STAT 2507 Assignment # 3 (Chapter 5 and 6) Fall 2013
Due in class: November 11, 2013

Last Name ——————————————- , **First Name** ——————-
Student # ——————— —————-

Total of marks=100.

Part I. Lab questions. Use only the blanks left to answer lab questions.

1. Suppose that Y is a binomial random variable with parameters $n = 16$ and $p = 0.5$. Generate $1,000$ observations from Y and answer the following questions:
   random 16 c1;
   binomial 16 0.5.
   (a) [5] What is the shape of the distribution of Y? **(Nearly) symmetric.**
   (b) [5] Use the *describe* command to obtain the sample mean $\bar{y}$ and the standard deviation s of these 1,000 observations. How many values (among the 1,000 generated) fall between $\bar{y} \pm 2s$? **956(random number).**
   (c) [5] Based on the Empirical Rule, approximately how many values of Y should fall between $np \pm 2\sqrt{np(1-p)}$? **Approximately 950.**

2. Suppose that a random variable X has a Poisson distribution with mean $\mu = 22$. Use the *cdf* command that works by typing
   cdf;
   poisson 22.
   to answer the following questions.
   (a) [5] $P(X < 15) =$ **0.0477**; $P(15 < X \leq 20) =$ **0.3100.**
   (b) [5] If you simulate 1,000 of X, then the expected number of values that are less than 15 (among the 1,000 simulated) would be close to **48.**

3. Suppose that Y has a hypergeometric distribution with parameters $N = 20$, $M = 8$, and $n = 5$.

(a) [5] Use the command
cdf;
hypergeometric 20 8 5.
to find $P(Y \leq 2) = \mathbf{0.7038}$ and $P(Y > 2) = \underline{\mathbf{1 - 0.7038 = 0.2962}}$.
(b) [5] Use the "inverse cdf" command which for a given number a between 0 and 1 yields the value of (?) that satisfies $P(Y \leq ?) = a$, and works by typing
invcdf a;
hypergeometric 20 8 5.
to find the value of c in $P(Y \leq c) = 0.942$. The value of c is **<u>3.</u>**

4. Suppose that Y has a normal distribution with mean μ=31 and variance σ^2=4.

(a) [5] Use the *cdf* command that gives you the value of $P(Y \leq y)$ and works by typing
cdf y;
normal 31 2.
to find $P(Y > 29) = \underline{\mathbf{0.8413}}$, and $P(25 \leq Y \leq 30) = \underline{\mathbf{0.3072}}$.

(b) [5] Use the *invcdf* command which for a given number a between 0 and 1 gives the value of (?) that satisfies $P(Y \leq ?) = a$, and works by typing
invcdf a;
normal 31 2.
to find the value of c in $P(Y \leq c) = 0.25$. The value of c is **<u>29.651</u>**.

<u>ALSO do the following four questions:</u>

5. [10] Suppose that in a large population the proportion of people that have a certain disease is 0.01. Use the Poisson approximation to find the probability that in a random group of 200 people at least four people will have the disease.

Solution:
Let X be the number of people having the disease among the 200 people in the random group. Then $X \sim \text{Bin}(200, 0.01)$. This distribution can be approximated by a Poisson distribution

with mean $\mu = np = 2$. If Y denotes a Poisson random variable with mean $\mu = 2$, then, the desired probability is $P(X \geq 4) = 1 - P(Y \leq 3) \approx 0.143$.

6. Assume that head sizes (circumference) of new recruits in the Canadian armed forces can be approximated by a normal distribution with a mean of 22.8 inches and a standard deviation of 1.1 inches.

   (a) [6] What proportion of recruits have head sizes between 22 and 23 inches?
   Solution:
   Let X be the head size, then $X \sim N(22.8, 1.1^2)$ so
   $P(22 < X < 23) = P(\frac{22-22.8}{1.1} < Z < \frac{23-22.8}{1.1}) = P(-.73 < Z < 0.18) = 0.5714 - 0.2327 = 0.3387$

   (b) [6] 5% of the head sizes exceed ———— inches?
   Solution:
   $P(Z > c) = 0.05 \rightarrow P(Z < c) = 0.95 \rightarrow c = 1.645 \rightarrow 1.645 = \frac{X-22.8}{1.1} \rightarrow X = 24.6095$

7. [12] A company manufactures washers, about 5% of which are defective. If a random sample of 100 washers are inspected, what is the probability that fewer than 4 are defective?
   Solution:
   X: # of defective, so X has binomial distribution with $n = 100,\ p = 0.05$. It can approximate by normal distribution since $np = 100(0,05) = 5,\ nq = 100(0.95) = 95$, so $X \sim N(5, 4.75)$
   $P(X < 4) = P(Z < \frac{4+0.5-5}{\sqrt{4.75}}) = P(Z < -0.23) = 0.4090$
   Also, since $np < 7$, Poisson approximation to Binomial is also correct.

8. A case of wine has 12 bottles, 3 of which contain spoiled wine. A sample of 4 bottles is randomly selected from the case.
   (a) [4] What is probability that the sample contains 2 bottles of spoiled wine?
   Solution:
   X is # of spoiled bottles of wine in the sample which has hypergeometric distribution with $N = 12, M = 3, n = 3$, so
   $P(X = 2) = \dfrac{C_2^3 C_1^9}{C_3^{12}}$

(b) [4] What is probability that all 4 of the sampled bottles are spoiled?

Solution:

$P(X = 4) = 0$ since there are just 3 spoiled bottles of wine.

(c) [4] What are the mean and variance of the number of spoiled bottles in the sample?

solution:

$\mu = n(\frac{M}{N}) = 4(\frac{3}{12} = 1, \ \sigma^2 = n(\frac{M}{N})(1 - \frac{M}{N})(\frac{N-n}{N-1}) = 4(3/12)(9/12)(8/11) = 0.545$

9. A certain brand of computer disks averages 0.1 missing pulse per disk. Let the random variable X denotes the number of missing pulses.

(a) [3] What is the distribution of X?

Solution:

X has Poisson distribution with $\mu = 0.1$.

(b) [3] Find the probability that the next inspected disk will have no missing pulse.

Solution:

$$P(X = 0) = \frac{e^{-0.1} 0.1^0}{0!} = e^{-0.1} = 0.9048$$

(c) [3] Find the probability neither of the next two disks inspected will contain any missing pulse. **Solution:**

$P(X = 0)P(X = 0) = e^{-0.1}e^{-0.1} = 0.8187$.