CHG 4305 Advanced Materials in Chemical Engineering
University of Ottawa
Solution of Assignment #4
Prof. A.Y. Tremblay

4.1 Solution:
This problem asks for us to determine the change in diameter of a cylindrical rod of Inconel 625,450.00 mm long and 30.000 mm in diameter when it is heated from 50°C to 250°C while its ends are maintained rigid. There will be two contributions to the diameter increase of the rod; the first is due to thermal expansion (which will be denoted as Δd_1), while the second is from Poisson's lateral expansion as a result of elastic deformation from stresses that are established from the inability of the rod to elongate as it is heated (denoted as Δd_2). The magnitude of Δd_1may be computed using a modified form of Equation 17.3 as

$$\Delta d_1 = d_o \alpha_l \left(T_f - T_0\right)$$
$$\Delta d_1 = (30mm)(12.8 \times 10^{-6} C^{-1})(250^o C - 50^o C)$$
$$\Delta d_1 = 0.0768mm$$

Combining equation 17.3, 7.8, 7.5, and equation 7.2 we get

$$\Delta d_2 = -d_o v \varepsilon_z = -d_o v \alpha_1 (T_o - T_f)$$
$$\Delta d_2 = -(30mm)(0.35)(12.8 \times 10^{-6} C^{-1})(50^o C - 250^o C)$$
$$\Delta d_2 = 0.02688mm$$

Total change in diameter can be calculated as the sum of the two changes in diameter

$$\Delta d = \Delta d_1 + \Delta d_2$$
$$\Delta d = 0.0768mm + 0.02688mm$$
$$\Delta d = 0.10368mm$$

4.2 Solution:
A) There is a cylinder within a cylinder and they are both expanding due to temperature differently because of being different metals but are being held together by a weld. Normally you could determine the change in the length of the two independent metals (1 for steel 316 and 2 for steel 405) by using the equations below.

$$\frac{\Delta l_1}{l_o} = \alpha_1 \Delta T \qquad \frac{\Delta l_2}{l_o} = \alpha_2 \Delta T$$

Since steel 405 and steel 316 are held together by the weld, both metals will expand equally with a force of compression on steel 316 equal to the force of tension on steel 405.

$$\frac{\Delta l_1}{l_o} = \frac{\Delta l_2}{l_o} \qquad\qquad F_1 = F_2$$

$$\frac{\Delta l_1}{l_o} = \alpha_1 \Delta T - \frac{F_1/A_1}{E_1} \qquad\qquad \frac{\Delta l_2}{l_o} = \alpha_2 \Delta T + \frac{F_2/A_2}{E_2}$$

Isolate for F and solve

$$\alpha_2 \Delta T + \frac{F_2/A_2}{E_2} = \alpha_1 \Delta T - \frac{F_1/A_1}{E_1}$$
$$\alpha_1 \Delta T - \alpha_2 \Delta T = \frac{F_2/A_2}{E_2} + \frac{F_1/A_1}{E_1}$$
$$(\alpha_1 - \alpha_2)\Delta T = F\left(\frac{1}{E_2 A_2} + \frac{1}{E_1 A_1}\right)$$
$$F = \frac{(\alpha_1 - \alpha_2)\Delta T}{\frac{1}{E_2 A_2} + \frac{1}{E_1 A_1}}$$

$$= \frac{(16 \times 10^{-6\,o}C^{-1} - 10.8 \times 10^{-6\,o}C^{-1})(370^oC - 25^oC)}{\frac{1}{(200 \times 10^9 Pa)(((1.06)^2 - (1.045)^2)\pi)} + \frac{1}{(193 \times 10^9 Pa)(((1.015)^2 - (1)^2)\pi)}}$$
$$F = 17{,}090{,}285.04N$$

B) Using the force calculated in part a), calculate out the stress

$$\sigma_1 = \frac{F}{A_1}$$
$$\sigma_1 = \frac{17{,}090{,}285.04N}{(((1.015)^2 - (1)^2)\pi)}$$
$$\sigma_1 = 179.98MPa$$

$$\sigma_2 = \frac{F}{A_2}$$
$$\sigma_2 = \frac{17{,}090{,}285.04N}{(((1.06)^2 - (1.045)^2)\pi)}$$
$$\sigma_2 = 172.28\ MPa$$

C) Solution for length at 350 °C.

Know: $\alpha_1 \Delta T - \frac{\frac{F_1}{A_1}}{E_1}$ and initial length =8 m use:

$$\frac{\Delta l_1}{l_o} = \alpha_1 \Delta T - \frac{\frac{F_1}{A_1}}{E_1}$$

Find $\frac{\Delta l_1}{l_o} = 0.00453$, so you can calculate the new length; get 8.036 m.

D) a) The change in thickness of the shells (Δt) will happen due to two particular effects, the thermal and mechanical expansion. For the steel 316 inner shell, the increase of temperature increases the thickness but the tension from the weld decreases the thickness. This can be seen below.

$$\Delta t_1 = \Delta t_{1,Thermal} + \Delta t_{1,Mechanical}$$
$$\Delta t_{1,Mechanical} = -t_o v \varepsilon_{z1\,M}$$

$$\Delta t_1 = t_o \alpha_l (T_f - T_0) - t_o v \varepsilon_{z1\,M}$$

If not welded and left to thermally expand then put under compression; Final length is less than the initial. So $\varepsilon_{z1\,M} = (l_{1f} - l_{1i})/l_0 < 0$

Also $\varepsilon_{z1\,M} = \sigma_1/E_1$

$$\Delta t_1 = (1.5cm)(16 \times 10^{-6\,o}C^{-1})(370^oC - 25^oC)$$

$$+(1.5cm)(0.3)(179.98MPa/193000MPa)$$

$$\Delta t_1 = 0.00828 + 0.00042 = 0.00870cm$$

As can be seen, there is an increase in the thickness of the inner 316 shell, mostly due to thermal expansion.

$$\Delta t_2 = \Delta t_{2,Thermal} + \Delta t_{2,Mechanical}$$
$$\Delta t_{2,Mechanical} = -t_o v \varepsilon_{z2\,M}$$

$$\Delta t_2 = t_o \alpha_l (T_f - T_0) - t_o v \varepsilon_{z2\,M}$$

If not welded and left to thermally expand then put under tension; Final length is greater than the initial. So $\varepsilon_{z2\,M} = (l_{2f} - l_{2i})/l_0 > 0$

Also $\varepsilon_{z2\,M} = \sigma_2/E_2$

$$\Delta t_2 = (1.5cm)(10.8 \times 10^{-6\,o}C^{-1})(370^oC - 25^oC)$$

$$-(1.5cm)(0.3)(172.28MPa/200000MPa)$$

$$\Delta t_2 = 0.00559 - 0.000388 = 0.0052cm$$

There is also an increase in thickness of the outer 405 shell mostly due to thermal expansion.

b) The welds hold the shells at the top and bottom of the reactor. In the hoop direction, the inside shell will expand to a greater extent than the outer shell. So they will deform close to the welds. Assuming that due to this deformation, the welds do not affect expansion at the midheight of the reactor, we can determine the expansion of the shells in the hoop direction. This is given by :

$$l_{f1} = l_{o1}(\alpha_1 \Delta T + 1)$$

$$l_{f2} = l_{o2}(\alpha_2 \Delta T + 1)$$

Use the outer radius of the 316 shell in determining l_{o1} and the inner radius of the 405 shell in determining l_{o2}

$$l_{f1} = (2 * \pi * 1.015)(16 \times 10^{-6} * 345 + 1)$$
$$l_{f1} = 6.4126$$

The news radius of the inner shell =6.4126 / 2π = 1.02059

$$l_{f2} = (2 * \pi * 1.045)(10.8 \times 10^{-6} * 345 + 1)$$
$$l_{f2} = 6.5903$$

The news radius of the outer shell = 6.5903 / 2π =1.04887

The thickness of the jacket at 370 °C =1.0488-1.02059= 0.028m= 2.8cm

E)

Since $\sigma_{316} = 179.98\ MPa\ < \sigma_{y,316} = 205\ MPa$ Therefore, the deformation is elastic
Since $\sigma_{304} = 172.28\ MPa\ > \sigma_{y,304} = 170\ MPa$ Therefore, the deformation is plastic

4.3 Solution:

(a) This germanium material to which has been added 5×10^{22} m^{-3} Sb atoms is *n*-type since Sb is a donor in Ge. (Antimony is from group VA of the periodic table--Ge is from group IVA.)

(b) Since this material is *n*-type extrinsic, Equation 12.16 is valid. Furthermore, each Sb will donate a single electron, or the electron concentration is equal to the Sb concentration since all of the Sb atoms are ionized at room temperature; that is $n = 5 \times 10^{22}$ m^{-3}, and, as given in the problem statement, $\mu_e = 0.1$ m^2/V-s. Thus:

$$\sigma = n|e|\mu_e$$
$$= (5 \times 10^{22}\ \text{m}^{-3})(1.602 \times 10^{-19}\ \text{C})(0.1\ \text{m}^2/\text{V-s})$$
$$= 801\ (\Omega\text{-m})^{-1}$$

4.4 Solution:

This problem asks that we determine the temperature at which the electrical conductivity of intrinsic Ge is 22.8 $(\Omega\text{-m})^{-1}$, using Equation 12.36 and $C=5.25\text{x}10^9$ $(K^{3/2})(\Omega\text{-m})^{-1}$. First of all, taking logarithms of Equation 12.36

$$\ln \sigma = \ln C - \frac{3}{2}\ln T - \frac{E_g}{2kT}$$

And, calculate ln C = 22.38. Using this and $\sigma = 22.8$ $(\Omega\text{-m})^{-1}$, the above equation takes the form

$$\ln 22.8 = 22.38 - \frac{3}{2}\ln T - \frac{0.67 \text{ eV}}{(2)(8.62 \times 10^{-5} \text{ eV/K})(T)}$$

In order to solve for T from the above expression it is necessary to use an equation solver.

The resulting solution is T = 375, which value is the temperature in K; this corresponds to T(ºC) = 375 – 273 = 102ºC.