Name (print): ____________________ **Student Number:** ____________________

Test #3 (/30)

Part A: Fill In The Blank/Multiple Choice (1 Mark Each

Question 1: Consider the following function: $f(x) = 3x^2 + x$. The average rate of change from x=0 to x=3 is:

a) 1 b) 28 c) 10 d) 30 e) 0

Question 2: Name any function whose derivative is $f'(x) = 2$ (ie what derives to make 2)

$f(x) = 2x$ or any function of the form $f(x) = 2x + c$

Question 3: What does the slope of the tangent tell you about the function at that point?

The slope of the tangent tells us if the function is increasing or decreasing at that point (or stationary).

Part B: Short Answer (Show all work)

Question 1: Determine the following limit: $\lim_{x\to\infty} \sqrt{x^2-1} - \sqrt{x^2+x}$ [4 Marks]

$$\lim_{x\to\infty} \sqrt{x^2-1} - \sqrt{x^2+x}\left(\frac{\sqrt{x^2-1}+\sqrt{x^2+x}}{\sqrt{x^2-1}+\sqrt{x^2+x}}\right)$$

$$\lim_{x\to\infty}\left(\frac{x^2-1-(x^2+x)}{\sqrt{x^2-1}+\sqrt{x^2+x}}\right)$$

$$\lim_{x\to\infty}\left(\frac{-1-x}{\sqrt{x^2-1}+\sqrt{x^2+x}}\right)$$

The highest power is x, so dividing by x gives us:

$$\lim_{x\to\infty}\left(\frac{-\frac{1}{x}-\frac{x}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}}\right)$$

$$\lim_{x\to\infty}\left(\frac{-\frac{1}{x}-1}{\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x}}}\right)$$

$$\left(\frac{0-1}{\sqrt{1-0}+\sqrt{1+0}}\right)$$

$$-\frac{1}{2}$$

[1 Mark for knowing the conjugate]
[1 Mark for simplifying the conjugate]
[1 Mark for dividing by the highest power]
[1 Mark for the final answer]

Question 2: From first principles **(do not use derivative rules)**, determine the derivative of the following function $f(x) = (x-1)^2 + 3$ [4 Marks]

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{(x+h-1)^2+3-[(x-1)^2+3]}{h}$$

$$\lim_{h\to 0}\frac{(x+h-1)(x+h-1)+3-[(x-1)(x-1)+3]}{h}$$

$$\lim_{h\to 0}\frac{x^2+xh-x+xh+h^2-h-x-h+1+3-[x^2-2x+1+3]}{h}$$

$$\lim_{h\to 0}\frac{x^2+2xh-2x+h^2-2h+4-x^2+2x-4}{h}$$

$$\lim_{h\to 0}\frac{2xh+h^2-2h}{h}$$

$$\lim_{h\to 0} 2x+h-2$$

$$2x-2$$

[1 Mark for knowing the definition]
[2 Marks for the correct expanding/simplification of the limit (deduct 0.5 for each minor error)]
[1 Mark for the final answer]

Question 3: Using the rules of differentiation, determine the derivatives of the following functions (do not expand and simplify):

$a)\ f(x) = 3\sqrt{x} + \frac{2}{x} - x^{-1.23} + 5$ [2 Marks]

$f(x) = 3x^{0.5} + 2x^{-1} - x^{-1.23} + 5$
$f'(x) = \frac{3}{2}x^{-0.5} - 2x^{-2} + 1.23x^{-2.23}$
[0.5 Marks for each correct derivative term]

$b)\ g(x) = \left(3e^x - Sin(x)\right)\left(4x^2 + 2Cos(x)\right)$ [3 Marks]

$g' = A'B + B'A$
$g'(x) = \left(3e^x - Cos(x)\right)(4x^2 + 2Cos(x))$
$+\left(3e^x - Sin(x)\right)(8x - 2Sin(x))$
[1 Mark knowing the product rule]
[2 Marks for the correct derivation (0.5 per expression)]

$c)\ i(x) = e^{(2x+3)^4}$ [3 Marks]

$i(x) = e^{[(2x+3)^4]}[(2x+3)^4]'$

$i(x) = e^{(2x+3)^4}(4([2x+3])^3)\ [2x+3]'$

$i(x) = 8e^{(2x+3)^4}(2x+3)^3$

[1 Mark for knowing chain rule]

[2 Marks for the correct derivation

(subtract 1 for each noticable error)]

$d)\ h(x) = \frac{Sin(x)-e^x}{Cos(x)}$ [3 Marks]

(for a bonus mark fully simplify the numerator)

$h' = \frac{T'B-B'T}{B^2}$ [1 Mark (or product with chain)]

$h'(x) = \frac{(Cos(x)-e^x)(Cos(x))-(Sin(x)-e^x)(-Sin(x))}{Cos^2(x)}$ [2 Marks]

$h'(x) = \frac{Cos^2(x)-e^xCos(x)+Sin^2(x)-Sin(x)e^x}{Cos^2(x)}$

$h'(x) = \frac{-e^xCos(x)-Sin(x)e^x+1}{Cos^2(x)}$ [1 Bonus Mark]

[1 Mark for knowing quotient rule (or product with chain)]

[2 Marks for correct derivative (subtract 1 for each noticable error)]

[1 Bonus if they expanded & used $\sin^2 x + \cos^2 x = 1$]

e) $j(x) = (x^2 Sin(x) + 3e^x)^4$ [3 Marks]

$j' = f'(g(x))g'(x)$
$j' = 4(x^2 Sin(x) + 3e^x)^3 (x^2 Sin(x) + 3e^x)'$
$j' = 4(x^2 Sin(x) + 3e^x)^3 (2x\, Sin(x) + x^2 Cos(x) + 3e^x)$

[1 Mark for knowing chain and product was in involved]
[1 Mark for the chain rule done properly]
[1 Mark for the product rule done properly]

Question 5: Determine the equation of the tangent line for the function $f(x) = 2e^{3x} + Cos(2x)$ at x=0.
[5 Marks]

$f'(x) = 6e^{3x} - 2Sin(2x)$
$f'(0) = 6(1) - 2(0)$
$f'(0) = 6$

$f(0) = 2(1) + 1 = 3$

$y = mx + b$
$3 = 6(0) + b$
$b = 3$
Thus the line is $y = 6x + 3$

[1 Mark for the correct derivative]
[1 Mark for knowing to find f'(0)]
[1 Mark for finding the y-value f(0)]
[1 Mark for knowing y=mx+b]
[1 Mark for finding the equation y=6x+3]