SYNTAX-SEMANTICS INTERFACE - NOTES

Alexis Wellwood & Catherine Huish

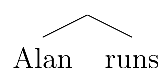
11 January 2008

1 Notes - 03 Jan 2008

1.1 Semantics 101

1.1.1 The interpretation function

A basic principle of semantic theorizing is compositionality. Simply put, this means that the semantic value of a sentence, also called its denotation, is a function of the values of its parts. We capture this notion by means of the interpretation function, $[\![\ ]\!]$, which assigns a value to lexical items and calculates values up to phrases and sentences. Consider the following sentence:



With simple sentences, the semantic value of the subject denotes an individual; in this case, 'Alan' will be represented by the constant term a. The value of the predicate is a set of individuals; here, 'runs' has the denotation {x : x runs}. So we say that $[\![S]\!] = t$ iff a $\in$ {x : x runs }, or, less technically, that S is true iff the denotation of the subject is a member of the denotation of the predicate. However, we'll soon find that this interpretation of subjects is literally impossible for quantificational subjects.

1.1.2 Logical rules

The first indication that subjects consisting of proper names must be treated differently than QP-subjects comes from consideration of predicates that logic tells us may never share members. Take the example where VP_1 = "is partially on this desk" and VP_2 = "is completely off this desk". That is, if one evaluates sentences by checking whether the subject is a member of the VP, then no matter what the denotation of the subject is, one of the sentences 'Subject VP_1' or 'Subject VP_2' will be false. Clearly, the following equation holds of these VPs:

$$[\![\mathrm{VP}_1]\!] \cap [\![\mathrm{VP}_2]\!] = \emptyset$$

A related yet independent argument is that, for any situation, the union of such disjoint VPs covers the entire domain. That is, given the above VPs, no matter what member of D the subject denotes, one of the sentences ‘Subject VP_1’ or ‘Subject VP_2’ will always be true. For these VPs, the following equation always holds:

$$[\![\mathrm{VP}_1]\!] \cup [\![\mathrm{VP}_2]\!] = D$$

Since this union covers all of D, the denotation of the subject must be in one of the VP denotations. For example, in a situation where Alexis is partially on the relevant desk, “Alexis is partially on this desk” is true, and “Alexis is completely off this desk” is false. Similarly, if Cat is completely off the desk, “Cat is partially on this desk” is false, and “Cat is completely off this desk” is true. One or the other proposition always holds of any given individual, at any given time.

1.1.3 Quantifier Phrase

Sentences with QP subjects do not obey these logical rules. In a situation where Alan is sitting on the desk, the sentence “Every professor at Concordia is partially on this desk” is false, and “Every professor at Concordia is completely off this desk” is also false. In the same situation, “Some professor at Concordia is partially on this desk” is true, but, since there are other professors at Concordia, “Some professor at Concordia is completely off this desk” is also true.

Where proper names and NPs generally denote individuals, a new semantics of QPs proposes that they denote sets of sets of individuals. Thus it cannot be the case that sentences with QP subjects are evaluated in the same way as we saw above, by checking whether the subject is a member of the VP. To do the semantics of QPs, we turn to viewing them as relations that hold between the set denoted by the NP and that denoted by the VP.

1.1.4 Generalized Quantifier Theory (aka, GQT)

The new theory that Qs denote relations was able to accurately account for our intuitions of QP constructions. For example, the sentence "Every boy runs" in GQT results in an interpretation of S that is true iff the denotation of the NP 'boy' ({ x : x is a boy}) is a subset of the VP 'runs' ({ x : x runs}). It was quickly discovered that any quantifier one can think of may be expressed as a relation in the same way:

$$\mathrm{R}_{every}(\mathrm{A,B}) = \mathrm{t} \text{ iff } [\![A]\!] \subseteq [\![B]\!]$$

$$\mathrm{R}_{two}(\mathrm{A,B}) = \mathrm{t} \text{ iff } |[\![A]\!] \cap [\![B]\!]| \geq 2 \text{ [and so on for any numeral]}$$

$$\mathrm{R}_{some}(\mathrm{A,B}) = \mathrm{t} \text{ iff } [\![A]\!] \cap [\![B]\!] \neq \emptyset$$

$$\mathrm{R}_{no}(\mathrm{A,B}) = \mathrm{t} \text{ iff } [\![A]\!] \cap [\![B]\!] = \emptyset$$

$$\mathrm{R}_{most}(\mathrm{A,B}) = \mathrm{t} \text{ iff } |[\![A]\!] \cap [\![B]\!]| > |[\![A]\!] - [\![B]\!]|$$

This new semantics for quantifiers helps to explain the intuitions of the section 'Logical rules', allowing us to capture the fact that QP-subjects yield different kinds of intuitions when considering complemented VPs, or VPs whose union covers the entire domain. That is, we can examine cases where the NP restrictor has members both in VP_1 and in VP_2, without any predetermination of truth or falsity.

GQT opened up new avenues of exploration, where for the first time we could ask questions like 'What is a possible natural language quantifier?', and, 'What properties, if any, are common to all natural language quantifiers?'

2 Notes - 08 Jan 2008

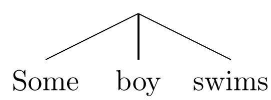
2.1 Generalized quantifiers, continued

What we call a generalized quantifier is the pairing of a quantifier and its NP restrictor. Take this sentence for example:

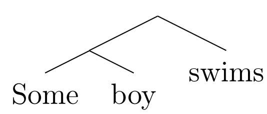
(1) Every boy swims.

Here, ‘Every’ denotes a relation, $\{\langle X,Y\rangle : X\subseteq Y\}$, where X is the set denoted by the NP restrictor, and Y is the set denoted by the predicate ‘swims’. For the Q ‘Every’, this is to ask whether every member of the set of boys is a member of the set of swimmers.

The ideal syntactic structure for our understanding of the quantifier as a relation would be something like the following:



With this, we could take the denotation of ‘boy’ and the denotation of ‘swims’ (two sets), form an ordered pair $\langle X, Y\rangle$, and ask if this pair is a member of R_{every}. Alas, syntactic evidence tells us the above does not correspond to the real structure of the sentence; the following is a more accurate depiction of the syntactic structure.

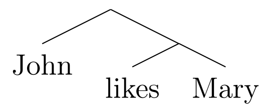


So, compositionally, we must first combine the restrictor with the denotation of the quantifier, so that the node immediately dominating ‘Some’ and ‘boy’ looks like this: $\{Z : \langle [\![\text{boy}]\!], Z\rangle \in R_{some}\}$. And herein lies the trouble: the thing that must combine with the predicate is a set of sets; for the sentence to be true, the predicate must be a member of the denotation of the *subject*, in direct contrast to the semantics we saw for proper names and predicates.

2.1.1 Note: Transitive verbs

Our analysis of quantifiers should not be very surprising, given what we know about the semantics of transitive verbs. In a sentence like (2), the denotation of the transitive verb combined with the denotation of Mary is a set of individuals, where we must ask if the subject is a member of that set.

(2)



In (2), the denotation of the node immediately dominating 'likes' and 'Mary' is the set $\{x : \langle x,m \rangle \in [\![\text{likes}]\!]\}$. The only difference here is that the second argument combines first with the relation 'likes', and then the first argument; the opposite order is true with quantificational constructions.

2.2 Universals

As was noted before, until GQT there were a number of questions about quantifiers that it hadn't even been possible to formulate, let alone begin to answer. One of the pressing questions was whether or not there was something universal about natural language quantifiers, i.e., if there were some properties common to all of them. And lo, a couple universals have been found, and are explored below.

2.2.1 Universal 1: Conservativity

Consider this expression, found to hold for all of the quantifiers we have seen so far:

$$\text{For all sets } X, Y,\ \mathrm{R}_\alpha(\mathrm{X,Y}) \equiv \mathrm{R}_\alpha(\mathrm{X,Y{\cap}X})$$

This equivalence means that if one side of the $\equiv$ is true, then the other side is true as well. That is, for all sets X and Y, if $\langle$X,Y$\rangle$ is a member of R_α (where α is a variable), then $\langle$X,Y$\cap$X$\rangle$ is also a member of R. You can see this intuitively with statements like, 'Every boy likes Mary and is a boy' or 'Some gardener waters the plants and is a gardener'. The hypothesis that this equivalence held for all natural language quantifiers (originally put forth by Barwise & Cooper), would be surprising if true - the gross majority of relations are *not* conservative.

To show that a relation has the property of being conservative means to *prove* that the relation has that property. In constructing a proof, one makes full use of all the relevant information at one's disposal (e.g. the definition of the quantifier in question, and the definition of the property one is claiming the quantifier possesses, and certain logical equivalences). It helps if one identifies at each stage of the proof just what one is trying to show. Also, note the overarching structure of the proof, as it is centered around proving an *equivalence*, which is equivalent to proving two if-then statements, first left to right, then right to left. The first step is to choose an arbitrary *instance* of X, Y, and if you prove it for an arbitrary instance, it will hold for any instance at all.

R_{every} is conservative. Proof:

$\Rightarrow$ Suppose that A, B are arbitrarily chosen sets such that $\langle A, B\rangle \in R_{every}$. From the definition of the quantifier, it follows that $A \subseteq B$. We know from logic that since $A \subseteq B$, it holds that $A \cap B = A$. This is equivalent to saying that $B \cap A = A$. Given that $A \subseteq A$, it holds by substitution that $A \subseteq B \cap A$. But this is exactly what we wanted to show: that $\langle A, B \cap A\rangle$ meets the condition on R_{every} – namely, that $A \subseteq B \cap A$. So $\langle A, B \cap A\rangle \in R_{every}$.

$\Leftarrow$ Suppose that $A, B \cap A$ are arbitrarily chosen sets such that $\langle A, B \cap A\rangle \in R_{every}$. So, by definition of the quantifier, $A \subseteq B \cap A$. By definition of the subset relation, every member of A is a member of $B \cap A$. In fact, as we saw above, $A = B \cap A$. And given this equation, it holds that $A \subseteq B$. So $\langle A, B\rangle \in R_{every}$.

Combining these two results, and since A, B were chosen arbitrarily, it holds for all sets X, Y that R_{every} is conservative.

Exercise: Try to construct a similar proof for the other quantifiers in (1.2.4). Also, is $\mathrm{R}_{hypothetical} = \{\langle \mathrm{X,Y}\rangle : \mathrm{a} \in \mathrm{X}\ \&\ \mathrm{b} \in \mathrm{Y}\}$ a possible natural language quantifier? That is, is it conservative?

Counterexample?: The case of 'only'

Barwise & Cooper initially admitted 'only' as a potential counterexample to conservativity as a universal property of natural language quantifiers, but quickly rejected it on the grounds that it does not behave like a determiner syntactically. For example, 'only' may pair with proper names ('Only John...' v. '*Some John...' etc.); it can modify the VP ('John only likes green ice cream' v. 'John *most likes ice cream'); and it may modify NPs with determiners ('Only those boys swim' v. '*Most those boys swim' or '*Every those boys swim').

Yet it does look like a quantifier, and it is assigned it a relational interpretation, i.e. $R_{only} = \{\langle X, Y \rangle : Y \subseteq X\}$). But if we try to construct a proof for conservativity, it fails; instead, we can easily construct an appropriate counterexample. To do so, we will demonstrate only that the right to left half of the equivalence doesn't hold, which is enough to say that 'only' is not conservative.

R_{only} *is not conservative:*

Let $D = \{a, b, c, d\}$

X (the set of girls) $= \{a, b, c\}$

Y (the set of partygoers) $= \{d\}$

Now, clearly none of the girls went to the party. Yet it is the case that $\langle X, Y \cap X \rangle \in R_{only}$: since $Y \cap X = \emptyset$ and the empty set is a subset of every set. That is, $Y \cap X \subseteq X$, and the condition on the relation is met. Yet it is not the case that $Y \subseteq X$, since $\{d\} \not\subseteq \{a, b, c\}$; therefore, $\langle X, Y \rangle \notin R_{only}$.

2.2.2 Universal 2: Permutability under Isomorphism

Consider the following equivalence:

For all sets X, Y, R_α(X,Y) $\equiv$ $R_\alpha(F_n(X), F_n(Y))$

[alternate characterization: For all functions F, R(F(X),F(Y)) $\equiv$ R(X,Y)]

Working through this, we require a background domain D, consisting of individuals; a set of all possible permutations on D ($\{f_1, f_2, ..., f_n\}$), where each f is a function mapping an individual of D to a (same or different) individual of D; and another set of functions, $\{F_1, F_2, ..., F_n\}$, which applies a particular f from the first set to subsets X, Y of D. A permutation function f is a *bijection*, that is, it maps each element of D to a *unique* element of D.

One may imagine this in the following way: if D $= \{a, b, c\}$, then when f assigns $a \mapsto b$, there are only two choices remaining for mapping b: either to a or c. If b is mapped to c, then c must be mapped to a, there are simply no options left. So a set, once the set permutation function has applied to it, has exactly the same cardinality as it had before the function was applied.

Now, if ⟨X,Y⟩ is a member of R_α (where α represents any Q), then any permutation of X and Y (so long as it is the same f_i applied to both sets; F_i applies f_i to X and f_i to Y) is also a member of R_α. This equivalence, too, holds of all the quantifiers we have seen so far. Again, to show that the property holds of a quantifier, we have to prove it; the following is a demonstration of a proof that permutability holds of the quantifier 'some'. Before proceeding, note the equation we discussed in class: namely, that $F(X \cap Y) = F(X) \cap F(Y)$.

R_{some} *is permutable. Proof:*

$\Rightarrow$ Suppose that F_k is an arbitrarily chosen set permutation function, and that A, B are arbitrarily chosen sets such that $\langle A, B\rangle \in R_{some}$. By definition of the quantifier, $|A \cap B| \neq \emptyset$. Since F_k is a bijection, it follows that $|A \cap B| = |F_k(A \cap B)|$. Given the equation stated above, it holds that $|F_k(A \cap B)| = |F_k(A) \cap F_k(B)|$. Now that we know $|A \cap B| = |F_k(A) \cap F_k(B)|$ and that $|A \cap B| \neq \emptyset$, it is obvious that $|F_k(A) \cap F_k(B)| \neq \emptyset$. So, by definition of R_{some}, $\langle F_k(A), F_k(B)\rangle \in R_{some}$.

$\Leftarrow$ Suppose that A, B are arbitrarily chosen sets and that F_k is an arbitrarily chosen set permutation function such that $\langle F_k(A), F_k(B)\rangle \in R_{some}$. So, by definition of the quantifier, $|F_k(A) \cap F_k(B)| \neq \emptyset$. By the equation noted above, $|F_k(A) \cap F_k(B)| = |F_k(A \cap B)|$. Again, since F_k is a bijection, it follows that $|F_k(A \cap B)| = |A \cap B|$. But this is exactly what we wanted to show: namely, that $|A \cap B| \neq \emptyset$. So, $\langle A, B\rangle \in R_{some}$.

Combining the two previous results, and since A, B, and F_k were chosen arbitrarily, it holds for all sets X, Y and all set permutation functions F_n that R_{some} is permutable.

Proving that this property holds for the other quantifiers we have defined should look very similar to this proof. Again, it didn't have to be the case that all natural language quantifiers should have this property; indeed, the vast majority of relations are *not* permutable. For example, this equivalence doesn't hold of $\mathrm{R}_{hypothetical}$ above. What's interesting here is that we now know that natural language quantifiers care about how many things are in a set, and what is shared between them, not their specific content.

One way of getting at the significance of this fact is through consideration of Britney Spears quantifiers. These quantifers are like $R_{hypothetical}$, in that they make reference to a specific individual in their definition – in this case, Britney Spears. Crucially, it could be any individual. By simply adding a condition of this type to the quantifiers we've already described, we create a class of possible quantifiers at least as large as those that actually occur in natural languages. Take the Britney Spears-isized quantifiers 'Some' and 'Every':

$$R_{BSsome} = \{\langle X, Y\rangle : \text{Britney Spears} \in X \ \& \ |X \cap Y| \neq \emptyset\}.$$

$$R_{BSevery} = \{\langle X, Y\rangle : \text{Britney Spears} \in X \ \& \ X \subseteq Y\}.$$

These quantifiers are not permutable (although they are conservative). For example, consider the following counterexample to the permutability of R_{BSsome}.

R_{BSsome} *is not permutable. Counterexample:*
Let $D = \{$Justin Timberlake, Britney Spears, Jonathan Rhys-Meyers$\}$
$[\![$girls that the boys like$]\!] = A = \{$Britney Spears$\}$
$[\![$danced$]\!] = B = \{$Justin Timberlake, Britney Spears$\}$

$$
\begin{array}{llll}
f: & \text{Britney Spears} & \mapsto & \text{Justin Timberlake} \\
 & \text{Justin Timberlake} & \mapsto & \text{Jonathan Rhys-Meyers} \\
 & \text{Jonathan Rhys-Meyers} & \mapsto & \text{Britney Spears} \\
F: & A & \mapsto & \{f(\text{Britney Spears})\} \\
 & & & (= \{\text{Justin Timberlake}\}) \\
 & B & \mapsto & \{f(\text{Justin Timberlake}), f(\text{Britney Spears})\} \\
 & & & (= \{\text{Jonathan Rhys-Meyers, Justin Timberlake}\})
\end{array}
$$

Consider the sentence, 'Some girl that the boys like danced'. Since Britney Spears $\in A$, and $|A \cap B| \neq \emptyset$, the condition on the quantifier is met. So, $\langle A, B \rangle \in R_{BSsome}$.

However, after the set permutation function applies, it is not the case that Britney Spears is a member of A, as the only member of that set is Justin Timberlake. Since this condition is false, it renders the compound statement false, even though it's true that that the intersection of A and B is non-empty. So $\langle F(A), F(B) \rangle \notin R_{BSsome}$.

The problem is that, as soon as a relation cares about the specific content of a set (and many, many do), permutability doesn't necessarily hold. Yet this property holds of all natural language quantifiers.

3 Notes - 10 Jan 2008

3.1 Universal 3: 'There is/are' constructions

A surprising syntactic fact found quite frequently across languages (though not always) is what kinds of quantifiers may follow 'There is/are' (crucially, this is the existential, not the locative, use of 'there'). Take the following example:

There is/are __ boy(s) in the garden.

Simple inspection should reveal that, in English, certain Q's may appear in the blank, and others not. This division forms the basis for the syntactic terms weak quantifiers (may appear after 'There is/are'; 'some', 'no', the numerals) and strong quantifiers (may not appear after 'There is/are'; 'every', 'most'). Given what we've seen so far with GQT, the explanation for the 'There is/are' facts may follow from the view of Q's as relations.

Consider some possible properties of relations:

symmetry	-	if R(x,y) = t then R(y,x) = t
transitivity	-	if R(x,y) = t & R(y,z) = t then R(x,z) = t
reflexivity	-	$\forall$x R(x,x) = t
asymmetry	-	if R(x,y) = t then R(y,x) = f
antisymmetry	-	if R(x,y) = t & x$\neq$y = t then R(y,x) = f

The quantifier 'Every', for example, is reflexive (and therefore not asymmetric), transitive, and antisymmetric (therefore not symmetric). The quantifier 'Most' has none of the above properties. So much for the 'strong' quantifiers. But what about 'some', 'no', and the numerals? Applying each of the definitions above to the 'weak' quantifiers, we come up with one property that they all share: symmetry.

With this discovery, a unifying property had been found that might help explain grammatical restrictions on quantifiers. Now whatever the account of the 'there is/are' phenomenon, this property must play a role as either a cause or consequent of that explanation.

Taken as a whole, our discussion of universals is meant to introduce GQT as a fruitful and interesting theory. We have begun to uncover the possibly innate limitations on what a natural language quantifier *can* be; by determining their properties, we are describing patterns in the mind. These interesting patterns and potential innate knowledge only became apparent after representing quantifiers as relations between sets.