

CHG 4305 Advanced Materials in Chemical Engineering
University of Ottawa
Solution of Assignment # 5
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5.1 The magnetic flux density within a bar of some material is 0.435 tesla at an H field of 3.44×10^5 A/m. Compute the following for this material: (a) the magnetic permeability and (b) the magnetic susceptibility. (c) What type(s) of magnetism would you suggest is (are) being displayed by this material? Why?

Solution

(a) The magnetic permeability of this material may be determined according to Equation 18.2 as

$$\mu = \frac{B}{H} = \frac{0.435 \text{ tesla}}{3.44 \times 10^5 \text{ A/m}} = 1.2645 \times 10^{-6} \text{ H/m}$$

(b) The magnetic susceptibility is calculated using a combined form of Equations 18.4 and 18.7 as

$$\begin{aligned}\mu_r &= \frac{\mu}{\mu_0} \\ \mu_r &= \chi_m + 1 \\ \chi_m &= \mu_r - 1 = \frac{\mu}{\mu_0} - 1\end{aligned}$$

$$= \frac{1.2645 \times 10^{-6} \text{ H/m}}{1.257 \times 10^{-6} \text{ H/m}} - 1 = 6.0 \times 10^{-3}$$

(c) This material would display both diamagnetic and paramagnetic behavior. All materials are diamagnetic, and since χ_m is positive and on the order of 10^{-3} , there would also be a paramagnetic contribution.

5.2 The magnetization within a bar of some metal alloy is 3.2×10^5 A/m at an H field of 50 A/m. Compute the following: (a) the magnetic susceptibility, (b) the permeability, and (c) the magnetic flux density within this material. (d) What type(s) of magnetism would you suggest as being displayed by this material? Why?

Solution

(a) This portion of the problem calls for us to compute the magnetic susceptibility within a bar of some metal alloy when $M = 3.2 \times 10^5$ A/m and $H = 50$ A/m. This requires that we solve for χ_m from Equation 18.6 as

$$\chi_m = \frac{M}{H} = \frac{3.2 \times 10^5 \text{ A/m}}{50 \text{ A/m}} = 6400$$

(b) In order to calculate the permeability we must first of all employ an altered form of Equation 18.7 as

$$\mu_r = (\chi_m + 1)$$

In addition, Equation 18.4 may be written as

$$\mu = \mu_r \mu_0$$

By substituting the expression for μ_r from the first equation above into the second equation and incorporating values for the parameters given in the problem statement or determined in part (a) yields

$$\begin{aligned} \mu &= \mu_r \mu_0 = (\chi_m + 1)\mu_0 \\ &= (6400 + 1)(1.257 \times 10^{-6} \text{ H/m}) = 8.05 \times 10^{-3} \text{ H/m} \end{aligned}$$

(c) The magnetic flux density may be determined using Equation 18.2 as

$$B = \mu H = (8.05 \times 10^{-3} \text{ H/m})(50 \text{ A/m}) = 0.40 \text{ tesla}$$

(d) This metal alloy would exhibit ferromagnetic behavior on the basis of the magnitude of its χ_m (6400), which is considerably larger than the χ_m values for diamagnetic and paramagnetic materials listed in Table 18.2.

5.3 A reactor component is fabricated from a titanium alloy that has a plane strain fracture toughness of $37 \text{ MPa}\sqrt{\text{m}}$. It has been determined that fracture results at a stress of 260 MPa when the maximum (or critical) internal crack length is 2 mm. For this same component and alloy, will fracture occur at a stress level of 350 MPa when the maximum internal crack length is 0.9 mm? Why or why not?

Solution

We are asked to determine if an aircraft component will fracture for a given fracture toughness ($37 \text{ MPa}\sqrt{\text{m}}$), stress level (350 MPa), and maximum internal crack length (0.9 mm), given that fracture occurs for the same component using the same alloy for another stress level and internal crack length. It first becomes necessary to solve for the parameter Y , using Equation 9.5, for the conditions under which fracture occurred (i.e., $\sigma = 260 \text{ MPa}$ and $2a = 2.0 \text{ mm}$). Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{37 \text{ MPa}\sqrt{\text{m}}}{(260 \text{ MPa})\sqrt{(\pi)\left(\frac{2 \times 10^{-3} \text{ m}}{2}\right)}} = 2.538$$

Now we will solve for the product $Y\sigma\sqrt{\pi a}$ for the other set of conditions, so as to ascertain whether or not this value is greater than the K_{Ic} for the alloy. Thus,

$$\begin{aligned} Y\sigma\sqrt{\pi a} &= (2.538)(350 \text{ MPa})\sqrt{(\pi)\left(\frac{0.9 \times 10^{-3} \text{ m}}{2}\right)} \\ &= 33.39 \text{ MPa}\sqrt{\text{m}} \end{aligned}$$

Therefore, fracture *will not* occur since this value ($33.39 \text{ MPa}\sqrt{\text{m}}$) is less than the K_{Ic} of the material, $37 \text{ MPa}\sqrt{\text{m}}$.

5.4 The fracture strength of a polycarbonate plate may be increased by etching away a thin surface layer. It is believed that the etching may alter surface crack geometry (i.e., reduce the crack length and increase the tip radius). Compute the ratio of the original and etched crack tip radii for a sixfold increase in fracture strength if half of the crack length is removed.

Solution

This problem asks that we compute the crack tip radius ratio before and after etching. Let

$$\begin{aligned}\rho_t &= \text{original crack tip radius, and} \\ \rho_t' &= \text{etched crack tip radius}\end{aligned}$$

Also,

$$\sigma_f' = \sigma_f$$

$$a' = \frac{a}{2}$$

$$\sigma_0' = 6\sigma_0$$

Solving for $\frac{\rho_t'}{\rho_t}$ from the following

$$\sigma_f = 2\sigma_0 \left(\frac{a}{\rho_t} \right)^{1/2} = \sigma_f' = 2\sigma_0' \left(\frac{a'}{\rho_t'} \right)^{1/2}$$

yields

$$\frac{\rho_t'}{\rho_t} = \left[\frac{\sigma_0'}{\sigma_0} \right]^2 \left[\frac{a'}{a} \right] = \left[\frac{6\sigma_0}{\sigma_0} \right]^2 \left[\frac{a/2}{a} \right] = 36/2 = 18$$