Solutions: Risk, Return, Prices, CAPM.

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1 Risk and Return

1.1 Questions

1. $E(r_{PF}) = 40\% \times 6\% + 25\% \times 10\% + 35\% \times 20\% = 11.9\%$.

2. Let x denote the proportion of your portfolio invested in A. x solves

$$x \times 7\% + (1 - x) \times 15\% = 10\% \Leftrightarrow x = 0.625$$

   You invest \$62,500 in the risky stock and \$37,500 in the risk-free asset.

3. Let x denote the proportion of your portfolio invested in the risky stock. x solves

$$x \times 11\% + (1 - x) \times 3\% = 9\% \Leftrightarrow x = 0.75$$

   You invest \$75,000 in the risky stock and \$25,000 in the risk-free asset. Similarly, to get a 15% expected return, solve

$$x \times 11\% + (1 - x) \times 3\% = 15\% \Leftrightarrow x = 1.50$$

   You invest \$150,000 in the risky stock and borrow \$50,000 at the risk-free rate.

4. The expected return r_A on stock A must be such that $25\% \times 4\% + 75\% \times r_A = 7\% \Leftrightarrow r_A = 8\%$. The current price of stock A must be at most $\$54/(1 + r_A) = \$50.00$.

5. $\sigma_{PF} = \sqrt{0.4^2 \times (10\%)^2 + 0.6^2 \times (18\%)^2 + 2 \times 0.4 \times 0.6 \times (-0.1) \times 10\% \times 18\%} = 11.14\%$.
   $\sigma_{PF} = \sqrt{0.3^2 \times (10\%)^2 + 0.4^2 \times (18\%)^2 + 2 \times 0.3 \times 0.4 \times (-0.1) \times 10\% \times 18\%} = 7.52\%$.

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6. Let x denote the proportion of your portfolio invested in the risky stock. To get a return of 14%, you need x to solve

$$x \times 11\% + (1 - x) \times 3\% = 14\% \Leftrightarrow x = 1.375$$

So you need to invest \$137,500 in the risky stock and borrow \$37,500 at the risk-free rate. The volatility of your portfolio is then

$$\sigma_{PF} = \sqrt{1.375^2 \times (15\%)^2} = 20.625\%.$$

So you can't get an expected return of 14% with a volatility below 20%.

7. Because A and B are not perfectly correlated ($\rho_{AB} < 1$), there is a benefit from diversifying your portfolio. Namely, you can achieve a total risk lower than 6%, with the same expected return (5%). If you sell 15 A stocks, and buy 3 B stocks. The expected return on your portfolio is still 5%, but its volatility is

$$\sigma_{PF} = \sqrt{0.85^2 \times (6\%)^2 + 0.15^2 \times (12\%)^2 + 2 \times 0.85 \times 0.15 \times (0.2) \times 6\% \times 12\%} = 5.74\%$$

1.2 Problem: Expected Return and Volatility of a Portfolio

Detailed computations are in the Excel File (WebCT).

1. $E(r_A) = 9.40\%$, $E(r_B) = 4.90\%$, $\sigma_A = 6.62\%$, $\sigma_B = 2.21\%$.

2. $\rho_{AB} = -0.37$

3. $E(r_{PF}) = 20\% \times 9.40\% + 80\% \times 4.90\% = 5.80\%$

$$\begin{aligned} \sigma_{PF} &= \sqrt{0.20^2 \times (6.62\%)^2 + 0.80^2 \times (2.21\%)^2 + 2 \times 0.20 \times 0.80 \times (-0.37) \times 6.62\% \times 2.21\%} \\ &= 1.77\% \end{aligned}$$

4. Graphically, the minimum variance portfolio seems to be for 17.5% invested in A and 82.5% in B (the exact number is 17.35% invested in A). See Excel File.

5. $E(r_{PF}) = 50\% \times 3.00\% + 10\% \times 9.40\% + 40\% \times 4.90\% = 4.40\%$

$$\begin{aligned} \sigma_{PF} &= \sqrt{0.10^2 \times (6.62\%)^2 + 0.40^2 \times (2.21\%)^2 + 2 \times 0.10 \times 0.40 \times (-0.37) \times 6.62\% \times 2.21\%} \\ &= 0.89\% \end{aligned}$$

2 CAPM

2.1 Questions

1. Nothing, the CAPM yields a relation between **systematic risk** (i.e. β) and expected returns. Total risk (i.e. volatility σ) is not the relevant measure of risk.

2. Same answer as above.

3. The CAPM relation is $E(r) = r_f + \beta[E(r_M) - r_f]$, so a higher β translates into a higher expected return. Stock B should therefore have a higher expected return than stock A. In words, investors require a higher expected return on B because the exposure of B's returns to systematic risk (i.e. market portfolio) is higher.

4. From the CAPM $E(r_S) = r_f + \beta[E(r_M) - r_f] = 3\% + 0.7 \times (10\% - 3\%) = 7.9\%$.

5. The β of a portfolio is just the weighted average of individual stocks βs. So $\beta_{PF} = 40\% \times (-0.1) + 25\% \times 0.6 + 35\% \times 0.5 = 0.285$. $E(r_{PF}) = 4\% + 0.285[9\% - 4\%] = 5.425\%$.

6. $\beta_{PF} = 40\% \times 1 + 60\% \times 0 = 0.4$.

7. If the expected return on your portfolio is 8%, then its systematic risk is $\beta_{PF} = (8\% - 4\%)(10\% - 4\%) = 0.67 < 0.7$. So you can build the required portfolio, for instance by investing in the market portfolio and the risk-free asset. Let x denote the fraction of your portfolio invested in the risky asset, x solves $x \times 4\% + (1 - x) \times 10\% = 8\% \Leftrightarrow x = 33.3\%$. Invest \$33,333 at the risk-free rate and \$66,667 in the market porfolio.

8. "Total risk (volatility) is not relevant for you. In particular, if you are well diversified, you should care only about the exposure of my firm's cash-flows to systematic risk. If the β of my firm is low, you might actually be willing to pay a higher price than you think for each stock".

9. Lutherans. According to the CAPM, no strategy can outperform the benchmark market portfolio over the long term. So you should just hold the market portfolio, that is, invest in a passive fund.

2.2 Problem: Expected return and β

1.
$$E(r_A) = 0.2 \times 17\% + 0.5 \times 12.50\% + 0.2 \times 3\% + 0.1 \times (-2\%) = 10.05\%,$$
$$E(r_M) = 0.2 \times 15\% + 0.5 \times 9\% + 0.2 \times 5\% + 0.1 \times (-1\%) = 8.40\%.$$

2. Implied β is $\beta_i = (10.05\% - 3.80\%)/(8.40\% - 3.80\%) = 1.36$.

3. From the table $cov(r_A, r_M) = 0.0026$ and $\sigma_M = 4.48\%$. So $\beta_A = cov(r_A, r_M)/\sigma_M^2 = 1.30$. If your analyst is correct, this means that A delivers a return that corresponds to a risk β of 1.36, whereas its actual risk is 1.30 only. So you should buy this stock and your (expected) α over the next year will be $0.06 \times (8.40\% - 3.80\%) = 0.276\%$

2.3 Problem: Pricing with the CAPM

1.

$$\begin{aligned}
E(r_A) &= 0.3 \times \left(\frac{150,000}{P} - 1\right) + 0.5 \times \left(\frac{100,000}{P} - 1\right) + 0.2 \times \left(\frac{90,000}{P} - 1\right) \\
&= \frac{0.3 \times 150,000 + 0.5 \times 100,000 + 0.2 \times 90,000}{P} - 1 \\
&= \frac{113,000}{P} - 1
\end{aligned}$$

2. From the table, $E(r_M) = 8.80\%$, $\sigma_M = 5.55\%$

$$\begin{aligned}
cov(r_A, r_M) &= 0.3 \times \left(\frac{150,000}{P} - 1 - \frac{113,000}{P} + 1\right) \times (15\% - 8.80\%) \\
&+ 0.5 \times \left(\frac{100,000}{P} - 1 - \frac{113,000}{P} + 1\right) \times (9\% - 8.80\%) \\
&+ 0.2 \times \left(\frac{90,000}{P} - 1 - \frac{113,000}{P} + 1\right) \times (-1\% - 8.80\%) \\
&= \frac{1,126}{P}
\end{aligned}$$

So $\beta_A = cov(r_A, r_M)/\sigma_M^2$ =366,060/P

3.

$$\begin{aligned}
E(r_A) = &\quad r_f + \beta_A[E(r_M) - r_f] \\
\Leftrightarrow \frac{113,000}{P} - 1 = &\quad 3.50\% + \frac{366,060}{P}(8.80\% - 3.50\%) \\
\Leftrightarrow \quad P = \$90,434 &
\end{aligned}$$

4. π' has the same β as π, therefore the same expected return $E(r_A) = 113,000/90.434-$

$1 = 24.95\%$. Therefore

$$V' = \frac{\$10,000}{24.95\%} = \$40,075.$$

2.4 Empirical Exercise: estimation of β.

The results are in the excel file "RiskReturn_SolutionsExcel".
Notice that, in order to estimate β, you need first to compute the monthly excess returns for the stock and the market PF. To do this, you first need to compute a monthly risk-free rate using $1 + r_f = (1 + r_f^m)^{12}$, where r_f is the annual risk-free rate and r_f^m is the monthly risk-free rate. Then you compute the excess return for, say november 2011, by computing the price change in november (price dec 1 - price nov 1) divided by the price nov 1, to which you subtract the risk-free rate for november 2011.
One concern with using the estimated β to derive (future) expected returns is that we use past data to estimate a parameter which, in the CAPM, is forward-looking. So implicitly, we assume that β is constant over time. Another concern is that the estimation is very imprecise.