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MAT1322 -- Winter 2016, Assignment 3
Praiyon Kumarasamy, 3/3/16 at 3:07 PM

Question 1: *Score 0/1*

Your response

The series

$$\sum_{n=2}^{\infty} \frac{3^{2n}}{2^{3n-1}}$$

is a geometric series.

(i) What is its initial term a ?

a = 3 (0%)

(ii) What is the common ratio r ?

r = No answer (0%)

(iii) What is its sum S ? Input 333 if the series is divergent.

S = No answer (0%)

Correct response

The series

$$\sum_{n=2}^{\infty} \frac{3^{2n}}{2^{3n-1}}$$

is a geometric series.

(i) What is its initial term a ?

a = 81/32

(ii) What is the common ratio r ?

r = 9/8

(iii) What is its sum S ? Input 333 if the series is divergent.

S = 333

Incorrect

Total grade: 0.0×1/3 + 0.0×1/3 + 0.0×1/3 = 0% + 0% + 0%

Comment:

The general term of the series is

$$\frac{3^{2n}}{2^{3n-1}} = 2\left(\frac{3^2}{2^3}\right)^n = 2\,(9/8)^n.$$

Hence we have a geometric series with common ratio

$r = 9/8$.

To determine the first term a , we let $n = 2$

in the expression for the general term of the series. Thus we obtain

$a = 2\,(9/8)^2 = 81/32$.

Since $\mid r \mid \geq 1$, the series is divergent.

Question 2: *Score 0/1*

Your response

Find the sum of the series

$$10 + \frac{30}{5} + \frac{90}{25} + \frac{270}{125} + \cdots$$

if it is convergent. Else, input 333.

Correct response

Find the sum of the series

$$10 + \frac{30}{5} + \frac{90}{25} + \frac{270}{125} + \cdots$$

if it is convergent. Else, input 333.

Incorrect

Sum = No answer (0%)

Sum = 25

Total grade: 0.0×1/1 = 0%

Comment:

Here we have a geometric series with initial term

$a = 10$ and common ratio $r = \frac{3}{5}$.

Since $\mid r \mid < 1$, the series is convergent, and its sum is

$$10 + \frac{30}{5} + \frac{90}{25} + \frac{270}{125} + \cdots = \frac{10}{1 - (\frac{3}{5})} = \frac{10}{\frac{2}{5}} = 25$$

Question 3: *Score 0/1*

Your response

Show that the series

$$\sum_{n=1}^{\infty} \frac{6}{n^2 + 7n + 12}$$

is a telescopic series. Calculate its sum and give an exact answer.

Sum = No answer (0%)

Correct response

Show that the series

$$\sum_{n=1}^{\infty} \frac{6}{n^2 + 7n + 12}$$

is a telescopic series. Calculate its sum and give an exact answer.

Sum = 3/2

Incorrect

Total grade: 0.0×1/1 = 0%

Comment:

We see here that $n^2 + 7n + 12 = (n + 3)(n + 4)$.

This allows us to use partial-fraction decomposition on the general term in order to obtain

$$\frac{6}{n^2 + 7n + 12} = \frac{6((n + 4) - (n + 3))}{(n + 3)(n + 4)}$$

$$= \frac{6}{n + 3} - \frac{6}{n + 4}$$

Therefore, the partial sum of the k first terms of the series is

$$s_k = \sum_{n=1}^{k} \frac{6}{n^2 + 7n + 12}$$

$$= \sum_{n=1}^{k} \frac{6}{n + 3} - \sum_{n=1}^{k} \frac{6}{n + 4}$$

$$= \left(\frac{6}{4} + \frac{6}{5} + \cdots + \frac{6}{k + 3}\right) - \left(\frac{6}{5} + \cdots + \frac{6}{k + 3} + \frac{6}{k + 4}\right)$$

$$= \frac{6}{4} - \frac{6}{k + 4}$$

This clearly shows that the series is telescopic. Finally, its sum is

$$\begin{aligned} s &= \lim_{k\to\infty} s_k \\ &= \lim_{k\to\infty}\left(\frac{6}{4}-\frac{6}{k+4}\right) \\ &= 3/2 \end{aligned}$$

Question 4: *Score 0/1*

True or False? Choose true only if both the argument and answer is correct.

The series

$$\sum_{n=1}^{\infty} \frac{5}{\sqrt{n^2+4}}$$

is divergent because

$$\frac{5}{\sqrt{n^2+4}} \geq \frac{5}{n}$$

Incorrect

for all $n \geq 1$ and

$$\sum_{n=1}^{\infty} \frac{5}{n} = 5\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

(harmonic series).

Your Answer:
Correct Answer: False

Comment: The inequality

$$\frac{5}{\sqrt{n^2+4}} \geq \frac{5}{n}$$

is not true for any value of $n \geq 1$.
However, for all $n \geq 1$, we have $n^2+4 \leq (1+4)n^2 = 5n^2$, hence $\sqrt{n^2+4} \leq \sqrt{5}n$, and

$$\frac{5}{\sqrt{n^2+4}} \geq \frac{5}{\sqrt{5}\,n}.$$

Since

$$\sum_{n=1}^{\infty} \frac{5}{\sqrt{5}n} = \frac{5}{\sqrt{5}}\sum_{n=1}^{\infty} \frac{1}{n} = \infty,$$

the series is divergent by virtue of the comparison test.

Question 5: *Score 0/1*

True or False? Choose true only if both the argument and answer is correct.

The series

$$\sum_{n=1}^{\infty} \frac{1}{4n^5+3n+2}$$

is divergent because

$$\frac{1}{4n^5+3n+2} \geq \frac{1}{3n}$$

Incorrect

for all $n \geq 1$ and

$$\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

(harmonic series).

Your Answer:

Correct Answer: False

Comment: We have on the contrary

$$\frac{1}{4n^5+3n+2} \leq \frac{1}{3n}$$

for all $n \geq 1$ but this information is useless since

$$\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n} = \infty.$$

We must use rather the estimation

$$0 \leq \frac{1}{4n^5+3n+2} \leq \frac{1}{4n^5}$$

valid for all $n \geq 1$.

Since

$$\sum_{n=1}^{\infty} \frac{1}{4n^5} = \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^5} < \infty$$

(series with general term $1 / n^p$ with $p = 5 > 1$), we conclude by the comparison test that the given series is convergent.

Question 6: *Score 0/1*

True or False? Choose true only if both the argument and answer is correct.

The series

$$\sum_{n=1}^{\infty} \frac{4e^{7n}}{e^{4n}+8}$$

is convergent because

$$\frac{4e^{7n}}{e^{4n}+8} \leq 4e^{3n}$$

Incorrect

for all $n \geq 1$ and

$$\sum_{n=1}^{\infty} 4e^{3n} = \frac{4e^3}{1-e^3} < \infty$$

(geometric series with common ratio e^3).

Your Answer:

Correct Answer: False

Comment: We have

$$\sum_{n=1}^{\infty} 4e^{3n} = \infty$$

which is a geometric series with common ratio $e^3 > 1$. In fact, the series is divergent because

$$\frac{4e^{7n}}{e^{4n}+8} \geq \frac{4e^{7n}}{(1+8)e^{4n}} = \frac{4}{9}e^{3n}$$

and

$$\sum_{n=1}^{\infty} \frac{4}{9}e^{3n} = \frac{4}{9}\sum_{n=1}^{\infty} e^{3n} = +\infty.$$

Question 7: *Score 1/1*

True or False? Choose true only if both the argument and answer is correct.

The series

$$\sum_{n=1}^{\infty} \frac{7+\cos^2(n)}{n^{5/2}+4}$$

is convergent because

$$0 \leq \frac{7+\cos^2(n)}{n^{5/2}+4} \leq \frac{8}{n^{5/2}+4} \leq \frac{8}{n^{5/2}}$$

Correct

for all $n \geq 1$ and

$$\sum_{n=1}^{\infty} \frac{8}{n^{5/2}} = 8\sum_{n=1}^{\infty} \frac{1}{n^{5/2}} < \infty.$$

(series with general term $1 / n^p$ with $p = 5 / 2 > 1$).

Your Answer: True

Comment: