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Department of
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CHG 2314

HEAT TRANSFER OPERATIONS

MIDTERM EXAM

DATE: **Thursday February 29, 2016, 17:30 – 18:50**

DURATION: **80 minutes**

PROFESSOR: **Dr. B. Kruczek**

1) Closed book examination
2) One double-sided reference page with your own notes/information is allowed. Your reference page is to be submitted with the exam booklet; it will be returned to you with the marked exam
3) Additional useful information is provided in the Appendix.
4) Do 2 out of 3 problems; each problem is worth 25 marks.
5) Please indicate which problems should be evaluated; if no indication is provided, the first two problems appearing in the exam booklet will be evaluated.
6) Calculator allowed: TI-30X or equivalent
7) Cell phones and all other electronic devices must be turned off and stored away from the desk
8) If you finish the exam before 18:35, you may leave the room. Otherwise, please wait till the end of the exam

Good luck!!!

1. The inner surface of a wall is exposed to a constant heat flux $q'' = 2500\ \text{W/m}^2$. The wall is 10 cm thick and has thermal conductivity $k = 13.6$ W/m K. The outer surface of the wall is exposed to ambient air at $T_\infty = 20^\circ\text{C}$; the heat transfer coefficient between ambient air and the outer surface is $h_o = 100\ \text{W/m}^2$ K. Neglecting radiation effects, determine the inner ($T_{s,i}$) and outer ($T_{s,o}$) surface temperatures of the wall.

   What would be $T_{s,i}$ and $T_{s,o}$ if 1500 pin fins per square meter of the wall were installed to its outer surface? The fins have thermal conductivity, $k = 170$ W/m K, and the diameter and length equal to $D = 1$ cm and $L = 5$ cm, respectively. In your calculation neglect the radiation effects and assume that the heat transfer coefficient between ambient air and the fin surface is the same as that between ambient air and the outer surface of the wall, i.e., $h_o = 100\ \text{W/m}^2$ K.

2. A long cylindrical rod of diameter 150 mm with thermal conductivity of 0.5 W/m K experiences a uniform heat generation $\dot{q}$ [W/m^3]. The rod is encapsulated by a circular sleeve having an outer diameter of 250 mm and a thermal conductivity of 8 W/m K. The thermal contact resistance between the rod and the sleeve $R''_{t,c} = 2 \times 10^{-4}\ \text{m}^2$ K/W. The outer surface of the sleeve is exposed to cross flow of air at 20°C with a convection coefficient of $40\ \text{W/m}^2$ K.

   a) Sketch the equivalent thermal circuit corresponding to steady-state conditions and label appropriate temperatures, resistances and heat flow.
   b) If the maximum temperature of the sleeve cannot exceed 100°C, what is the maximum allowable volumetric heat generation $\dot{q}$ within the rod?
   c) What is the corresponding maximum temperature of the rod?

3. A solid steel sphere ($\rho = 7832\ \text{kg/m}^3$, $c = 559$ J/kg·K, $k = 48.8$ W/m·K) 300 mm in diameter, is coated with a dielectric material that for a unit surface area, has a thermal resistance of $R''_{t,c} = 0.01\ \text{m}^2\cdot\text{K/W}$. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_\infty = 100^\circ\text{C}$ and $h = 40\ \text{W/m}^2$ K.

   a) Shaw that lumped thermal capacitance method is applicable to predict the coated sphere temperature as a function of time in the cooling bath.
   b) Estimate the time required for the inside temperature of the sphere to reach 200°C?
   c) What is the corresponding temperature of the dielectric coating at that time?

Hint: Neglect the effect of energy storage in the dielectric material, since its thermal capacitance ($\rho c V$) is small compared to that of the steel sphere.

Appendix

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M\tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

TABLE C.3 One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere

	Temperature Distribution	
Plane Wall	$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$	(C.22)
Circular Rod	$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.23)
Sphere	$T(r) = \frac{\dot{q}r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.24)
	Heat Flux	
Plane Wall	$q''(x) = \dot{q}x$	(C.25)
Circular Rod	$q''(r) = \frac{\dot{q}r}{2}$	(C.26)
Sphere	$q''(r) = \frac{\dot{q}r}{3}$	(C.27)
	Heat Rate	
Plane Wall	$q(x) = \dot{q}xA_x$	(C.28)
Circular Rod	$q(r) = \dot{q}\pi Lr^2$	(C.29)
Sphere	$q(r) = \frac{\dot{q}4\pi r^3}{3}$	(C.30)