

ECON 4020 A Mid-Term Examination

Tuesday, October 15, 2013

Prof. K. G. Armstrong

Name: _____

Instructions: Answer all questions in the space provided.

30 pts

1. Consider the production possibilities set

10 pts

$$Y = \{(y_1, y_2) : y_2 \leq ay_1^2\}, \text{ where } a < 0.$$

(a) Draw the graph of Y .

3 pts

(b) Show whether or not Y is closed, convex, and satisfies free disposal.

3 pts

(c) Characterize the set of efficient points of Y .

1 pt

(d) What returns to scale property does Y exhibit, if any?

1.5 pts

- (e) If $(p_1, p_2) = (-4a, 1)$, which production plan will a profit-maximizing firm select from Y and how much profit will it make? 1.5 pts

2. Consider the function $\pi : \mathbb{R}_{++}^3 \rightarrow \mathbb{R}_+$ defined by 12 pts

$$\pi(p, w_1, w_2) = \frac{p^2}{4(w_1 + w_2)}.$$

- (a) Assuming that $\pi(\cdot)$ is a valid profit function, find the associated output supply and input demand functions. 2 pts

- (b) Verify that $\pi(\cdot)$ satisfies the monotonicity property of a profit function. 2 pts

- (c) Verify that $\pi(\cdot)$ satisfies the homogeneity property of a profit function. 1 pt

- (d) Verify that $\pi(\cdot)$ satisfies the curvature property of a profit function. 5 pts

(e) What is the functional form of the underlying production function? 2 pts

3. Consider the transformation function 8 pts

$$T(y, x_1, x_2) = y^2 - x_1^2 - x_2^2 .$$

(a) For y fixed, draw the graph of the input requirement set. 3 pts

(b) Derive the associated cost function. 2 pts

(c) For w_1 and w_2 fixed, draw the graphs of total cost, average total cost and marginal cost. 3 pts

(d) For extra credit: Write down a different transformation function that is equivalent to $T(\cdot)$. 1 pt