CHG 4305 Advanced Materials in Chemical Engineering
University of Ottawa
Assignment #6
Prof. A.Y. Tremblay
Due Date: 13:00h December 7th, 2015.

6.1 For the following pairs of alloys that are coupled in seawater, predict the possibility of corrosion; if corrosion is probable, note which metal/alloy will corrode.

(a) Aluminum and magnesium

(b) Zinc and a low-carbon steel

(c) Brass (60 wt% Cu–40 wt% Zn) and Monel (70 wt% Ni–30 wt% Cu)

(d) Titanium and 304 stainless steel

(e) Cast iron and 316 stainless steel

<u>Solution</u>

This problem asks, for several pairs of alloys that are immersed in seawater, to predict whether or not corrosion is possible, and if it is possible, to note which alloy will corrode. In order to make these predictions it is necessary to use the galvanic series, Table 16.2. If both of the alloys in the pair reside within the same set of brackets in this table, then galvanic corrosion is unlikely. However, if the two alloys do not lie within the same set of brackets, then that alloy appearing lower in the table will experience corrosion.

(a) For the aluminum-magnesium couple, corrosion is possible, and magnesium will corrode.

(b) For the zinc-low carbon steel couple, corrosion is possible, and zinc will corrode.

(c) For the brass-monel couple, corrosion is unlikely inasmuch as both alloys appear within the same set of brackets.

(d) For the titanium-304 stainless steel pair, the stainless steel will corrode, inasmuch as it is below titanium in both its active and passive states.

(e) For the cast iron-316 stainless steel couple, the cast iron will corrode since it is below stainless steel in both active and passive states.

6.2 An electrochemical cell is constructed such that on one side a pure nickel electrode is in contact with a solution containing Ni^{2+} ions at a concentration of 4×10^{-3} M. The other cell half consists of a pure Fe electrode that is immersed in a solution of Fe^{2+} ions having a concentration of 0.15 M. At what temperature will the potential between the two electrodes be +0.135 V?

Solution

This problem asks for us to calculate the temperature for a nickel-iron electrochemical cell when the potential between the Ni and Fe electrodes is +0.135 V. On the basis of their relative positions in the standard emf series (Table 16.1), assume that Fe is oxidized and Ni is reduced. Thus, the electrochemical reaction that occurs within this cell is just

$$Ni^{2+} + Fe \rightarrow Ni + Fe^{2+}$$

Thus, Equation 16.20 is written in the form

$$\Delta V = \left(V_{\text{Ni}}^0 - V_{\text{Fe}}^0\right) - \frac{RT}{nF} \ln \frac{[\text{Fe}^{2+}]}{[\text{Ni}^{2+}]}$$

Solving this expression for T gives

$$T = -\frac{nF}{R}\left[\frac{\Delta V - \left(V_{\text{Ni}}^0 - V_{\text{Fe}}^0\right)}{\ln \dfrac{[\text{Fe}^{2+}]}{[\text{Ni}^{2+}]}}\right]$$

The standard potentials from Table 16.1 are $V_{\text{Fe}}^0 = -0.440$ V and $V_{\text{Ni}}^0 = -0.250$ V. Therefore,

$$T = -\frac{(2)(96,500 \text{ C/mol})}{8.31 \text{ J/mol-K}}\left[\frac{0.135\ V - \left\{-0.250\ V - (-0.440\ V)\right\}}{\ln\left(\dfrac{0.15\, M}{4 \times 10^{-3}\ M}\right)}\right]$$

$$= 352.4 \text{ K} = 79.29°\text{C}$$

6.3 A continuous and aligned fiber-reinforced composite is to be produced consisting of 25 vol% UHMWPE fibers and 75 vol% polyethylene matrix; the mechanical characteristics of these two materials are as follows:

	Modulus of Elasticity [GPa]	Tensile Strength [MPa]
UHMWPE fiber	172	2900
PE	0.2	10

The stress on the polyethylene matrix when the UHMWPE fibers fail is 8 MPa.

For this composite, compute the following:

(a) The longitudinal tensile strength

(b) The longitudinal modulus of elasticity

Solution

This problem calls for us to compute the longitudinal tensile strength and elastic modulus of an UHMWPE fiber-reinforced polyethylene composite.

(a) The longitudinal tensile strength is determined using Equation 15.17 as

$$\sigma_{cl}^* = \sigma'_m(1 - V_f) + \sigma_f^* V_f$$

$$= (8 \text{ MPa})(0.75) + (2900 \text{ MPa})(0.25)$$

$$= 731.0 \text{ MPa}$$

(b) The longitudinal elastic modulus is computed using Equation 15.10a as

$$E_{cl} = E_m V_m + E_f V_f$$

$$= (0.2 \text{ GPa})(0.75) + (172 \text{ GPa})(0.25)$$

$$= 43.15 \text{ GPa}$$

6.4 The composite described in 6.3 above is in the form of a square, flat sheet, of dimension 0.5 mm thick x 1 m x 1 m. It is subjected to a longitudinal load of 50,000 N.

(a) Calculate the fiber–matrix load ratio.
(b) Calculate the actual loads carried by both fiber and matrix phases.
(c) Compute the magnitude of the stress on each of the fiber and matrix phases.
(f) What strain is experienced by the composite?
(d) What is the length of the composite in the longitudinal direction?

The longitudinal load is released and a load is applied in the transverse direction.

(e) Determine the maximum transverse load that can be sustained by the composite before it fails at a stress of 8 MPa?
(f) What is the strain of the composite in the transverse direction under this load?
(g) What is the final length of the composite in the transverse direction under this load?

Solution

First, find the cross-sectional area of the flat sheet.

A_c = (0.5mm)(1000mm) = 500mm^2

The problem stipulates that the cross-sectional area of a composite, A_c, is 500 mm^2, and the longitudinal load, F_c, is 50,000 N for the composite described in Problem 15.8. Also, for this composite

$V_f = 0.25$

$V_m = 0.75$

E_f = 172 GPa

E_m = 0.2 GPa

(a) First, we are asked to calculate the F_f/F_m ratio. According to Equation 15.11

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{(172\ \text{GPa})(0.25)}{(0.2\ \text{GPa})(0.75)} = 286.7$$

Or, $F_f = 286.7 F_m$

(b) Now, the actual loads carried by both phases are called for. From Equation 15.4

$$F_f + F_m = F_c = 50{,}000\ \text{N}$$

$$286.7F_m + F_m = 50{,}000 \text{ N}$$

which leads to

$$F_m = 173.8 \text{ N}$$

$$F_f = F_c - F_m = 50{,}000 \text{ N} - 173.8 \text{ N} = 49{,}826 \text{ N}$$

(c) To compute the stress on each of the phases, it is first necessary to know the cross-sectional areas of both fiber and matrix. These are determined as

$$A_f = V_f A_c = (0.25)(500 \text{ mm}^2) = 125 \text{ mm}^2$$

$$A_m = V_m A_c = (0.75)(500 \text{ mm}^2) = 375 \text{ mm}^2$$

Now, the stresses are determined using Equation 7.1 as

$$\sigma_f = \frac{F_f}{A_f} = \frac{49{,}826 \ N}{(125 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 398.6 \times 10^6 \text{ N/m}^2 = 398.6 \text{ MPa}$$

And

$$\sigma_m = \frac{F_m}{A_m} = \frac{173.8 \text{ N}}{(375 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 0.4635 \times 10^6 \text{ N/m}^2 = 0.4635 \text{ MPa}$$

(d) The strain on the composite is the same as the strain on each of the matrix and fiber phases; applying Equation 7.5 to both matrix and fiber phases leads to

$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{0.4635 \, MPa}{0.2 \times 10^3 \, MPa} = 2.3175 \times 10^{-3}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{398.6 \, MPa}{172 \times 10^3 \, MPa} = 2.3175 \times 10^{-3}$$

(e) In the transverse direction the stress on the composite is the same as the stress on each of the matrix and fiber phases. Therefore, the force can be calculated with the stress at failure (8 MPa)

$$\sigma_c = \sigma_m = \sigma_f = \sigma$$

$$F = \sigma A = (8 \times 10^6\ N/\mathrm{m}^2)(0.0005\ \mathrm{m})(1.00\mathrm{m}) = 4{,}000\ \mathrm{N}$$

(f) The Young's Modulus for the composite can be calculated in the transverse direction using <15.16>.

$$E_{ct} = \frac{E_m E_f}{V_m E_f + V_f E_m} = \frac{(0.2\ \mathrm{GPa})(172\ \mathrm{GPa})}{(0.75)(172\ \mathrm{GPa}) + (0.25)(0.2\ \mathrm{GPa})} = 0.2666\ \mathrm{GPa}$$

From this, strain can be found:

$$\varepsilon_{ct} = \frac{\sigma}{E_{ct}} = \frac{8\ \mathrm{MPa}}{266.6\ \mathrm{MPa}} = 0.030$$

(g)Final length of composite in the transverse direction can be calculated:

$$\Delta l = \frac{\varepsilon_{ct}}{l_o} = \frac{0.03}{1\mathrm{m}} = 0.03\mathrm{m}$$

Therefore, the final length of the composite in the transverse direction is 1.03m.

6.5 It is necessary to fabricate an aligned and discontinuous carbon fiber-epoxy matrix composite having a longitudinal tensile strength of 1800 MPa using 0.47 volume fraction of fibers. Compute the required fiber fracture strength, assuming that the average fiber diameter and length are 7×10^{-3} mm and 3.0 mm, respectively. The fiber-matrix bond strength is 42 MPa, and the matrix stress at fiber failure is 11 MPa. The values of tensile strength for fibers range from 1.5 to 4.8 GPa (Table 15.4 Callister).

Solution

In this problem, for an aligned and discontinuous carbon fiber-epoxy matrix composite having a longitudinal tensile strength of 1900 MPa we are asked to compute the required fiber fracture strength. To begin, since the value of σ_f^* is unknown, calculation of the value of l_c in Equation 15.3 is not possible, and, therefore, we are not able to decide which of Equations 15.18 and 15.19 to use. Thus, it is necessary to substitute for l_c in Equation 15.3 into Equation 15.18, solve for the value of σ_f^*, then, using this value, solve for l_c from Equation 15.3. If $l > l_c$, we use Equation 15.18, otherwise Equation 15.19 must be used. *Note:* the σ_f^* parameters in Equations 15.18 and 15.3 are the same. Realizing this, and substituting for l_c in Equation 15.3 into Equation 15.18 leads to

$$\sigma_{cd}^* = \sigma_f^* V_f \left[1 - \frac{l_c}{2l}\right] + \sigma_m'\left(1 - V_f\right) = \sigma_f^* V_f \left[1 - \frac{\sigma_f^* d}{4\tau_c l}\right] + \sigma_m'\left(1 - V_f\right)$$

$$= \sigma_f^* V_f - \frac{\sigma_f^{*2} V_f d}{4\tau_c l} + \sigma_m' - \sigma_m' V_f$$

This expression is a quadratic equation in which σ_f^* is the unknown. Rearrangement into a more convenient form leads to

$$\sigma_f^{*2}\left[\frac{V_f d}{4\tau_c l}\right] - \sigma_f^*(V_f) + \left[\sigma_{cd}^* - \sigma_m'\left(1 - V_f\right)\right] = 0$$

Or

$$a\sigma_f^{*2} + b\sigma_f^* + c = 0$$

where

$$a = \frac{V_f d}{4\tau_c l}$$

$$= \frac{(0.47)(7.0\times10^{-6}\,m)}{(4)(42MPa)(3.0\times10^{-3}\,m)} = 6.528\times10^{-6}\ (\text{MPa})^{-1}$$

Furthermore,

$$b = -V_f = -0.47$$

And

$$c = \sigma_{cd}^{*} - \sigma_m'(1 - V_f)$$

$$= 1800\ \text{MPa} - (11\ \text{MPa})(1\text{–}0.47) = 1794.2\ \text{MPa}$$

Now solving the above quadratic equation for σ_f^* yields

$$\sigma_f^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.47)\pm\sqrt{(-0.47)^2 - (4)\left[6.528\times10^{-6}(MPa)^{-1}\right](1794.2.4MPa)}}{(2)\left[6.528\times10^{-6}(MPa)^{-1}\right]}$$

This yields the two possible roots as

$$\sigma_f^*(+) = 67{,}953\ \text{MPa}$$

$$\sigma_f^*(-) = 4044.6\ \text{MPa}$$

Upon consultation of the magnitudes of σ_f^* for various fibers and whiskers in Table 15.4, only $\sigma_f^*(-)$ is reasonable. Now, using this value, let us calculate the value of l_c using Equation 15.3 in order to ascertain if use of Equation 15.18 in the previous treatment was appropriate. Thus

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(4044.6 \text{ MPa})(0.007 \text{ mm})}{(2)(42 \text{ MPa})} = 0.3371 \text{ mm}$$

Since $l > l_c$ (3.0 mm > 0.34 mm), our choice of Equation 15.18 was indeed appropriate, and σ_f^* = 4044.6 MPa.