

Assignment 3 Solutions
CHG4305 Advanced Materials in Chemical Engineering
Fall 2015

Question 1:

- a) We can calculate the outer radius of a thick-walled vessel using the equation given in class. Be sure to divide the maximum shear stress by the safety factor, 3.

$$\frac{p_i}{S} = \frac{1}{2} \left(1 - \left(\frac{r_i}{r_o} \right)^2 \right)$$

$$\frac{36}{130} = \frac{1}{2} \left(1 - \left(\frac{15.5}{r_o} \right)^2 \right)$$

$$r_o = 23.2 \text{ cm}$$

$$t = r_o - r_i = 23.2 - 15.5 = 7.7 \text{ cm}$$

Calculating for σ_l yields a stress of 29 MPa which is well below $390/3=130$ MPa, and as such this thickness should be sufficient for stress in the lateral direction.

To calculate the weight, we must subtract the inner sphere and cylinder volumes from the outer volumes.

$$V = V_o - V_i = \pi r_o^2 l - \pi r_i^2 l + \frac{4}{3} \pi r_o^3 - \frac{4}{3} \pi r_i^3 = 0.205 \text{ m}^3$$

$$m = \rho V = (7860)(0.205) = 1614 \text{ kg}$$

- b) A similar process to the first part yields $t = 18.0$ cm, $V = 0.639 \text{ m}^3$ and $m = 1724$ kg
- c) With two layers wrapped in the hoop direction, and 1 longitudinally, the maximum stress of 800 MPa should be further decreased (after dividing by the safety factor) by a factor of 2/3 in the hoop direction.

$$S = \frac{800 \cdot \frac{2}{3}}{3} = 177.8 \text{ MPa}$$

For the AFRE we use $r_i=16.5$ cm (due to the 1cm aluminum shell), we can calculate the thickness as 4.9cm, $V=0.127 \text{ m}^3$ and $m=203$ kg. For the aluminum shell ($r_i=15.5$ cm, $r_o=16.5$ cm), we get a volume of 0.021 m^3 and $m=58$ kg. Combining, we get $m=261$ kg.

Calculating for σ_l at this thickness yields a stress of 53 MPa which is well below $[800 \cdot (1/3)]/3=89$ MPa, which represents the maximum stress in the lateral direction accounting for the 1 layer wrapped longitudinally.

- d) Steel = (1614)(0.7) = \$1130
 Aluminum = (1724)(2) = \$3449
 AFRE: = (203)(4)+(57.5)(2) = \$928
- e) Price: Aluminum (\$3449) > Steel (\$1130) > AFRE/Aluminum (\$928)
 Weight: Aluminum (1724kg) > Steel (1614kg) > AFRE/Aluminum (261kg)

Question 2:

To use equation 7.19 in the textbook we need to convert from engineering stress and strain.

- (1) $\sigma_T = \sigma(1+\epsilon)$
 (2) $\epsilon_T = \ln(1+ \epsilon)$

Engineering Stress (MPa)	Engineering Strain	True Stress (MPa) calculated using (1)	True Strain Calculated Using (2)
240	0.194	286.6	0.177
260	0.296	337.0	0.259
?	0.25	?	0.223

The relationship between the true stress and strain can be modelled by (3) $\sigma_T = K\epsilon_T^n$ so we must find the parameters K and n.

$$\ln \sigma_T = \ln K + n \ln \epsilon_T$$

$$\ln \left(\frac{\sigma_{T2}}{\sigma_{T1}} \right) = n \ln \left(\frac{\epsilon_{T2}}{\epsilon_{T1}} \right)$$

$$n = \frac{\ln \left(\frac{\sigma_{T2}}{\sigma_{T1}} \right)}{\ln \left(\frac{\epsilon_{T2}}{\epsilon_{T1}} \right)} = \frac{\ln \left(\frac{337.0}{286.6} \right)}{\ln \left(\frac{0.259}{0.177} \right)} = 0.4263$$

We can now calculate K using (3), and get K = 599 MPa. Then using (3) for the true strain of 0.223 we get a true stress of 316 MPa.

$$\sigma_T = 316 \text{ MPa}$$

$$\sigma = \sigma_T / (1+\epsilon) = 316 / (1.25) = 253 \text{ MPa}$$

Therefore, 253 MPa of engineering stress would be required to produce 0.25 engineering strain.

Question 3:

a) $\sigma = \frac{3FL}{2bd^2} = \frac{3(300)(0.05)}{2(0.011)(0.006)^2} = 57 \text{ MPa}$

b) $\Delta y = \frac{FL^3}{48EI} = \frac{FL^3}{4Ebd^3} = \frac{270(0.05)^3}{4(69E9)(0.006)(0.011)^3} = 0.0515 \text{ mm}$