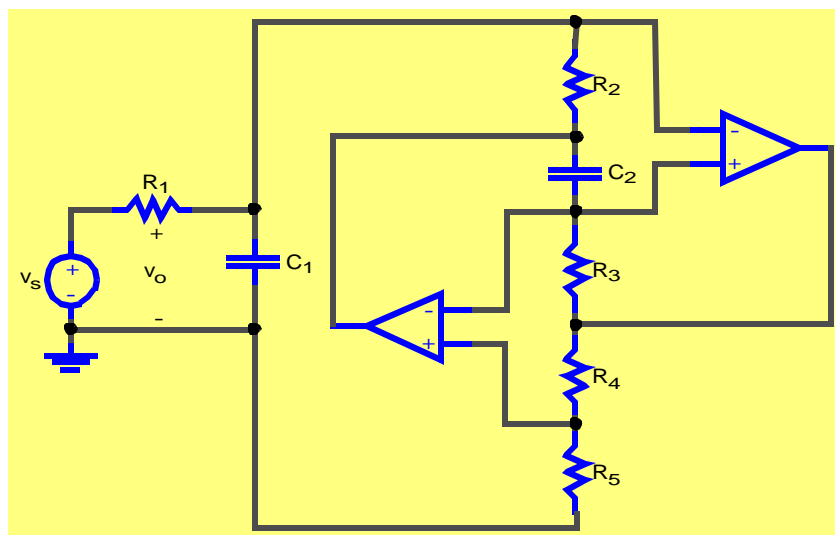


## CIRCUIT THEORY

### ELG-2130



### Laboratory Manual

*D. Makrakis*

2004

*(from previous versions by O. Yang, N. D. Georganas and P. Payeur)*

---

## 1 Acknowledgements

---

This document has been prepared from previous versions by professors Oliver Yang, Nicolas D. Georganas and Pierre Payeur. Most of the credit on the content of this laboratory manual must be given to them.

## 2 General objectives of the laboratory experiments

---

- To learn the theory, the properties and the design of linear electrical circuits through experiments, analysis and discussion of their results.
- To learn various techniques of measuring electrical parameters, such as voltage, current, power and resistance.
- To become familiar with the operation and the characteristics of various electrical measurement instruments.
- To learn how to design circuits performing certain desired functions, such as signal filtering.

## 3 Laboratory preparation

---

- Even though the experiments are conducted on a team basis, EACH student is expected to understand the goals and the procedure of each experiment to be performed.
- EACH student MUST read the instructions and reference material before showing up in the lab.
- EACH student MUST have finished all the preparation questions, drawn any required diagrams of connections and done any calculations before coming to the lab.

## 4 Accuracy and precision

---

Each student is expected to familiarize himself/herself with the basic concepts of accuracy and precision that are discussed in the introductory notes of this lab manual. Each experiment must provide adequate accuracy and precision, to the level that is required for analysis and investigation. Statements of accuracy estimates should accompany measurements and calculations where required. Note that inaccurate results may indicate an effect, which is not present at all. Excessive accuracy, on the other hand, is a waste of time and effort.

## 5 Laboratory reports

---

The format specified below must be respected.

- One report per experiment per group.
- EACH student within a group MUST write at least one report in order to get the grades at the end of the semester.
- Use 8.5"x11" paper only.
- Even though lab reports might be handwritten, clarity and neatness are required. Marks can be lost for reports that are not presented in a convenient way.
- Groups MUST use the official title page provided for each experiment. Every field must be filled with the complete identification (complete name and student number) for each member of the group. Incomplete identification of any member will lead to a mark equal to 0 and will not be reviewed.
- The lab reports are normally due 2 weeks after the last session of the lab.

---

## 6 How to write a laboratory report

---

In addition to the documentation of measurement results, one must allow the other people to make use of the results later, and therefore add an adequate amount of information to make the report comprehensive. For this reason, a laboratory report must be prepared and written carefully.

During lab measurements it is necessary to indicate precisely the conditions of the experiment, so that if the necessity arises to duplicate some measurements sometime later, one is sure to have the same experimental conditions. For this reason, it is compulsory to note the model and serial numbers of the equipments used. It might also help to draw a block diagram with all the setup connections. The following is a list of what a lab report should contain:

- Title
- Names (complete) and student numbers.
- Date
- Objectives
- List of instruments (models with serial numbers)
- Circuit diagrams
- Methods and/or measurement techniques
- Measurements results (where possible, tabulate computed values from theory beside the measured values)
- Graphs (draw your graphs carefully, identify them with a title, legends and axes captions)
- Error analysis
- Discussion of results (include general observations from you tables and graphs, comparison with theoretical values, explanation of physical behaviors and of any abnormalities)
- Conclusion.

## 7 Safety regulations in the measurement laboratory

---

You must observe the following general safety precautions in your measurement laboratory:

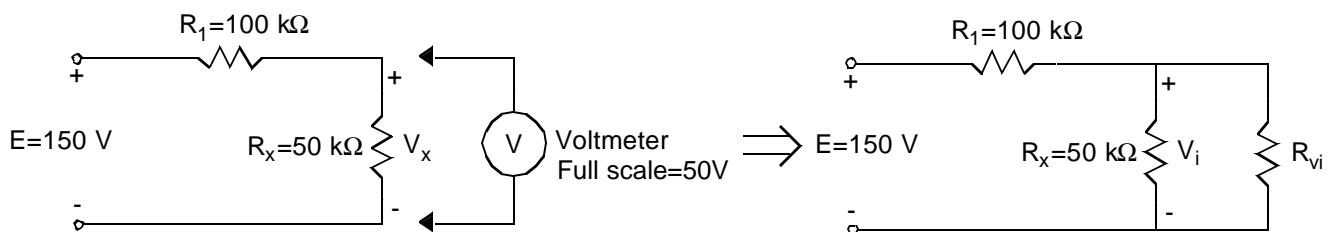
1. The bench is for experimental equipment only. Do not leave coats, sweaters, briefcases and other irrelevant paraphernalia on the bench.
2. Keep your workspace tidy and set aside all equipment and leads that are not actually part of the test being conducted.
3. Move around slowly to avoid knocking things over, and never try to catch a falling object.
4. Make sure that there are at least three persons in the lab all the time.
5. Use the proper power cord and correct fuse. Replace the power cord if it is cracked or broken or has any pins missing. Make sure that all devices are using a three-wire power cable when powered from a 120V outlet.
6. Never try to work with a high voltage ( $> 15\text{V}$ ) circuit when tired. Some circuit elements such as capacitors and inductors may produce high voltages even when the power supply voltage is low.
7. Never handle “live” equipment when hands, feet, or body are wet or perspiring, when standing on a wet floor or on a metal surface.
8. While manipulating a circuit with an applied voltage or current, put one hand in your pocket or behind your back.

9. Before touching a non-insulated wire, disconnect the voltage and ground the high voltage points.
10. If it is necessary to touch equipment (e.g. when checking for overheated motors): i) first check for electrical potential with a voltmeter, ii) always use the back of the hand to avoid freezing to the conductor in the event of accidental shock.
11. Before making a connection to the power supplies, check and make sure that the circuits are properly connected, that the sources are the ones intended, that the polarities are correct, and that the meters chosen have adequate ranges to register the possible maxima.
12. Observe the polarity of the meter probes so that the pointers do not deflect the wrong way. Note that if any component is improperly chosen, it may be destroyed before the power can be shut off again.
13. Do not remove the covers and panels of the instruments.
14. If faulty equipment operation is suspected, or if the equipment has been subjected to any damage, have the equipment operation verified by authorized personnel.
15. Identify the emergency power cut-off switches in your lab.
16. No metallic jewelry (no rings, watches, metal chains or bracelets) shall be worn when working on live electrical equipment.

## 8 Inaccuracy due to device loading

A voltmeter is always connected in parallel with the element of which we want to measure the voltage drop. The connection changes the circuit because the voltmeter now provides another path for the current to flow. This interaction between the circuit and the voltmeter constitutes one type of *loading effect* on which the sensitivity of the voltmeter would depend.

**FIGURE 1** Loading effect of a voltmeter.



Let us consider the circuit of figure 1 where an internal voltage source of  $E = 150 \text{ V}$  is applied to two resistors in series. We want to investigate what happens to the voltage drop  $V_x$  across the resistance  $R_x = 50 \text{ k}\Omega$  when two voltmeters, one having a sensitivity of  $S_1 = 1 \text{ k}\Omega/\text{V}$  and the other having a sensitivity of  $S_2 = 20 \text{ k}\Omega/\text{V}$ , are used to measure  $V_x$ .

We shall first determine the reading of each instrument and their error relative to the voltage drop when the circuit is undisturbed (i.e. no loading effect). By a voltage divider equation, the voltage without loading is:

$$V_x = \frac{50k}{100k + 50k} \cdot 150 = 50 \text{ V}$$

Next we determine the internal resistance  $R_{vi}$  of each voltmeter assuming a setting with full-scale deflection is chosen. From the product of the sensitivity and the full scale deflection (the range), we have:

---

---

$$R_{v1} = \frac{1 \text{ k}\Omega}{\text{V}} \cdot 50 \text{ V} = 50 \text{ k}\Omega$$

$$R_{v2} = \frac{20 \text{ k}\Omega}{\text{V}} \cdot 50 \text{ V} = 1 \text{ M}\Omega$$

With the presence of the voltmeter in parallel with the 50 k $\Omega$  resistor, the new voltage drop, due to loading effect from the voltmeter, becomes:

$$V_i = \frac{R_{ei} \cdot E}{R_1 + R_{ei}} \quad i = 1, 2$$

where  $R_{ei}$  is the equivalent resistance of the two resistors  $R_x$  and  $R_{vi}$  in parallel, and  $R_{ei} = \frac{R_{vi}R_x}{R_{vi} + R_x}$ . Applying the equation for  $V_i$ , we obtain:

$$V_1 = \frac{25\text{k} \cdot 150}{100\text{k} + 25\text{k}} = 30 \text{ V}$$

$$V_2 = \frac{47.6\text{k} \cdot 150}{100\text{k} + 47.6\text{k}} = 48.4 \text{ V}$$

which are different from the original value. Thus, the voltage has changed due to the loading effect of the voltmeter that creates another path for the current to circulate outside of the 50 k $\Omega$  resistor. The relative error for each voltmeter is then:

$$e_1 = \frac{50\text{V} \angle 30\text{V}}{50\text{V}} = 40\%$$

$$e_2 = \frac{50\text{V} \angle 48.4\text{V}}{50\text{V}} = 3.2\%$$

**LABORATORY 1**

# **DIRECT CURRENT MEASUREMENTS AND RESISTIVE NETWORKS**

## **1.1 Objectives**

---

1. To become familiar with the following measurement devices: ammeter, voltmeter, ohmmeter.
2. To learn typical errors in measurements and their correction methods.
3. To become familiar with the behavior of potentiometers and their influence on circuit parameters.
4. To verify experimentally the Kirchhoff's laws.
5. To verify experimentally the design concept of a voltage and current dividers.

## **1.2 Preparation**

---

1. Read and understand the introductory notes on measurements and their errors provided in the previous pages.
2. Read and understand the course notes on resistive circuits.
3. Study and understand the color code of resistors given at the end of this lab.
4. Read and understand the experiment procedure below.
5. Answer the preparation questions.

---

## Objectives

---

### 1.3 Parts and equipment

---

- 1 voltage/current source
- 2 multimeters
- 1 voltmeter
- 1 ammeter
- 6 resistors: one each of  $470\ \Omega$ ,  $4.7\ \text{k}\Omega$ ,  $47\ \text{k}\Omega$ ,  $75\ \Omega$ ,  $47\ \Omega$  and  $18\ \Omega$ .
- 1 potentiometer (variable resistor).

### 1.4 Notes

---

When placing a meter in a circuit, you should remember the following rules:

- Always connect a voltmeter in parallel to the circuit to be measured.
- Always connect an ammeter in series with the circuit to be measured.
- Use the correct terminal polarity so that the pointer does not deflect the wrong way.
- When using a meter with a scale selection, start at the highest possible scale and then decrease the setting (increase the sensitivity) to achieve as close as possible a full scale deflection; this will give the minimum relative instrumental error.
- Take the necessary steps to correct the systematic error of the instrument when required.

### 1.5 Measurement of a resistance

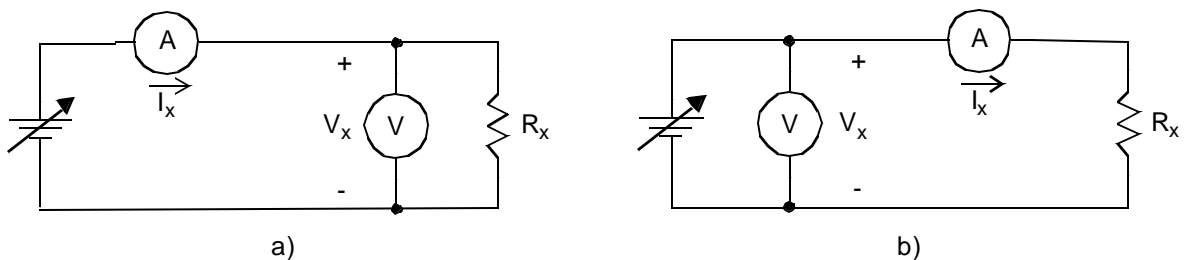
---

#### PREPARATION QUESTION:

Consider the circuit of figure 2, for  $R_x=470\ \Omega$ . Determine the maximum value of the source voltage so that the power dissipated in the resistor does not exceed the rating (use  $250\ \text{mW}$ ) given by the manufacturer.

---

**FIGURE 2** Measurement of a resistance.



#### EXPERIMENT STEPS:

1. Familiarize yourselves with the ammeter and the voltmeter by measuring power sources with known values.
2. Set up the circuit of figure 2a with  $R_x=470\ \Omega$ . Use a voltage source with a value that is slightly lower ( $\sim 2$  Volts less) than the value you found in the preparation question. Take a habit of using the red wires for the positive voltage and the black ones for the reference (ground). This will help you track down connection errors in more complicated circuits.

---

## Objectives

---

Note: To avoid blowing the fuses frequently, it is recommended to use a digital meter for voltage measurements and an analog meter for current measurements.

- Obtain a proper range of the voltmeter and the ammeter to give the largest deflexion. Take and record the readings and the chosen range of the voltmeter and the ammeter. Write these values in table 1 below.
- Repeat step 3 for  $R_x=4.7\text{ k}\Omega$  and  $47\text{ k}\Omega$ . Use the same voltage every time, but decrease the source voltage to zero before each repetition.
- Repeat steps 3 and 4 with the circuit shown in figure 2b.
- For both cases (figures 2a and 2b) calculate the corrected values of the different resistors by taking into account the current flowing through the voltmeter and the voltage drop across the ammeter.
- Measure the resistors with the analog ohmeter. Use a range that gives you the largest deflection.
- Evaluate the errors for the three methods (figures 2a, 2b and ohmeter), and compare the three methods with respect to the best one.

---

**TABLE 1** Results from the measurement of resistances in circuit 4a.

Nominal value $R_x$	Measured value $V_x$ [V]	Measuring range [V/div]	Measured value $I_x$ [mA]	Measuring range [mA/div]	Ohmeter value $R_o'$ [ $\Omega$ ]
470 $\Omega$					
4.7 k $\Omega$					
47 k $\Omega$					

---

**TABLE 1** Results from the measurement of resistances in circuit 4b.

Nominal value $R_x$	Measured value $V_x$ [V]	Measuring range [V/div]	Measured value $I_x$ [mA]	Measuring range [mA/div]	Ohmeter value $R_o$ [ $\Omega$ ]
470 $\Omega$					
4.7 k $\Omega$					
47 k $\Omega$					

### QUESTIONS:

- What is a multimeter?
- Between the two circuits shown in figure 2, which one is more appropriate for measuring big resistance values and for small resistance values? Explain in terms of the loading effects of the meters.
- What is the power dissipated in  $R_x=4.7\text{ k}\Omega$  and  $47\text{ k}\Omega$ ?

## 1.6 Potentiometer

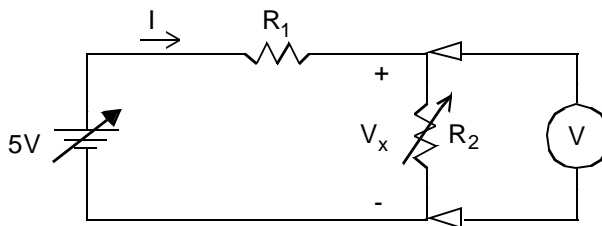
---

A potentiometer is a resistor whose resistance value can be adjusted within a given range. This device is widely used to fine-tune electronic circuits or as control input such as the volume button on a TV or a radio receiver. This experiment will let you observe the behavior of a potentiometer in a circuit and the effect it has on the voltage and the current.

---

**Objectives**

---

**FIGURE 3** A resistive circuit containing a potentiometer.**EXPERIMENT STEPS:**

1. Set up the circuit of figure 3 with  $R_1=47\text{ k}\Omega$  and  $R_2$  as a potentiometer.
2. For different values of  $R_2$  from its minimum to its maximum, measure the voltage  $V_x$ . Applying the Ohm's law, calculate the current  $I$  for each value of  $R_2$ . Fill in the table 2 and draw a plot of  $V_x$  and  $I$  as a function of  $R_x$ .  
Note: For each setting of the potentiometer,  $R_2$ , make sure to measure the exact value of the resistance with the ohmmeter. Unplug the potentiometer from the circuit to make this measurement.
3. Discuss your results from the graphs.

**TABLE 2** Results from the measurements on a potentiometer.

$R_2$ [W]	$V_x$ [V]	$I$ [A]

**QUESTIONS:**

1. Draw a plot of  $V_x$  as a function of  $I$ . Is this curve linear or not?
2. Determine what is the effect of the resistor  $R_1$  and of the potentiometer  $R_2$  on the relationship between  $V_x$  and  $I$ .

**1.7 Kirchhoff's laws**

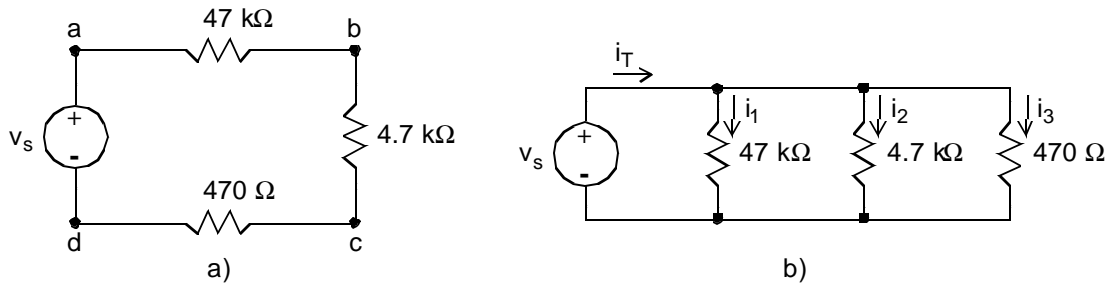
The power balance equation says that the instantaneous power supplied by active elements of the circuit must be equal to the instantaneous power absorbed by the passive elements of the circuit.

---

## Objectives

---

**FIGURE 4** Series and parallel circuits to verify Kirchhoff's laws.



### PREPARATION QUESTIONS:

1. Using a voltage divider, calculate the voltage drop across each resistor of figure 4a.
2. Using a current divider, calculate the current circulating through each resistor of figure 4b.

### EXPERIMENT STEPS:

1. Set up the circuit of figure 4a. Adjust  $V_s$  to 5V with an analog voltmeter and leave it connected.
2. Use a digital voltmeter to measure the voltages  $V_{ab}$ ,  $V_{bc}$ ,  $V_{cd}$  and  $V_{ad}$ . While making each measurement, observe any change in the reading of the first meter.
3. Set up the circuit shown in figure 4b with the same source  $V_s$ .
4. Use an analog ammeter to measure  $I_T$  and leave it connected for the remainder of the experiment.
5. Measure  $I_1$ ,  $I_2$  and  $I_3$ . While making each measurement, observe any change in the reading of the first meter.

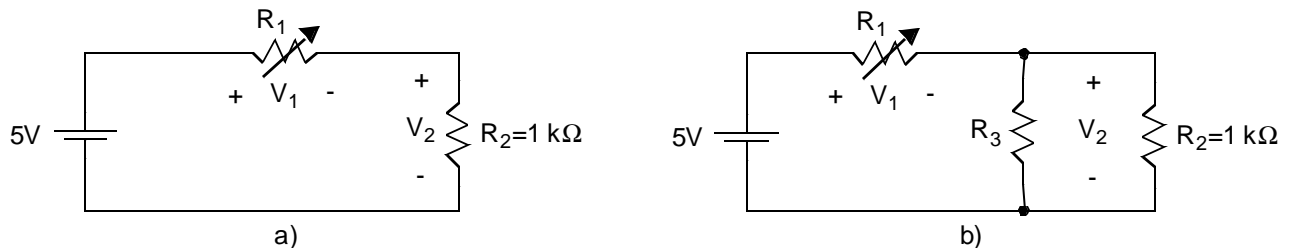
### QUESTION:

1. Do the measurements verify Kirchhoff's Voltage Law and Current Law respectively? Justify and explain any discrepancies from theory.

## 1.8 Voltage divider

---

**FIGURE 5** Circuits on which to verify the voltage divider concept.



### PREPARATION QUESTIONS:

For these questions, assume the internal resistance of the voltmeter to be infinite and that of the battery to be negligible.

1. Determine a value for  $R_1$  in figure 5a such that  $V_2 = 1.0$  volt.

---

2. Determine  $V_2$  in figure 5b for  $R_2=1\text{ k}\Omega$ ,  $R_3=10\text{ k}\Omega$  and the value of  $R_1$  found in question 1.

EXPERIMENT STEPS:

1. Set up the circuit in figure 5a.
2. Verify by measurement the design of the first preparation question. Discuss and explain any discrepancies.
3. Set up the circuit of figure 5b, where  $R_2$  is loaded by a parallel resistor  $R_3$ .
4. Verify by measurement the result of the second preparation question.

QUESTION:

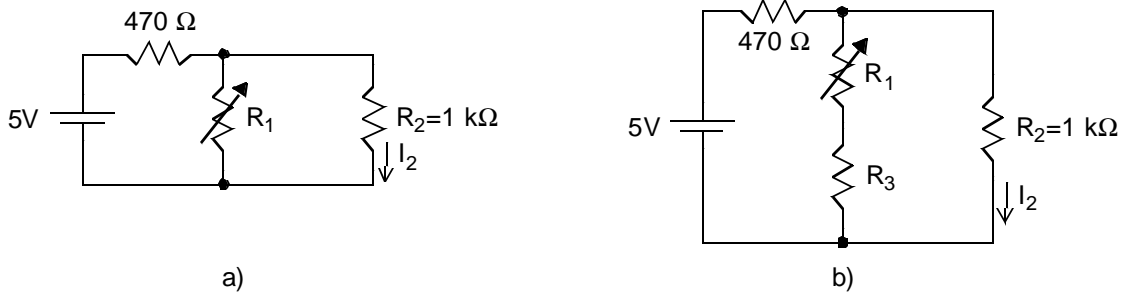
1. Comment on the effects of  $R_3$  on the voltage divider as it ranges from 0 to  $\infty$ .

---

## 1.9 Current divider

---

**FIGURE 6** Circuits on which to verify the current divider concept.



PREPARATION QUESTIONS:

For these questions, assume the internal resistance of the voltmeter to be infinite and that of the battery to be negligible.

1. Determine a value for  $R_1$  in figure 6a such that  $I_2=3\text{ mA}$ .
2. Determine  $I_2$  in figure 6b for  $R_2=1\text{ k}\Omega$ ,  $R_3=10\text{ k}\Omega$  and the value of  $R_1$  found in question 1.

EXPERIMENT STEPS:

1. Set up the circuit in figure 6a.
2. Verify by measurement the design of the first preparation question. Discuss and explain any discrepancies.
3. Set up the circuit of figure 6b where  $R_1$  is loaded by a resistor  $R_3$  in series.
4. Verify by measurement the result of the second preparation question.

QUESTION:

1. Comment on the effects of  $R_3$  on the current divider as it ranges from 0 to  $\infty$  .

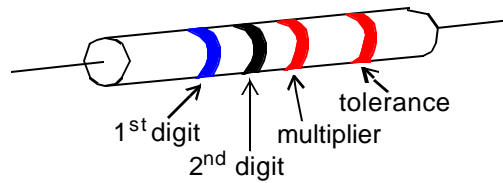
**1.10 Annexe: Resistor color codes**

---

In order to indicate the resistance value of a resistor, a standard color code is usually painted on the device. Each strip has a specific signification and the color refers to a numerical value.

---

**FIGURE 7** Significance of each strip on a resistor color code.



---

**FIGURE 8** Resistor color code.

color	digit	multiplier	tolerance
silver		0.01	10%
gold		0.1	5%
black	0	1	
brown	1	10	
red	2	$10^2$	2%
orange	3	$10^3$	
yellow	4	$10^4$	
green	5	$10^5$	
blue	6	$10^6$	
violet	7	$10^7$	
gray	8	$10^8$	
white	9		
none			20%

For example, a resistor having successively the following color strip painted on the device: [yellow, violet, orange, gold] has a resistor value of:

$$R = (4, 7, \times 10^3, 5\%) = 47 \times 10^3 \pm 5\% = 47 \text{ k}\Omega \pm 5\%$$

**SITE**  
**University of Ottawa**

**ELG-2130**  
**Circuit Theory**

***Laboratory Report # 1***

***Direct Current Measurements and Resistive Networks***

Presented to:

Prof. \_\_\_\_\_

By:

Author

Name	Student #

Other team members

Name	Student #

Date: \_\_\_\_\_

## LABORATORY 2      OSCILLOSCOPE AND CIRCUIT THEOREMS

---

### 2.1 Objectives

1. To introduce the operation of an oscilloscope as a measuring instrument.
2. To introduce the functions of a function generator as a signal source.
3. To verify experimentally the Thévenin equivalent circuit.
4. To verify experimentally the maximum power transfer property.

---

### 2.2 Preparation

1. Read and understand the course notes on circuit theorems.
2. Read and understand the experiment procedure below.
3. Answer the preparation questions.

---

### 2.3 Parts and equipment

- 1 voltage source
- 1 multimeter
- 1 2-channel oscilloscope

---

## Magnitude measurement

---

- 1 function generator
- 1 resistance box (must have a selection of  $10\Omega$ )
- 1  $\mu\text{F}$  capacitor.
- resistors:  $470\Omega$ ,  $500\Omega$ ,  $1\text{k}\Omega$ .

### 2.4 Magnitude measurement

---

The oscilloscope makes possible the direct measurement of the magnitude (peak value) of any periodic signal waveforms. Most of the other AC instruments, including the digital meters, indicate effective values (RMS).

1. Set up a sinusoid with a frequency and magnitude of your choice.
2. Measure the peak-to-peak value.
3. Measure the voltage of this signal using a voltmeter.
4. Comment and relate the two measured voltage values.
5. Repeat the same steps for a triangular waveform.

### 2.5 Phase measurement

---

Given two sinusoidal signals of the same frequency that are not synchronised in time as shown in figure 9.

$$v_1 = V_1 \cos(\omega t)$$

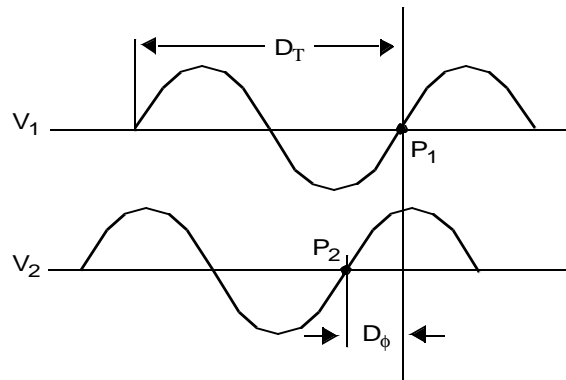
$$v_2 = V_2 \cos(\omega t + \phi)$$

where  $\omega = 2\pi f$ , and  $\phi$  is their phase difference that corresponds to the time shift between the two signals.

---

**FIGURE 9**

Two sinusoidal signals that are not in phase.



In order to measure the phase,  $\phi$ , one can apply the following procedure. Using an oscilloscope, apply the signals  $v_1$  and  $v_2$  to each of the two inputs  $Y_A$  and  $Y_B$ . Adjust the two displays to have the same time base. Let  $D_\phi$  and  $D_T$  be the distances measured on the screen of the oscilloscope as shown in figure 9. Then,

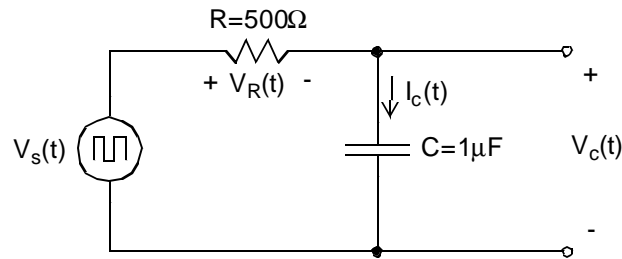
$$\phi_o = 360^\circ \left| \frac{D_\phi}{D_T} \right|$$

If  $P_2$  is on the left of  $P_1$ , the phase of  $v_2$  is positive with respect to  $v_1$ , i.e.  $\phi = \phi_o$ . Otherwise, the phase is negative, i.e.  $\phi = \angle\phi_o$ .

**EXPERIMENT STEPS:**

1. Set up the RC circuit shown below. Use a 1 V peak offset square wave (i.e. with the minimum value = 0 V) for  $V_s(t)$ . Set the frequency to 100 Hz.

**FIGURE 10** RC circuit for phase measurement.



2. Display  $V_s(t)$  and  $V_c(t)$  simultaneously on the oscilloscope. Then observe their phase difference.
3. Measure the phase difference applying the procedure described above.

**2.6 Thévenin equivalent circuit**

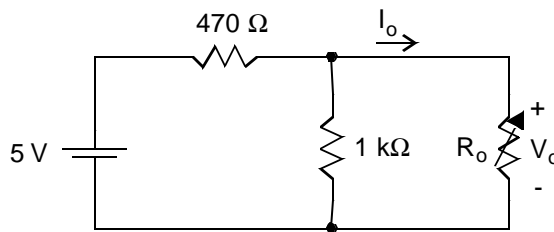
**PREPARATION QUESTIONS:**

1. Determine a value of  $R_o$  in figure 11, in order to obtain  $V_o = 2.02V$ .
2. Apply the Thévenin's theorem to determine the internal source resistance  $R_{th}$  as seen by the load  $R_o$ .
3. Determine the value of  $R_o$  that will make the maximum power to be dissipated in it.

**EXPERIMENT STEPS:**

1. Set up the circuit of figure 11. Use a variable resistance box to implement  $R_o$ .

**FIGURE 11** Application of the Thévenin's theorem.



2. Adjust  $R_o$  to obtain  $V_o = 2.02 V$ . Check the meter reading accurately. Compare the actual value of  $R_o$  with that of the preparation question.
3. Select a range of different  $R_o$  values. For each  $R_o$  setting, measure the corresponding voltage  $V_o$  and current  $I_o$ .
4. Plot  $V_o$  versus  $I_o$ .
5. Determine from the graph the equivalent Thévenin voltage ( $V_{th}$ ) and resistance ( $R_{th}$ ). Draw the Thévenin's equivalent circuit and compare it with your preparation. Justify any discrepancies.
6. From the set of measurements obtained in part 3, plot the power dissipated in the resistor versus the resistance  $R_o$ .

- 
7. Determine the value of  $R_o$  that gives the largest power dissipation. Does this value agree with your design? Justify your observations.

## 2.7 Superposition Theorem

---

1. At the circuit of figure 11, measure the voltage  $V_o$ , current  $I_o$ , as well as the voltages across and currents passing through the resistors of  $470\ \Omega$  and  $1k\Omega$
2. Leave the circuit of figure 11 as is and replace the 5 V source with a 10 V one.
3. Measure all voltages and currents measured in (1). Compare their value with the value they had when the 5V source was used.
4. Add another 5 V source in series with the 10 V one.
5. Measure all voltages and currents measured in (1). Find a relation between these values and the values they had when the 5 V and 10 V sources were applied individually.

**SITE**  
**University of Ottawa**

**ELG-2130**  
**Circuit Theory**

***Laboratory Report # 2***

***Oscilloscope and Circuit Theorems***

Presented to:

Prof. \_\_\_\_\_

By:

Author

Name	Student #

Other team members

Name	Student #

Date: \_\_\_\_\_

## LABORATORY 3      RESPONSE OF RL AND RLC CIRCUITS

---

### 3.1 Objectives

1. To experiment and become familiar with circuits containing energy storage elements.
2. To design first-order circuits with a given step response.
3. To measure the step response of second-order circuits.

---

### 3.2 Preparation

1. Read and understand the course notes on first and second order circuits.
2. Read and understand the experiment procedure below.
3. Answer the preparation questions.

---

### 3.3 Parts and equipment

- 1 digital ohmmeter
- 1 2-channel oscilloscope
- 1 function generator
- 1 resistance box (must have a selection of  $10\Omega$ )

- 1 2  $\mu\text{F}$  capacitor
- 1 100 mH inductor

### 3.4 Measurement of first and second order circuits

When measuring the responses of second-order and higher-order systems, the techniques are similar. Complications arise only from the fact that the forms of the responses can be much more complex than in the simple first-order case, so that there will be more parameters to measure.

As an example, consider the response of a series RLC circuit to a step input voltage of  $E$  volts. If the circuit is underdamped, the capacitor voltage  $V_c$  will have a constant component plus a decaying oscillation, as follows:

$$V_c = E \left[ \frac{E}{\sqrt{1 - z^2}} e^{-\alpha t} \sin(\omega_d t + \phi) \right]$$

where  $z = (R\sqrt{C/L})/2$ ,  $\alpha = z\omega_o$ ,  $\omega_d = \omega_o\sqrt{1 - z^2}$ ,  $\omega_o = 1/\sqrt{LC}$  and  $\sin\phi = \sqrt{1 - z^2}$ . Note that  $z$  is called the damping factor and is less than one for an underdamped system.

FIGURE 12 Response of an underdamped circuit.

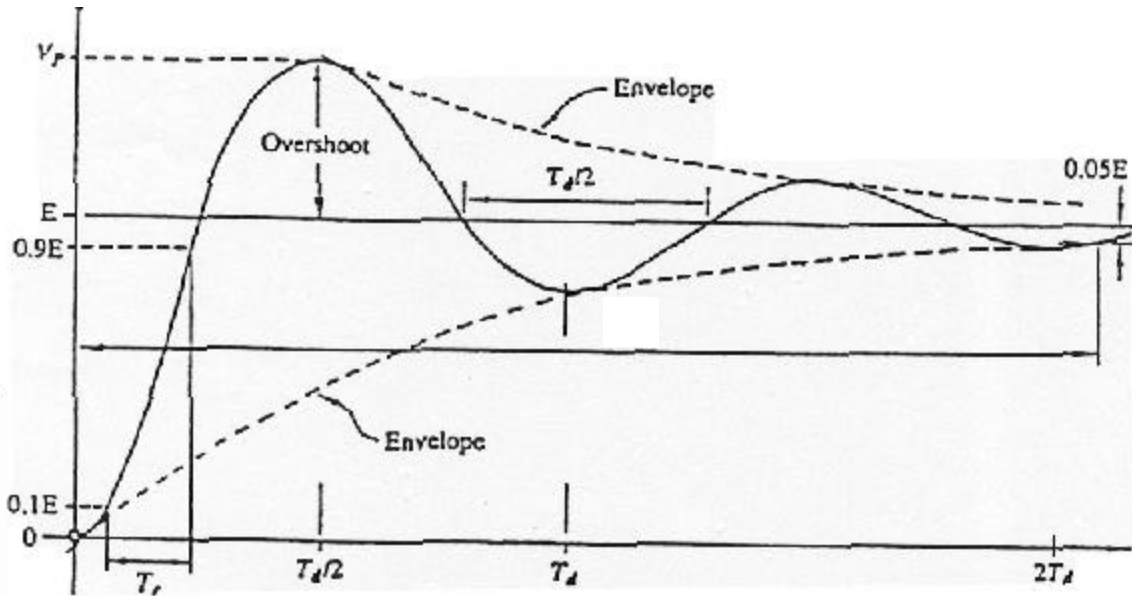


Figure 12 shows a typical response of this type and some of the parameters that may be measured. The final value and the 10-90% rise time ( $T_r$ ) are still applicable, but because of the oscillation there are some other things to consider. The frequency of the oscillation and the “ringing angular frequency” ( $\omega_d = 2\pi f_d = 2\pi/T_d$ ), can be found by measuring the period of oscillation ( $T_d$ ). For accuracy, as many half-periods as possible should be averaged. The time-constant ( $1/\alpha$ ) of the exponential decay of the oscillation can be determined by sketching the “envelope” of the signal and measuring it as though it was a first-order response. The percentage of the first overshoot would be given by the relation:

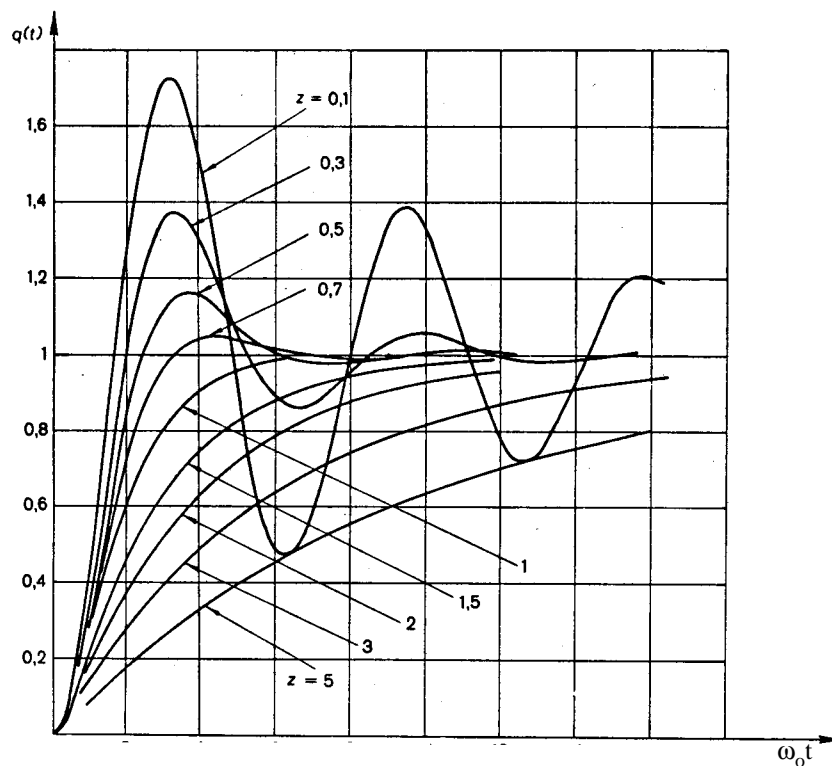
$$\% \text{ overshoot} = \frac{V_p \angle E}{E} \times 100 = e^{\frac{\angle z \pi}{\sqrt{1 - z^2}}} \times 100\%$$

which is a function of  $z$  only. Thus, one can see that all systems with the same damping factor will have the same overshoot, regardless of natural frequency. From these two measurements (ringing frequency and overshoot) it is possible to derive  $z$  and  $\omega_o$ , the damping factor and the undamped natural angular frequency of the system.

For oscillatory systems, a measure of the speed of response that is more meaningful than the rise time is the “settling time”; the time required for the response to converge towards the final value, within some specified percentage of the total change ( $E$  in this example). A conservative estimate based on the exponential decay of the envelope of the oscillation gives three time-constants ( $3/z\omega_o$  seconds) for a 5% criterion.

For critically damped or overdamped systems, where there is no overshoot or oscillation, measurements of rise time and settling time are meaningful and straightforward, but the determination of  $z$  and  $\omega_o$  requires the solution of two exponential equations, obtained by measurements of the response taken at two significantly different times. A more convenient method of finding these two parameters for any damping is to compare the shape and the time scale of the observed response with those of standard response curves such as those given in figure 13.

**FIGURE 13** Step response of a linear system.



It will be evident that a permanent record of the response would be helpful for the analysis. It is generally not difficult to draw a graph of the response point by point from the oscilloscope. Get the desired repetitive display, a square-wave input is required, and its size and polarity must be recorded along with the circuit response. In a very lightly damped system the oscillations will persist for a long time and a very low input frequency may be required.

---

### 3.5 Measurements on a first-order circuit

---

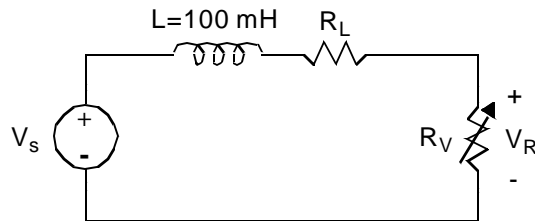
#### PREPARATION QUESTIONS:

Consider the first-order circuit below with a total resistance of  $500\ \Omega$  ( $R_V + R_L = 500\ \Omega$ ).

---

**FIGURE 14**

First-order circuit (RL)



1. Determine the value of an inductor that will give a time constant of 0.2 ms.
2. Derive and plot the response  $V_R(t)$  and  $V_L(t)$  of the circuit to a 1 V peak offset square wave (i.e. with minimum value = 0 V). Use a frequency of 400 Hz. Use the same scales and show the initial conditions (states at  $t=0$ ) clearly on both graphs.

#### EXPERIMENT STEPS:

1. Measure the resistance of the inductor  $R_L$  with a digital ohmmeter. Use the variable resistor  $R_V$  to provide a total resistance of  $500\ \Omega$  when connected in series with the inductor.
2. Set up the RL circuit of figure 14. Use a 1 V peak offset square wave (i.e. with minimum value = 0 V) for  $V_s(t)$  and a frequency of 400 Hz.
3. Measure the rise time, and the pulse width of  $V_R(t)$ .
4. Switch to a sine wave. Display simultaneously  $V_s(t)$  and  $V_R(t)$ . Do your results agree with the theoretical predictions on the phase  $\phi = \text{atan}(\angle\omega L/R)$ ? Explain.  
Note: This measurement requires some planning as the probes and the function generator all have a common ground. Otherwise, you will be surprised with your results.
5. Measure the phase difference between  $V_s(t)$  and  $V_L(t)$ . Compare with the theoretical value given by:

$$\phi_T = \text{atan}\left(\frac{\omega L R_V}{\omega^2 L^2 + R_L^2 + R_L R_V}\right)$$

6. Vary the resistance and comment on the changes in  $V_L(t)$ .

**3.6 Measurements on a second-order circuit**

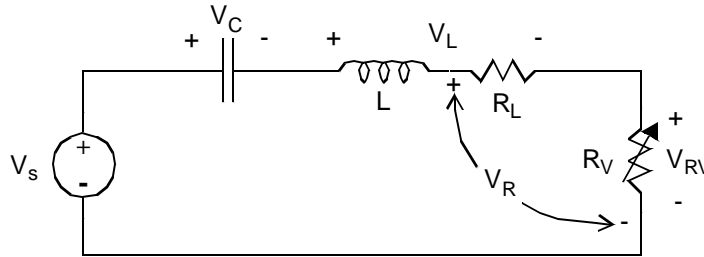
---

PREPARATION QUESTIONS:

Consider the series RLC circuit below, where  $R = R_V + R_L = 500\ \Omega$ ,  $L = 100\ \text{mH}$  and  $C = 0.2\ \mu\text{F}$ .

---

**FIGURE 15** Second-order circuit (RLC).



1. Sketch  $V_R(t)$ ,  $V_C(t)$  and  $V_L(t)$  in response to a step input of 5V.  
Suggestion: Use a computer program to plot the equations that you derived.
2. How would you determine  $V_{RV}(t)$ ?

EXPERIMENT STEPS:

1. Set up the series RLC circuit of figure 15 with  $R_L + R_V = 500\ \Omega$ ,  $L = 100\ \text{mH}$  and  $C = 0.2\ \mu\text{F}$ .
2. Display the response  $V_{RV}(t)$  to a  $V_s(t)$  that has a 5V peak offset square wave (i.e. with the minimum value = 0V) at 200 Hz.
3. Compare waveforms with the theoretical ones, obtained in your preparation.
4. Determine  $T_r$ , % overshoot, the ringing angular frequency  $\omega_d$  and the final value for  $V_C(t)$ . Compare with the theoretical values.
5. Vary the resistance R and observe the changes. Quantify your observations mathematically.
6. Observe the waveforms of  $V_L(t)$  and  $V_R(t)$ , sketch and discuss these results.
7. Experiment with a sine wave in replacement of the square wave. Observe the relationships in magnitude and phase between the two signals. Discuss your observations.

**SITE**  
**University of Ottawa**

**ELG-2130**  
**Circuit Theory**

***Laboratory Report # 3***

***Response of RL and RLC Circuits***

Presented to:

Prof. \_\_\_\_\_

By:

Author

Name	Student #

Other team members

Name	Student #

Date: \_\_\_\_\_

**LABORATORY 4**    **AC CIRCUITS:  
VOLTAGES AND  
CURRENTS IN THE  
COMPLEX DOMAIN -  
POWER TRANSFER**

**4.1 Objectives**

---

1. To see how complex numbers are used in a real ac circuit.
2. To measure complex voltages and calculate complex currents.
3. To observe and measure the active and reactive power in a circuit.
4. To find a load such that the reactive power is eliminated.

**4.2 Preparation**

---

1. Read and understand the course notes on sinusoidal steady-state analysis (AC and AC steady state power).
2. Read and understand the experimental procedure below (2 parts).

### 4.3 Voltages and Currents in Complex Domain

---

Design an experiment to measure voltages and calculate currents in the complex domain, and then interpret them in the time domain.

Hints: Use only capacitors and resistors in your design. They present less problems than inductors. Use 4-5 components in a simple circuit configuration. Work with frequencies in the interval [1 kHz -10 kHz]. Avoid using electrolytic capacitors in this experiment. They also present problems in this experiment. Measure all voltages with respect to a ground point.

#### **Step -1:**

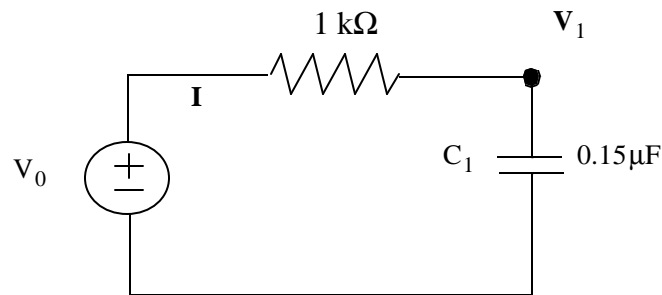
1. Construct the circuit of figure 16.
2. What are the magnitude and phase of the voltage at node  $V_1$  if  $V_0$  is a sinusoidal wave with no DC offset, 5V magnitude (ie. 10V peak-to-peak), and: (1) a frequency of 1 kHz; (2) a frequency of 2 kHz?

Are the phase shifts positive or negative with reference to the source voltage? Use phasors to solve for the voltage at  $V_1$ , as well as the current through the loop, for both cases. What is the phase relationship between the voltage  $V_1$  and the current through the capacitor?

---

**FIGURE 16**

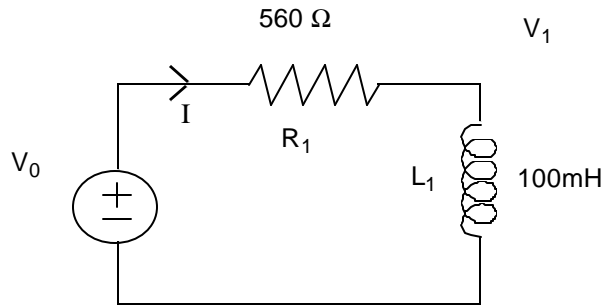
AC RC circuit



#### **Step -2: :**

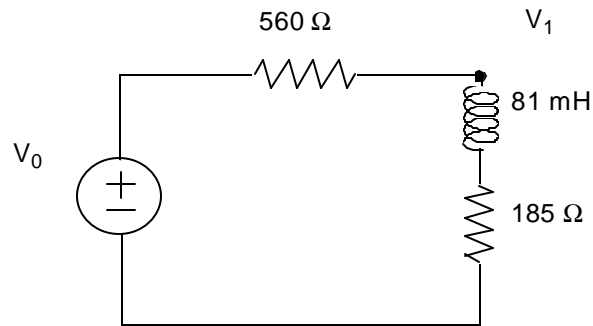
1. Construct the circuit of figure 17.
2. What are the magnitude and phase of the voltage at node  $V_1$  if  $V_0$  is a sinusoidal wave with no DC offset, 5V magnitude, and a frequency of 500 Hz? Is the phase shift positive or negative with reference to the source voltage?
3. Use phasors to solve for the voltage at  $V_1$ , as well as the current through the loop. What is the phase relationship between the voltage  $V_1$  and the current through the inductor?
4. Next, solve for  $V_1$  again, but model the 100 mH inductor as a series combination of an 81 mH inductor and a  $185\ \Omega$  resistor, as shown in the circuit of figure 18. Does this calculation resemble more closely your observed results than the previous solution? Explain.

FIGURE 17 AC RL circuit



---

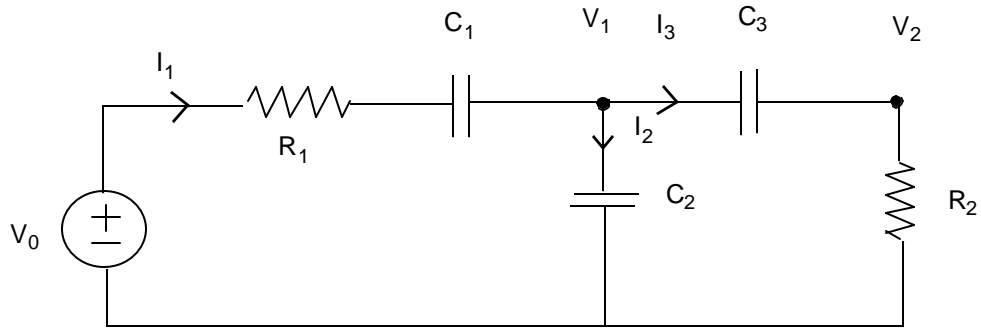
FIGURE 18 Alternative implementation of RL circuit



**Step -3:**

1. Construct the circuit of figure 19, where all components have different values. Use capacitor values between  $0.01 \mu\text{F}$  and  $0.5 \mu\text{F}$ , and set the magnitude of  $V_0$  to  $5 \text{ V}$ .
2. Measure the magnitude and phase of the voltages  $V_1$ ; and  $V_2$ , when  $V_0$  has the frequency of  $1 \text{ kHz}$ . Repeat for  $V_0$  at the frequency of  $2 \text{ kHz}$ .
3. Use phasors to solve for  $V_1$ ,  $V_2$ ,  $I_1$ ,  $I_2$  and  $I_3$ , for both cases.

FIGURE 19

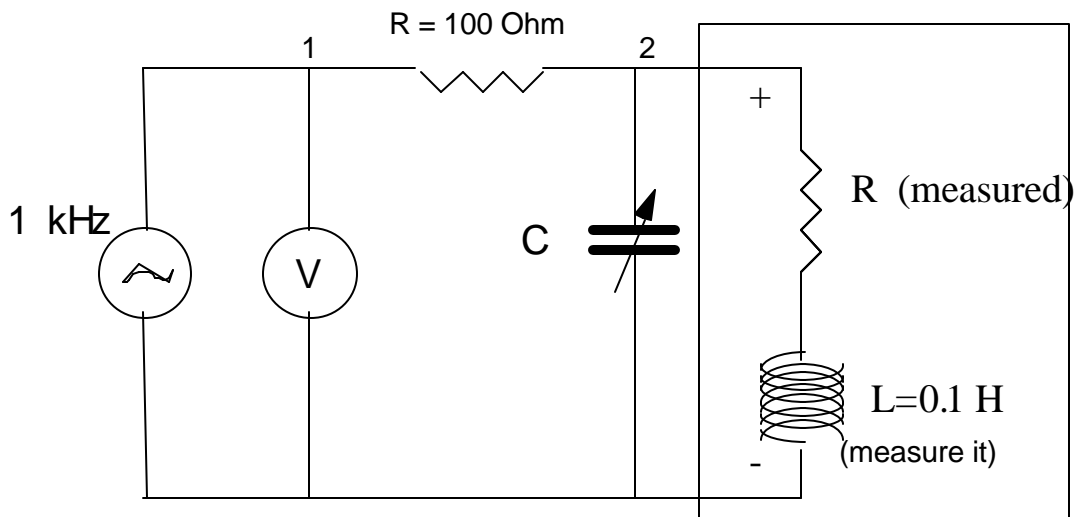


#### 4.4 Transfer of Power in AC Circuits

Implement the circuit shown in figure 20:

FIGURE 20

Circuit for the study of Complex Power Transfer



1. Calculate the reactive power in the circuit, in the terminals of the load, by voltage measurements.
2. Insert a capacitor and calculate the necessary capacitance  $C$  in order to nullify the reactive power transferred at the terminals of the load, and connect it across the charge terminals. Measure the active and reactive power.

Hints: Do not forget that the inductor has an internal resistance. For part 2, measure the voltage across nodes 1 and 2, as you vary  $C$ . When they will be in phase,  $C$  has compensated the reactive part of  $L$ . Calculate the current.

---

**SITE**  
**University of Ottawa**

**ELG-2130**  
**Circuit Theory**

***Laboratory Report # 4***

***AC Circuits: Voltages and Currents in the Complex Domain - Power Transfer***

Presented to:

Prof. \_\_\_\_\_

By:

Author

Name	Student #

Other team members

Name	Student #

Date: \_\_\_\_\_

---

## Appendix

# OSCILLOSCOPE

### Operation of an oscilloscope

An oscilloscope is both an instrument of observation and measurement. Its multiple applications originate from the fact that it can display on its screen the Cartesian representation of various waveforms. This is accomplished by displaying a variable signal on the vertical axes (Y) as a function of another variable signal on the horizontal axis (X). Some common examples are the amplitudes of voltages (or currents) (Y) as a function of time (X).

You must observe the following safety precautions during your test and measurement operation.

- *Do not intensify unnecessarily*: The brightness of the spots or traces on the viewing area must not be increased excessively. Excessively intensified spots or traces may irritate an operator. They may also result in burning the phosphorescence coating of the CRT in prolonged operation.
- *Do not apply an excessively high input voltage*: Each input connector has a rated maximum allowable input voltage. The following values give an order of magnitude for these ratings: max. input: 300V (DC + peak AC), max. trigger in: 150 V (DC + peak AC).

#### EXPERIMENT STEPS:

1. Before you initialize and experiment with your scope, examine the front panel and identify each of the controls and connectors.
2. If it's not already done, turn off the POWER switch and connect the power cord to the line receptacle.
3. Set the controls as follows:
  - MODE (vertical): CH1
  - MODE (sweep): AUTO
  - POSITION Y (vertical): mid-position
  - POSITION X (horizontal): mid-position
4. Switch on the oscilloscope and allow some time (~30 sec) for it to warm up. Then one can see a horizontal beam (the CH1 trace) appear on the screen. If not, adjust the vertical (Y) position and horizontal (X) position knobs properly, the beam will show up.
5. Set the trace to the center of the viewing area by the vertical POSITION control.
6. Adjust horizontal (X) position and the horizontal gain to let the line occupy the whole length of the screen but without surpassing it too much.

- 
7. Set the TIME/DIV switch to 1 mSEC.
  8. Set brightness of the trace to the desired degree by the INTEN control.
  9. Adjust the FOCUS control to make the trace line thin and clear.

Many kinds of signals can be measured with an oscilloscope: DC, AC and mixed signals. To observe these signals correctly, an adequate signal input coupling must be selected with the AC-GND-DC switch.

Also, for accurate measurement of signal waveforms, it is essential to display adequate amplitude of the waveforms on the viewing area. An excessively small or large signal compared with the viewing area makes measurement difficult and inaccurate. If the signal to be measured is small, it needs to be amplified; if it's large, it needs to be attenuated. The sensitivity is controlled by the VOLTS/DIV switch.

10. Set the controls to the following values:

- MODE (vertical): CH1
- CH1 VOLTS/DIV: 0.2 V
- MODE (horizontal): x1
- SEC/DIV: 0.5 ms
- TRIGGER MODE: AUTO
- TRIGGER SOURCE: CH1

11. Make sure that your probe is connected to the channel 1 (CH1) input BNC, and its tip is attached to the CAL OUT (or PROBE ADJUST) terminal. Turn on your scope, set the CH1 input coupling switch to GND, and align the trace along the center horizontal graticule line using the CH1 POSITION control. Then switch to AC input coupling.

12. Now you can use the horizontal system of your scope to look at the probe adjustment signal. Move the waveform with the horizontal POSITION control until one rising edge is lined up with the center vertical graticule line. Examine the screen to see where the leading edge of the next pulse crosses the horizontal centerline of the graticule. Count the major and the minor graticule markings along the center horizontal graticule between the successive pulse leading edges. Note that number.

13. Change the sweep speed to 0.2 ms and line up a rising edge with the vertical graticule line on the left edge of the screen. Again count divisions to the next rising edge. Because the switch was changed from 0.5 ms to 0.2 ms, the waveform will look two and one-half times as long as before. But the signal itself hasn't changed. This is just a question of rendering of the same signal on different horizontal scales.

14. Turn the SEC/DIV switch back to 0.5 ms and set the MAG switch to x10. The trace is now a 10x magnification of the original trace. That is, its sweep speed is ten times faster. For example, the sweep speed of the magnified trace is now 0.05 ms per division, while the sweep speed of the original (unmagnified) trace remains 0.5 ms.

#### QUESTION:

With the help of the ADD and CH2 INVERT functions, explain how you would make differential measurements, i.e. measure the difference between 2 signals.

## *Operation of the function generator*

A function generator is a device which acts like a source of signals. It can generate signals such as sine, triangular and square waveforms, negative and positive pulses and DC levels.

---

### PREPARATION QUESTION:

Define the following terms: 1) frequency, 2) rise time, 3) duty cycle.

### EXPERIMENT STEPS:

1. Before you initialize and experiment with your function generator, examine the front panel and identify each of the controls and connectors.
2. Switch on the function generator and allow some time for it to warm up.
3. Adjust FREQUENCY dial to 1.0 and rotate FREQUENCY MULT to 1k.
4. Set ATTENUATION to 0.
5. Turn DC OFFSET and SYMMETRY to OFF.
6. Rotate the FUNCTION dial and select a sine waveform.
7. Connect the FUNCTION OUT connector of the function generator to a vertical (CH1 or CH2) input channel of the oscilloscope.
8. Set the triggering system of the oscilloscope to correspond to the vertical input channel, and adjust the triggering level to freeze the sine wave on the screen. If you do not see any display, repeat and understand the steps about the operation of the oscilloscope. This should lead you to a visible signal on the scope. Now use the FUNCTION switch of the function generator to obtain various waveforms.
9. Now that you have got a visible waveform on your oscilloscope, experiment with different frequencies. Use the FREQUENCY dial and the FREQUENCY MULT of the function generator to sweep through a broad spectrum of frequency (from very low to very high) of the signal, and observe the waveforms on the screen.
10. The frequency,  $f$ , of the signal can be obtained by calculating the inverse of the period,  $T$ , i.e.  $f = 1/T$ . For various frequencies of the sine wave, measure the period  $T_0$  as seen on the scope using its horizontal grid. Don't forget to take into account the SEC/DIV setting that gives you the scale of the grid. Calculate the corresponding frequency and compare it with that indicated on the rotary FREQUENCY dial of the function generator.
11. Use the FUNCTION switch of your function generator to select other signals (square and triangular waves) and repeat the previous step.

### QUESTION:

Comment on the accuracy of the frequency dial of the function generator.