

Multiple Choices Response

Q1	Q2	Q3	Q4	Q5
D	C	C	F	A

1. Let $x, y > 0$. Which of the following statements is false?

~~(a)~~ $\ln(xy) = \ln(x) + \ln(y)$. *true*

(b) $e^{x+y} = e^x e^y$. *true*

(c) $\ln\left(\frac{x}{y}\right) = \frac{\ln(x)}{\ln(y)}$. *false*

(d) $\sqrt{x} = x^{\frac{1}{2}}$. *true*

A. (b)

~~B. (a)~~

C. (d)

D. (c)

E. None of the mentioned above.

~~F. All the mentioned above.~~

2. Find the solution of the equation $\ln(x+2) + \ln(x-1) = 2\ln(2)$

A. $\{2, -3\}$

B. $\{-2, 1\}$

C. $\{2\}$

$x+2 > 0$
 $x > -2$

$x-1 > 0$
 $x > 1$

D. $\{-3\}$

E. $\{2, 3\}$

F. None of the above.

~~$\ln(x^2 + 1x - 2) = \ln(2^2)$~~

$x^2 + x - 2 = 4$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

~~$x = -3$~~ $x = 2$
rejected

3. Find the set of solutions for the following inequality $3 + |2x - 5| \leq 12$

- A. $[-2, +\infty)$
- B. $(7, +\infty)$
- C. $[-2, 7]$

- D. $(-\infty, -2]$
- E. $[7, -\infty)$
- F. $(-\infty, 7]$

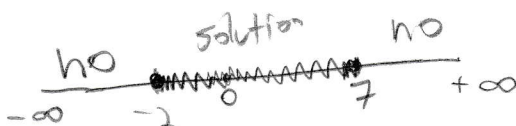
$$|2x - 5| \leq 9$$

$$[-2, 7]$$

$$2x - 5 \leq 9 \quad \text{or} \quad 2x - 5 \leq -9$$

$$2x \leq 14 \quad \text{or} \quad 2x \leq -4$$

$$x \leq 7 \quad \text{or} \quad x \leq -2$$



4. Find the domain of the function $f(x) = \frac{\sqrt{x-2}}{2x+4}$

- A. \mathbb{R}
- B. $\mathbb{R} - \{-2\}$
- C. $\mathbb{R} - \{-2, 2\}$

$$x - 2 \geq 0 \implies x \geq 2$$

$$2x + 4 \neq 0 \implies 2x \neq -4 \implies x \neq -2$$

- D. $\mathbb{R} - \{-4\}$
- E. $(-2, 2]$
- F. $[2, \infty)$

$$[2, +\infty)$$

5. Let $f(x) = \frac{1}{\sqrt{x+1}}$ and $g(x) = x^2 - 1$ then $f(g(x)) =$

- A. $\frac{1}{|x|}$
- B. $\frac{x^2 - 1}{\sqrt{x+1}}$
- C. $\frac{1}{x+1} - 1$

$$f(g(x)) = \frac{1}{\sqrt{(x^2 - 1) + 1}}$$

- D. $\sqrt{x+1} - 1$
- E. $\frac{1}{\sqrt{x^2 - 1}}$
- F. Non of the above.

$$= \frac{1}{\sqrt{x^2}}$$

$$= \frac{1}{|x|}$$

Question 6 [12 pts]

In a wildlife conservation park, every year 25% of the population of elephants either die of natural causes, are eaten by predators or are poached. In the meantime, there are 500 new elephants born every year. The DTDS that models the number of elephants in the wildlife conservation park at year t , denoted by x_t , is given by $x_{t+1} = 0.75x_t + 500$. $r = 0.75$ $C = 500$

(a) What is the updating function of the DTDS? $f(x) = 0.75x + 500$

(b) Find the fixed point of the DTDS if there is any. $x^* = 2000$ elephants

since $r \neq 1$,

$$x^* = \frac{C}{1-r} = \frac{500}{1-0.75} = 2000$$

(c) Find the general solution formula for the DTDS for a general initial value x_0

$$x_t = (0.75)^t (x_0 - 2000) + 2000$$

$$x_t = (r)^t (x_0 - x^*) + x^*$$

$r = 0.75$ $x^* = 2000$

(d) A researcher counted 1000 elephants today. What should we expect the population to be

$x_0 = 1000$
after 15 years?
 $t = 15$

$$x_{15} = 1987 \text{ elephants}$$

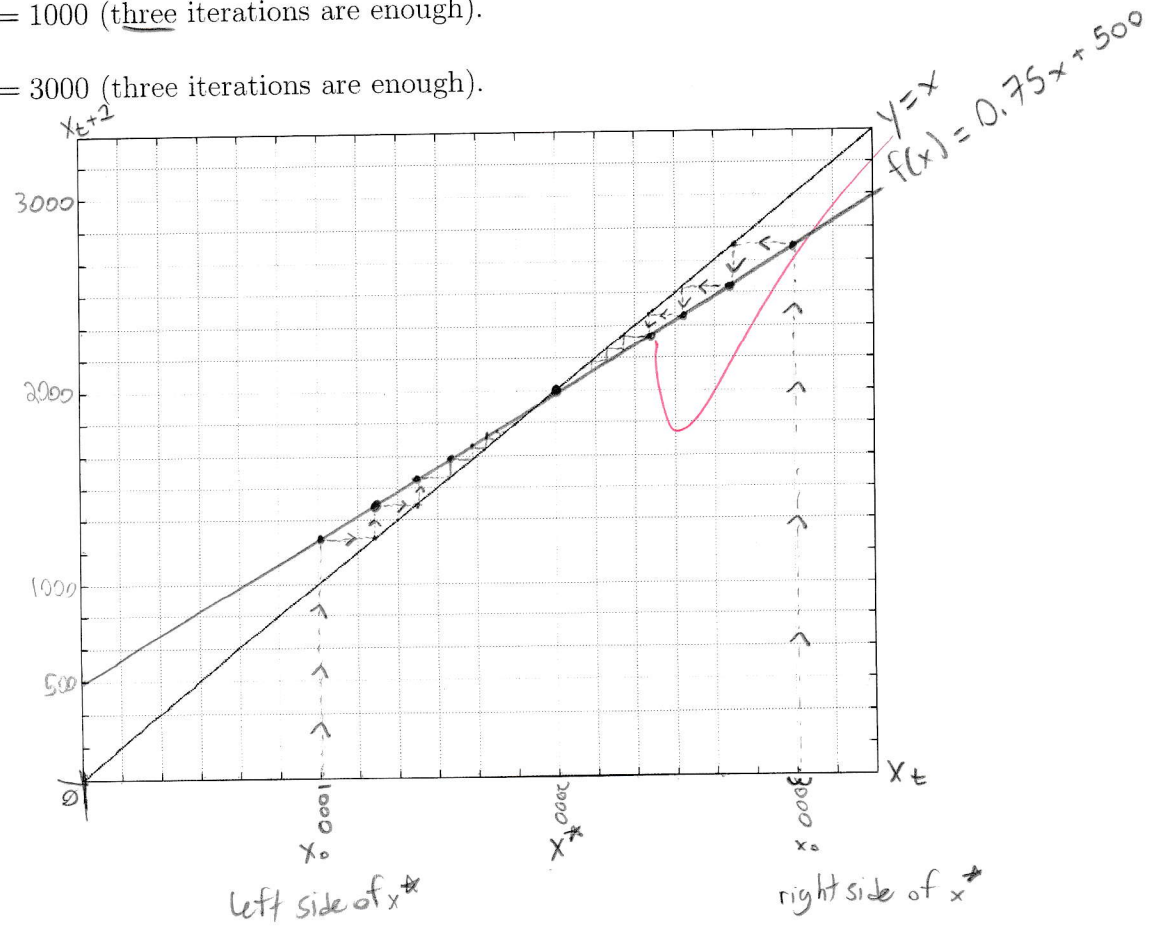
$$x_{15} = (0.75)^{15} (1000 - 2000) + 2000$$

$$x_{15} = 1986.6365 \dots$$

$$x_{15} \approx 1987 \text{ elephants}$$

(e) Draw the cobweb diagram of the dynamical system with

- $x_0 = 1000$ (three iterations are enough).
- $x_0 = 3000$ (three iterations are enough).



- (f) Determine the stability of the equilibrium point.
Justify your answer with **two different ways of reasoning.**

The equilibrium point $x^* = 2000$ is stable b/c:
 - "r" value is $0 < 0.75 < 1$ $|r| < 1$ ✓
 - both x_0 values on the left and right side of the x^* point cobweb towards x^* .

- (g) Given that the population today is 1000 elephants. How long would it take for the population to reach 1950 elephants? about 10.4 years

$x_0 = 1000$

$x_t = 1950$

$$x_t = (0.75)^t (x_0 - 2000) + 2000$$

$$1950 = (0.75)^t (1000 - 2000) + 2000$$

$$-50 = (0.75)^t (-1000)$$

$$0.05 = (0.75)^t$$

$$\ln(0.05) = \ln(0.75^t)$$

$$\ln(0.05) = t \cdot \ln(0.75)$$

$$\frac{\ln(0.05)}{\ln(0.75)} = t$$

$t = 10.4133 \dots$

- (h) **(Bonus)** [3 pts] The conservation park is not satisfied with the results and would like to increase the steady state of the elephant population to 3125 elephants. They do not have control on the number of the newborns. However, they are willing to increase the security and build fences to protect the elephants from predators and poachers. What is the percentage of annual decrease that they can allow to reach a steady state of 3125 elephants? 16%

$x^* = 3125$ $c = 500$

Since $r \neq 1$, $x^* = \frac{c}{1-r}$

$$3125 = \frac{500}{1-r}$$

$$3125 - 3125r = 500$$

$$\frac{2625}{3125} = \frac{3125r}{3125}$$

$$0.84 = r$$

$$1 - 0.84 = 0.16 \times 100\% = \boxed{16\%}$$

Question 7 [8 pts]

Consider the nonlinear DTDS $x_{t+1} = \frac{3x_t}{1+2x_t}$ used in fisheries management.

(a) What is the updating function? $f(x) = \frac{3x}{1+2x}$

(b) Given that $x_0 = 0.8$. Find x_1, x_2 and x_3 .
 $x_1 = 0.923$ $x_2 = 0.973$ $x_3 = 0.991$

$x_0 = 0.8$
 $x_1 = \frac{3(0.8)}{1+2(0.8)} = \frac{2.4}{2.6} = \frac{12}{13}$ or 0.923
 $x_2 = \frac{3(\frac{12}{13})}{1+2(\frac{12}{13})} = \underline{0.973}$
 $x_3 = \frac{3(0.973)}{1+2(0.973)} = \underline{0.991}$

(c) Find the fixed point(s) if any. $x^* = 0$ $x^* = 1$

$$x^* = \frac{3x^*}{1+2x^*}$$

$$x^* + 2(x^*)^2 = 3x^*$$

$$0 = 3x^* - x^* - 2(x^*)^2$$

$$0 = 2x^* - 2(x^*)^2$$

$$0 = 2x^*(1 - x^*)$$

$\therefore 2x^* = 0 \Rightarrow x^* = 0$

$\therefore 1 - x^* = 0 \Rightarrow x^* = 1$

(d) In a different research on the same DTDS $x_{t+1} = \frac{3x_t}{1+2x_t}$, we obtained $x_5 = 0.75$. However, we lost some data due to computer malfunction. Estimate x_4 from x_5 . Do not use the values from part (b). This is a different research. $x_4 =$ 0.5 find inverse

inverse of $f(x)$:

$$y = \frac{3x}{1+2x}$$

$$y + 2xy = 3x$$

$$y = 3x - 2xy$$

$$y = x(3 - 2y)$$

$$\frac{y}{3-2y} = x$$

$f^{-1}(x) = \frac{x}{3-2x}$

if we plug in $x_5 = 0.75$ into the inverse function, we will get the previous step, x_4 .

$$x_4 = \frac{0.75}{3 - 2(0.75)}$$

$$x_4 = \frac{0.75}{1.5}$$

$x_4 = 0.5$

(This page is intentionally left blank. You may use it as scrap paper - please do not detach)

*Parallel lines have so much in common...
it is a shame they'll never meet!*