MAT 2322 C Assignment # 3

due Thursday, February 11th

- The assignments are due at the beggining of the lecture.
- Late assignments will not be accepted.

1. Find the area of the part of the surface $z = f(x,y) = 6x^{3/2} + 4y^{3/2} + 2$ that sits over the square $0 \le x \le 4$, $0 \le y \le 4$.

2. Evaluate $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos(x+y+z)\,dx\,dy\,dz$

Find the volumes of the indicated solids:
3. The region in the first octant bounded by the coordinate planes, the plane $y+z=2$ and the parabolic cylinder $x = 4 - y^2$.
4. The tetrahedron in the first octant bounded by the coordinate planes and the plane passing through the points $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$.

Evaluate the following integrals by switching the order of integration:
5. $\displaystyle\int_1^4\int_0^1\int_{2y}^2 \frac{2\cos(x^2)}{\sqrt{z}}\,dx\,dy\,dz$
6. $\displaystyle\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi\, e^{2x}\,\sin(\pi y^2)}{y^2}\,dx\,dy\,dz$

MAT 2322 Assignment # 3: Solutions

1. the area is $A = \int_0^4\int_0^4 \sqrt{1+(f_x)^2+(f_y)^2}\,dx\,dy$

$$= \int_0^4\int_0^4 \sqrt{1+(9x^{1/2})^2+(6y^{1/2})^2}\,dx\,dy$$

$$= \int_0^4\int_0^4 (1+81x+36y)^{1/2}\,dx\,dy$$

$$= \left(\frac{1}{81}\right)\left(\frac{2}{3}\right)\int_0^4 \left(\left(1+81x+36y\right)^{3/2}\Big|_0^4\right)dy$$

$$= \frac{2}{243}\int_0^4 \left((325+36y)^{3/2} - (1+36y)^{3/2}\right)dy$$

$$= \left(\frac{2}{243}\right)\left(\frac{1}{36}\right)\left(\frac{2}{5}\right)\left[(325+36y)^{5/2} - (1+36y)^{5/2}\Big|_0^4\right]$$

$$= \boxed{\frac{1}{10935}\left(469^{5/2} - 325^{5/2} - 145^{5/2} + 1\right)} \simeq \boxed{238.3}$$

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2. $\int_0^\pi\int_0^\pi\int_0^\pi \cos(x+y+z)\,dx\,dy\,dz$

$$= \int_0^\pi\int_0^\pi \left(\sin(x+y+z)\Big|_0^\pi\right)dy\,dz$$

$$= \int_0^\pi\int_0^\pi \left(\sin(\pi+y+z) - \sin(y+z)\right)dy\,dz$$

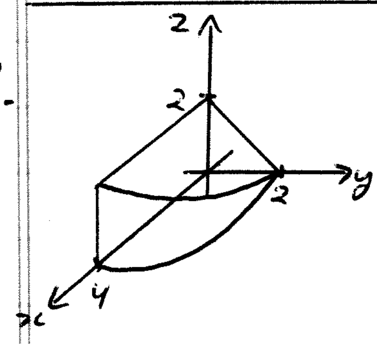
$$= \int_0^\pi \left(-\cos(\pi+y+z) + \cos(y+z)\Big|_0^\pi\right)dz$$

$$= \int_0^\pi \left(-\cos(2\pi+z) + \cos(\pi+z) + \cos(\pi+z) - \cos(z)\right)dz$$

$$= \int_0^{\pi} \left(2\cos(\pi+z) - 2\cos z\right)dz$$

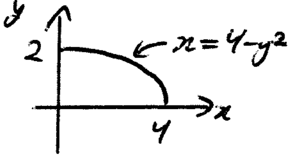
$$= 2\sin(\pi+z) - 2\sin z \Big|_0^{\pi} = \boxed{0}$$

3.



top is given by $z = 2-y$

so z goes 0 to $2-y$



then x goes 0 to $4-y^2$
and y from 0 to 2

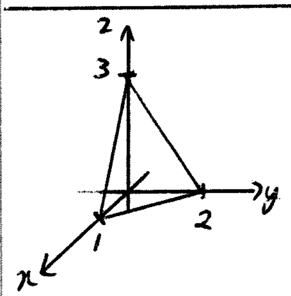
then $$V = \int_0^2 \int_0^{4-y^2} \int_0^{2-y} dz\,dx\,dy$$

$$= \int_0^2 \int_0^{4-y^2} (2-y)\,dx\,dy = \int_0^2 (2-y)(4-y^2)\,dy$$

$$= \int_0^2 (8-4y-2y^2+y^3)\,dy = 8y - 2y^2 - \tfrac{2}{3}y^3 + \tfrac{1}{4}y^4 \Big|_0^2$$

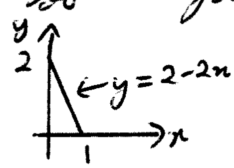
$$= 16 - 8 - 16/3 + 4 - 0 = 12 - 16/3 = \boxed{20/3}$$

4.



top is plane $z = 3 - 3x - \frac{3}{2}y$

so z goes from 0 to $3-3x-\frac{3}{2}y$



then y goes from 0 to $2-2x$
and x from 0 to 1

then $$V = \int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} dz\,dy\,dx$$

$$= \int_0^1 \int_0^{2-2x} \left(3-3x-\tfrac{3}{2}y\right)dy\,dx = \int_0^1 \left((3-3x)(2-2x) - \tfrac{3}{4}(2-2x)^2\right)dx$$

$$=\int_0^1 \left(6-6x-6x+6x^2-\frac{3}{4}(4-4x-4x+4x^2)\right)dx$$

$$=\int_0^1 (3-6x+3x^2)\,dx = 3x-3x^2+x^3\Big|_0^1 = \boxed{1}$$

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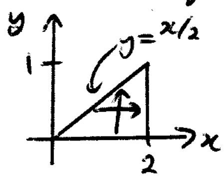
5. $$\int_1^4\int_0^1\int_{2y}^2 \frac{2\cos(x^2)}{\sqrt{z}}\,dx\,dy\,dz = \left(\int_1^4 \frac{dz}{\sqrt{z}}\right)\left(\int_0^1\int_{2y}^2 2\cos(x^2)\,dx\,dy\right)$$



x goes 2y to 2
then y 0 to 1

$$= \left(2\sqrt{z}\Big|_1^4\right)\left(\int_0^2\int_0^{x/2} 2\cos(x^2)\,dy\,dx\right)$$

so y goes 0 to x/2
then x 0 to 2

$$= 2(2-1)\int_0^2 x\cos(x^2)\,dx$$

$$= \sin(x^2)\Big|_0^2 = \boxed{\sin(4)}$$

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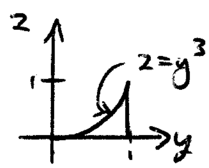
6. $$\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi e^{2x}\sin(\pi y^2)}{y^2}\,dx\,dy\,dz$$



y $\sqrt[3]{z}$ to 1
z 0 to 1
so z from 0 to y^3
and y 0 to 1

$$=\left(\int_0^{\ln 3} e^{2x}\,dx\right)\left(\int_0^1\int_{\sqrt[3]{z}}^1 \frac{\pi\sin(\pi y^2)}{y^2}\,dy\,dz\right)$$

$$=\left(\frac{1}{2}e^{2x}\Big|_0^{\ln 3}\right)\left(\int_0^1\int_0^{y^3}\frac{\pi\sin(\pi y^2)}{y^2}\,dz\,dy\right)$$

$$=\frac{1}{2}(9-1)\int_0^1 \frac{\pi\sin(\pi y^2)}{y^2}(y^3-0)\,dy$$

$$=4\int_0^1 \pi y\sin(\pi y^2)\,dy = -2\cos(\pi y^2)\Big|_0^1$$

$$= -2(\cos\pi-\cos 0) = \boxed{4}$$