MATH 3705A
Test 1 Solutions
January 21, 2010

[Marks] Questions 1-4 are multiple choice. Circle the correct answer. Only the answer will be marked.

[3] 1. $\mathcal{L}\{e^{-2t}\cos(3t)\} =$

(a) $\dfrac{s}{(s-2)^2+9}$ (b) $\dfrac{s-2}{s^2+9}$ (c) $\dfrac{s-2}{(s-2)^2+9}$ (d) $\dfrac{s+2}{(s+2)^2+9}$ (e) None of these

Solution: (d)

[3] 2. $\mathcal{L}\{u(t-3)e^{-2t}\} =$

(a) $\dfrac{e^{-3(s+2)}}{s+2}$ (b) $\dfrac{e^{-3s}}{s+2}$ (c) $\dfrac{e^{3s}}{s+2}$ (d) $\dfrac{e^{-3(s-2)}}{s+2}$ (e) None of these

Solution: (a)

[3] 3. $\mathcal{L}^{-1}\{\dfrac{6s}{(s^2+9)^2}\} =$

(a) $t\sin(3t)$ (b) $t\cos(3t)$ (c) $-t\sin(3t)$ (d) $-t\cos(3t)$ (e) None of these

Solution: (a)

[3] 4. $\mathcal{L}^{-1}\{\dfrac{se^{-s}}{s^2-2s+4}\} =$

(a) $e^t\cos(\sqrt{3}t) + \dfrac{1}{3}e^{3t}\sin(\sqrt{3}t)$

(b) $u(t-1)\{e^{t-1}\cos[\sqrt{3}(t-1)] + e^{t-1}\sin[\sqrt{3}(t-1)]\}$

(c) $u(t-1)e^{t-1}\{\cos[\sqrt{3}(t-1)] + \dfrac{1}{\sqrt{3}}\sin[\sqrt{3}(t-1)]\}$

(d) $u(t-1)\{e^{t-1}\cos[\sqrt{3}(t-1)] + \dfrac{1}{3}e^{t-1}\sin[\sqrt{3}(t-1)]\}$

(e) None of the above

Solution: (c)

[9] 5. Solve the initial-value problem $y'' - y' - 6y = 0,\ y(0) = 0,\ y'(0) = 5$.

Solution:

$[s^2Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] - 6Y(s) = 0 \ \Rightarrow$

$(s^2 - s - 6)Y(s) - 5 = 0 \ \Rightarrow \ Y(s) = \dfrac{5}{s^2-s-6} = \dfrac{5}{(s+2)(s-3)} = \left[\dfrac{1}{s-3} - \dfrac{1}{s+2}\right]$

by partial fractions, hence $y(t) = \left(e^{3t} - e^{-2t}\right)$.

[9] 6. Solve the initial-value problem $y'' + 4y' + 13y = 0,\ y(0) = 2,\ y'(0) = 4$.

<u>Solution:</u>

$$[s^2Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 13Y(s) = 0 \ \Rightarrow$$

$$(s^2 + 4s + 13)Y(s) - 2s - 12 = 0 \ \Rightarrow\ Y(s) = 2\frac{s+6}{s^2+4s+13} = 2\frac{(s+2)+4}{(s+2)^2+9} =$$

$$2\frac{s+2}{(s+2)^2+9} + \frac{8}{3}\frac{3}{(s+2)^2+9} \ \Rightarrow\ y(t) = 2e^{-2t}\cos(3t) + \frac{8}{3}e^{-2t}\sin(3t).$$