \frac{\rho c D}{6U} = \frac{7832 \cdot 559 \cdot 0.3}{6 \cdot 15.4}$

$\therefore \tau = 14,215 \left[\frac{\cancel{kg/m^3} \cdot \cancel{J/kg\,K} \cdot \cancel{m}}{\frac{\cancel{J}}{s\,\cancel{m^2 K}}} = s\right]$

rearranging:

$$t = -\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)\tau = -\ln\left(\frac{200-100}{500-100}\right) \cdot 14,215 = 19,706 \text{ s}$$

c) corresponding temp of coating

Coating provides a resistance (temperature drop) but no heat capacity; i.e. cannot store the energy; thus at any time: $\frac{T_s - T_c}{R''_{t,c}} = h(T_c - T_\infty)$

The only unknown is $T_{s,o}$. Rearranging:

$$(T_s - T_c) = h R''_{t,c}(T_c - T_\infty) \Rightarrow T_c(1 + hR''_{t,c}) = T_s + hR''_{t,c} T_\infty$$

$$\therefore T_{s,o} = \frac{T_{s,i} + hR''_{t,c} T_\infty}{1 + hR''_{t,c}} = \frac{200 + 40 \cdot 0.04 \cdot 100}{1 + 40 \cdot 0.04} = 138.5°C$$