Exercises: Risk, Return, Prices, CAPM.

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*N.B. anytime a "correlation" between two securities is mentioned, it stands for the correlation between those securities' **returns**.*

1 Risk and Return

1.1 Questions

1. You own a portfolio that is 40% invested in stock A, 25% in stock B and 35% in stock C. The expected returns on these stocks are respectively 6%, 10% and 20%. What is the expected return of your portfolio?

2. You own \$100,000 that you can invest in two stocks, A and B. The expected returns on A and B are 7% and 15% respectively. How much should you invest in each security to get a 10% expected return on your portfolio?

3. You own \$100,000 that you want to invest. You can invest or borrow at the risk-free rate (3%). In addition, you can invest in a risky stock that has a 11% expected return. Can you build a portfolio that has a 9% expected return? Same question with a 15% expected return?

4. You own \$100,000 and want to invest 25% of your portfolio at the risk-free rate (4%) and the remaining 75% in one risky stock. You are considering stock A. Your estimate of A's price in one year is \$54.00 and A does not pay dividends. You want the total expected return on your portfolio to be 7%. What is the maximum price you are willing to pay for each A stock today?

5. Stock A has a volatility of 10%, stock B has a volatility of 18%. The correlation between A and B is -0.1. What is the volatility of a portfolio that is 40% invested in A and 60% invested in B? Same question if your portfolio is 30% invested in A, 40% invested in B and 30% invested in a risk-free security.

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6. You own \$100,000 that you want to invest. You can invest or borrow at the risk-free rate (3%). In addition you can invest in a risky stock that has a 11% expected return and a 15% volatility. Can you compose a portfolio that has a 14% expected return and volatility lower than 20%?

7. One of your (wealthy) friends owns 100 A stocks for a total market value of \$10,000. Expected return on A is 5%, volatility is 6%. B stocks have a current price of \$500, their expected return is 5% and their volatility is 12%. The correlation between A and B is 0.2. Your friend has been advised to sell 15 of his A stocks to buy B stocks but argues that this reallocation is pointless as it would not improve his expected return and B stocks are more risky than A stocks. What can you tell him?

1.2 Problem: Expected Return and Volatility of a Portfolio

Information on securities A and B are given in the table below.

State of the economy	Probability	r_A	r_B
Boom	0.2	+20.00%	+6.00%
Normal	0.5	+10.00%	+3.00%
Recession	0.2	+3.00%	+6.00%
Depression	0.1	-2.00%	+10.00%

The second column gives the probability of each state of the economy in the next year. r_A and r_B are annual returns of securities A and B depending on the state of the economy.

1. Compute the expected return and the volatility for securities A and B.

2. Compute the correlation between A and B.

3. Using your previous computations, derive the expected return and volatility of a portfolio that is 20% invested in A and 80% in B. Compare with expected return and volatility for security A and B.

4. Using Excel, graph the expected return and volatility of all portfolios composed of A and B (in positive proportions). Use that graph to give an estimate of the composition of the minimum-variance portfolio.

5. The risk-free rate (return on a US treasury bond) is 3.00%. You invest \$50,000 in treasury bonds, \$10,000 in A and \$40,000 in B. Compute the expected return and volatility of your portfolio.

2 CAPM

2.1 Questions

1. Stock A has a 10% volatility and stock B a 18% volatility. According to the CAPM, what can you say about the expected return on A relative to the expected return on B?

2. Stock A has a 8% expected return and stock B a 12% expected return. According to the CAPM, what can you say about the volatility of A relative to the volatility of B?

3. Stock A has a β of 0.8 and stock B a β if 1.2. According to the CAPM, what can you say about the expected return on A relative to the expected return on B?

4. The expected return on the market portfolio is 10%. The risk-free rate is 3%. What is the expected return on a stock that has a β of 0.7?

5. You own a portfolio that is 40% invested in stock A, 25% in stock B and 35% in stock C. The βs of these stocks are respectively -0.1, 0.6 and 0.5. What is the β of your portfolio? Suppose that the expected return on the market portfolio is 9% and and the risk-free rate is 4%, what is the expected return on your portfolio?

6. You own a portfolio that is 40% invested in the market portfolio, and 60% invested in a risk-free security. What is the β of your portfolio?

7. You own \$100,000 that you want to invest. The expected return on the market portfolio is 10%, the risk-free rate is 4%. Can you compose a portfolio that has an expected return higher than 8% and a β lower than 0.7?

8. You want your (wealthy) friend to invest in your new company that provides actual financial training to Wall Street traders. He argues that given the volatility of future cash-flows (20%) and his low tolerance for risk, he'd be willing to pay only a low price for each stock of your firm. What can you tell him?

9. Read the article from the Economist, *"Faith and the Market"*. Which of the Catholics or the Lutherans are more likely to use the CAPM as a selling point to attract investors?

2.2 Problem: Expected return and β

You are a fund manager and you try to generate "abnormal" performance (αs). Your financial analyst gives you information on stock A and and the market portfolio.

State of the economy	Probability	r_A	r_M
Boom	0.2	+17.00%	+15.00%
Normal	0.5	+12.50%	+9.00%
Recession	0.2	+3.00%	+5.00%
Depression	0.1	-2.00%	-1.00%

The risk-free rate is 3.80%. r_A and r_M are annual returns of security A and the market portfolio depending on the state of the economy.

1. Using information in the table above only, compute the expected returns on stock A and the market portfolio.

2. Using the result from the previous question and the CAPM equation, compute the implied β of stock A.

3. Compute directly (i.e. using information in the table only) the β of stock A. Compare to the result from the previous question. If you trust your analyst and the CAPM what should you do?

2.3 Problem: Pricing with the CAPM

You are a financial analyst and you try to value an asset A (i.e. find its market price). That asset generates a single cash-flow in one year, which depends on the state of the economy.

State of the economy	Probability	CF from A	r_M
Boom	0.3	\$150,000	+15.00%
Normal	0.5	\$100,000	+9.00%
Depression	0.2	\$90,000	-1.00%

The risk-free rate is 3.5%. r_M is the annual returns on the market portfolio depending on the state of the economy. Let P denote the value (or price) of project A and β_A its exposure to systematic risk.

1. Using information in the table above, write the expected return on A as a function of P.

2. Write β_A as a function of P (*Hint: express A's return in each state of the economy as a function of P, then using your answer to the previous question, compute the covariance between A and the market portfolio as a function of P*).

3. Using the CAPM equation and your answers to the previous questions, compute P.

4. Suppose that project π' has exactly the same exposure to market risk β_π than project π. Project π' will generate an expected cash-flow of \$10,000 each year from next year on. What is the value V' of π'?

2.4 Empirical Exercise: estimation of β.

This exercise shows you (roughly) how the CAPM β of an asset can be derived based on historical data.

Open the Excel file named "Beta". You will find monthly prices for a stock (General Electric) and for the Dow Jones Industrial Average, for the period december 2001 to december 2011. You will also find the yield (i.e. annual interest rate) on a 30-year treasury bond for the same period.

Based on the CAPM, the model you are trying to estimate has the following specification,

$$r_A^t = r_f + \beta(r_M^t - r_f^t) + \epsilon^t \Leftrightarrow r_A^t - r_f^t = \beta(r_M^t - r_f^t) + \epsilon^t$$

where r_A^t is the monthly return of the stock in month t, r_M^t is the average return of the market index in month t, r_f^t is the monthly risk free rate and ϵ^t is an error term. Your objective is to find the β that "minimizes" the error terms ϵ^t. In other words, you need to estimate the coefficient of the regression of the stock's excess return (i.e. total return - risk-free rate) on the market excess return, based on historical returns.

In Excel, the function "slope" will compute this coefficient for you. Slope has two arguments, the first one is the series of historical data for the dependent variable, that is, the stock excess return, $r_A - r_f$, the second one is the series of historical data for the explanatory variable, that is, the market excess return, $r_M - r_f$.

1. Compute a ten-year β, that is, a β based on the last 10 years of past returns.
2. Compute a five-year β, that is, a β based on the last 5 years of past returns.
3. Compare the two results, comment. If you were to use one of these β to compute the expected return of the General Electric stock, what would be your concern(s)?