MAT 2322 C Assignment # 2

due Thursday, February 4th

- The assignments are due at the beggining of the lecture.
- Late assignments will not be accepted.

Evaluate the iterated integrals:

1. $\displaystyle\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx$
2. $\displaystyle\int_{-1}^{2}\int_0^{\pi/2} y\,\sin x\,dx\,dy$
3. $\displaystyle\int_1^4\int_1^e \frac{\ln x}{xy}\,dx\,dy$

Sketch the region of integration, reverse the order of integration and evaluate the integrals:

4. $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx$
5. $\displaystyle\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy$
6. $\displaystyle\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\,dx\,dy$

Sketch the region of integration, switch to polar coordinates and evaluate the integrals:

7. $\displaystyle\int_0^2\int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy$
8. $\displaystyle\int_{-1}^0\int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$
9. $\displaystyle\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy$

MAT 2322 Assignment #2: Solutions

1. $\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx = \int_0^{\ln 2}\int_1^{\ln 5} e^{2x}e^{y}\,dy\,dx$

$= \left(\int_0^{\ln 2} e^{2x}\,dx\right)\left(\int_1^{\ln 5} e^{y}\,dy\right) = \left(\frac{1}{2}e^{2x}\Big|_0^{\ln 2}\right)\left(e^{y}\Big|_1^{\ln 5}\right)$

$= \frac{1}{2}\left(e^{2\ln 2} - 1\right)\left(e^{\ln 5} - e\right) = \frac{1}{2}(4-1)(5-e) = \boxed{\frac{3}{2}(5-e)}$

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2. $\int_{-1}^{2}\int_0^{\pi/2} y \sin x\,dx\,dy = \left(\int_{-1}^{2} y\,dy\right)\left(\int_0^{\pi/2}\sin x\,dx\right)$

$= \left(\frac{1}{2}y^2\Big|_{-1}^{2}\right)\left(-\cos x\Big|_0^{\pi/2}\right) = \left(\frac{1}{2}(4-1)\right)(-0+1) = \boxed{3/2}$

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3. $\int_1^4\int_1^e \frac{\ln x}{xy}\,dx\,dy = \left(\int_1^4 \frac{1}{y}\,dy\right)\left(\int_1^e \frac{\ln x}{x}\,dx\right)$

$= \left(\ln y\Big|_1^4\right)\left(\frac{1}{2}(\ln x)^2\Big|_1^e\right) = (\ln 4 - 0)\left(\frac{1}{2}(1)^2 - 0\right) = \frac{1}{2}\ln 4 = \boxed{\ln 2}$

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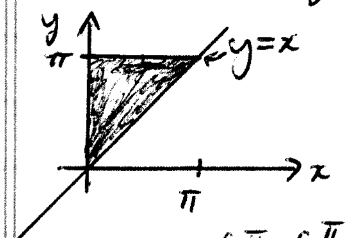
4. $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx$

y goes from x to π
then x from 0 to π



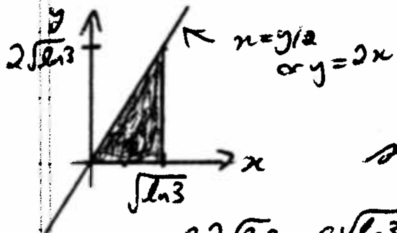
so then x goes 0 to y and y 0 to π

thus $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx = \int_0^{\pi}\int_0^{y} \frac{\sin y}{y}\,dx\,dy$

$= \int_0^{\pi} \frac{\sin y}{y}\left(x\Big|_0^{y}\right)dy = \int_0^{\pi}\frac{\sin y}{y}(y-0)\,dy = \int_0^{\pi}\sin y\,dy$

$= -\cos y\Big|_0^{\pi} = -\cos(\pi) + \cos(0) = \boxed{2}$

5. $\displaystyle\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy$ so x goes from $y/2$ to $\sqrt{\ln 3}$ then y from 0 to $2\sqrt{\ln 3}$



so then y goes 0 to $2x$, x 0 to $\sqrt{\ln 3}$

so $\displaystyle\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy = \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2}\, dy\, dx.$

$\displaystyle = \int_0^{\sqrt{\ln 3}} e^{x^2} \left(y \Big|_0^{2x} \right) dx = \int_0^{\sqrt{\ln 3}} 2x e^{x^2}\, dx$

$\displaystyle = e^{x^2} \Big|_0^{\sqrt{\ln 3}} = e^{\ln 3} - e^0 = 3 - 1 = \boxed{2}$

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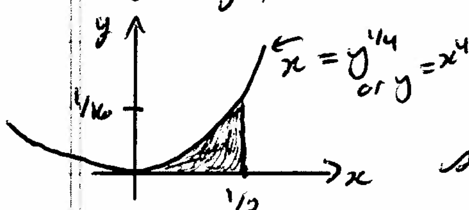
6. $\displaystyle\int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\, dx\, dy$ x goes from $y^{1/4}$ to $1/2$ then y from 0 to $1/16$



so y goes from 0 to x^4, then x from 0 to $1/2$

so $\displaystyle\int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\, dx\, dy = \int_0^{1/2} \int_0^{x^4} \cos(16\pi x^5)\, dy\, dx$

$\displaystyle = \int_0^{1/2} \cos(16\pi x^5) \left(y \Big|_0^{x^4} \right) dx = \int_0^{1/2} x^4 \cos(16\pi x^5)\, dx$

$\displaystyle = \left(\frac{1}{5}\right)\left(\frac{1}{16\pi}\right) \sin(16\pi x^5) \Big|_0^{1/2} = \frac{1}{80\pi}\left(\sin(\pi/2) - 0\right) = \boxed{1/80\pi}$

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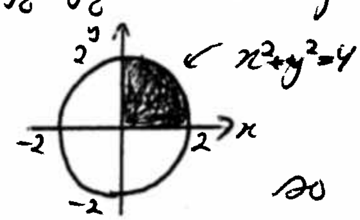
7. $\displaystyle\int_0^{2} \int_0^{\sqrt{4-y^2}} (x^2+y^2)\, dx\, dy$ so x goes from 0 to $\sqrt{4-y^2}$ then y from 0 to 2



so r goes from 0 to 2, θ from 0 to $\pi/2$

then $\int_0^2\int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy = \int_0^{\pi/2}\int_0^2 (r^2)\,r\,dr\,d\theta$

$= \left(\int_0^{\pi/2} d\theta\right)\left(\int_0^2 r^3\,dr\right) = (\pi/2)\left(\frac{1}{4}r^4\Big|_0^2\right) = (\pi/2)(4) = \boxed{2\pi}$

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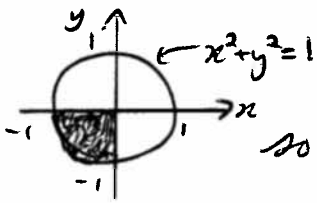
8. $\int_{-1}^0\int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$ so y goes from $-\sqrt{1-x^2}$ to 0 then x from -1 to 0



so r goes 0 to 1, θ from π to $3\pi/2$

and $\int_{-1}^0\int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx = \int_{\pi}^{3\pi/2}\int_0^1 \frac{2}{1+r}\,r\,dr\,d\theta$

$= \left(\int_{\pi}^{3\pi/2} d\theta\right)\left(\int_0^1 \frac{2r}{1+r}\,dr\right) = (\pi/2)\int_0^1 \frac{2(1+r)-2}{1+r}\,dr$

$= (\pi/2)\int_0^1\left(2 - \frac{2}{1+r}\right)dr = (\pi/2)\left(2r - 2\ln(1+r)\Big|_0^1\right)$

$= (\pi/2)\left((2-2\ln(2)) - 0\right) = \boxed{\pi(1-\ln 2)}$

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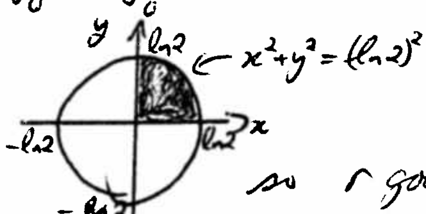
9. $\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy$ so x goes from 0 to $\sqrt{(\ln 2)^2-y^2}$ then y from 0 to $\ln 2$



so r goes from 0 to $\ln 2$ and θ 0 to $\pi/2$

then $\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy = \int_0^{\pi/2}\int_0^{\ln 2} e^r\,r\,dr\,d\theta$

$= \left(\int_0^{\pi/2} d\theta\right)\left(\int_0^{\ln 2} r e^r\,dr\right) = (\pi/2)\left(re^r - e^r\Big|_0^{\ln 2}\right)$

$= (\pi/2)\left(((\ln 2)(2) - 2) - (0-1)\right) = \boxed{(\pi/2)(2\ln 2 - 1)}$

$= \boxed{(\pi/2)(\ln 4 - 1)}$