STUDY GUIDE for MATH 1225

METHODS of CALCULUS

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Study Guide

for

Math 1225

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Contents

Exponential and Logarithmic Functions

The table below lists the exponential rules that you should be familiar with.

Rule	Example	Explanation
$b^x b^y = b^{x+y}$	$2^3 2^4 = 2^7$	When multiplying two terms with a common base, add the exponents.
$\frac{b^x}{b^y} = b^{x-y}$	$\frac{4^5}{4^3} = 4^2$	When dividing terms with a common base, subtract the exponents.
$(b^x)^y = b^{xy}$	$\left(5^2\right)^4 = 5^8$	If you have a number raised to a power, raised to *another* power, then just multiply the exponents.
$b^0 = 1$	$5^0 = 1$	Any number raised to the power of zero is equal to 1.
$b^1 = b$	$7^1 = 7$	Any number raised to the power of one is itself.

You'll also want to memorize the following logarithm rules.

Rule	Example	Explanation
$\log_b x + \log_b y = \log_b xy$	$\log_3 5 + \log_3 7 = \log_3 35$	When adding two logarithms with the same base, just multiply the terms inside.
$\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$	$\log_3 6 - \log_3 2 = \log_3 3$	When subtracting two logarithms with the same base, just divide the terms inside.
$\log_b x^y = y \log_b x$	$\log_3 8 = \log_3 2^3 = 3 \log_3 2$	If you have an exponent on the term inside the logarithm, it can be dropped down in front.[1]
$\log_b b = 1$	$\log_5 5 = 1$	If the base of a logarithm is the same as the number inside, the result is 1.
$\log_b 1 = 0$	$\log_3 1 = 0$	Regardless of the base, if the number inside is a 1, the result is 0.

One final exponential rule is that $x^{a/b}$ means "raise x to the power of a and then take the b^{th} root of the result." $8^{2/3}$ means "take the square of 8 and cube root the result."

$$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

Or, we could have done things in the reverse order:

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$$

Usually this second way is easier – take the root first and then raise it to the necessary power.

[1] Don't confuse this with the fact that the derivative of x^n is nx^{n-1}. This isn't differentiation yet - it's a separate rule that only applies to logarithms.

Differentiation

The derivative is defined in terms of limits

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

The derivative can be expressed in many ways: $\frac{dy}{dx}$, "the slope of the tangent line", y', $f'(x)$. Likewise, the second derivative can be expressed in many ways: $\frac{d^2y}{dx^2}$, $f''(x)$ or y''.

The derivative gives us information about how the function is acting – specifically, the first derivative tells us if $f(x)$ is increasing or decreasing. The second derivative tells us about the concavity of our original function. The different cases are summarized below:

$f(x)$ is increasing	$f'(x) > 0$
$f(x)$ is decreasing	$f'(x) < 0$
$f(x)$ is concave up	$f''(x) > 0$
$f(x)$ is concave down	$f''(x) < 0$

The derivative of a function is a formula that represents the "slope of the tangent line" at any point.

A *critical point* is a value of x where $f'(x) = 0$.

An *inflection point* is a value of x where $f''(x) = 0$.

Make sure you're familiar with the following derivatives:

$f(x)$	$f'(x)$	Example
k (*constant*)	0	$f(x) = 5; f'(x) = 0$
x^n	nx^{n-1}	$f(x) = x^5;\ f'(x) = 5x^4$
$\ln x$	$\frac{1}{x}$	
$\log_b x$	$\frac{1}{x \ln b}$	$f(x) = \log_3 x;\ f'(x) = \frac{1}{x \ln 3}$
e^x	e^x	
b^x	$b^x \ln b$	$f(x) = 3^x;\ f'(x) = 3^x \ln x$

Don't mistake this chart to mean that (for example) b^x is *equal* to $b^x \ln b$. They're related to each other in a special way[2], but they're not the same.

When differentiating a root or a fraction (like $\sqrt{x}$ or $\frac{1}{x^3}$), you'll want to rewrite these with numerical exponents (like $x^{1/2}$ and x^{-3}).

In later sections, we'll also include the derivatives of sin, cos and tan.

[2] the rate of change of one is equal to the other

Differentiation Rules

Whenever you're asked to find the derivative *at* a certain point, always differentiate first and then substitute the number in for x.

It's possible to have a question involving some combination of the product rule, quotient rule and chain rule. It's also possible that a question won't involve *any* of these rules.

The **Product Rule** is used to differentiate products of two functions.

$$(f(x) \cdot g(x))' = f'(x)g(x) + g'(x)f(x)$$

In words: The derivative of the first times the second plus the derivative of the second times the first. Since the terms are being added, the order that we add the terms isn't important here.

The **Quotient Rule** is used to differentiate quotients of two functions. In other words, when one function is divided by another.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Since we're subtracting on the numerator, you must be careful about the order of the terms.

You can memorize the quotient rule with the rhyme:
"Low-dee-high minus high-dee-low; draw the line and square below."
Low stands for $g(x)$. High stands for $f(x)$. Dee means "derivative of".

The **Chain Rule** is used whenever you have a function inside of another function. The rule says to differentiate the *outer* function and then to multiply by the derivative of the *inner* function.

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

Many students struggle with the chain rule. I find it easier to think of it less in terms of the formula and more in terms of the idea: differentiate the outside (and leave the inside untouched) and then multiply by the derivative of the inside. Here's how the chain rule interacts with each of our other formulas.

$f(x)$	$f'(x)$	Example
$(\textit{stuff})^n$	$n(\textit{stuff})^{n-1} \cdot (\textit{derivative of stuff})$	$f(x) = (\ln x)^5$; $f'(x) = 5(\ln x)^4 \cdot \frac{1}{x}$
$e^{\textit{stuff}}$	$e^{\textit{stuff}} \cdot (\textit{derivative of stuff})$	$f(x) = e^{x^2}$; $f'(x) = e^{x^2} \cdot 2x$
$b^{\textit{stuff}}$	$b^{\textit{stuff}} \ln b \cdot (\textit{derivative of stuff})$	$f(x) = 5^{3x+x^7}$; $f'(x) = 5^{3x+x^7} \ln 5 \cdot (3 + 7x^6)$
$\ln(\textit{stuff})$	$\frac{1}{\textit{stuff}} \cdot (\textit{derivative of stuff})$	$f(x) = \ln(x^2+4)$; $f'(x) = \frac{1}{x^2+4} \cdot 2x$
$\log_b(\textit{stuff})$	$\frac{1}{(\textit{stuff}) \ln b} \cdot (\textit{derivative of stuff})$	$f(x) = \log_3(3x^2+1)$; $f'(x) = \frac{1}{(3x^2+1)\ln 3} \cdot (6x)$

Logarithmic Differentiation

Logarithmic differentiation must be used whenever differentiating a function that has an x in both the base **and** the exponent. A few examples are:

$$x^x \qquad (x+5)^{\ln x} \qquad \left(e^x+5x-x^3+3\right)^{e^x+1} \qquad \left(\sqrt{x}+5\right)^{\sqrt{x}}$$

When performing logarithmic differentiation, you'll always be following the steps outlined below.

Example	Steps
Find the derivative of $y=(x^2+5)^x$	
$\ln y = \ln(x^2+5)^x$	Take the ln of both sides
$\ln y = x\ln(x^2+5)$	Drop the exponent down in front (this is one of our logarithm rules)
$\frac{1}{y}\cdot y' = 1\cdot\ln(x^2+5)+\frac{1}{x^2+5}\cdot 2x\cdot x$	Differentiate both sides of the equation. On the left, we'll be using the chain rule (keeping in mind that the derivative of y is y'. On the right, we have a product rule.
$\frac{y'}{y}=\ln(x^2+5)+\frac{2x^2}{x^2+5}$	Simplify.
$y'=y\left(\ln(x^2+5)+\frac{2x^2}{x^2+5}\right)$	Cross multiply the y term from the denominator of the left side to the numerator of the right side. Our goal is to have y' isolated.
$y'=(x^2+5)^x\left(\ln(x^2+5)+\frac{2x^2}{x^2+5}\right)$	Look back at the original problem and replace y with our original equation. This is our final answer.

Antidifferentiation and Integration

Instead of asking "what's the derivative of $f(x)$?", we're asking the opposite question - what function, when differentiated, leaves me with $f(x)$? With differentiation, x^2 turns into $2x$. With antidifferentiation, we're going in the other direction – starting with $2x$ and changing it into x^2.

Antidifferentiation is also called integration. Both are denoted with a squiggle at the beginning $\int$ and a dx at the end. The dx indicates which variable is being antidifferentiated. Some questions may use a u instead of an x. In that case, we'd have a du at the end of our antiderivative. The squiggle and the dx just say to "find the antiderivative of what's between us."[3]

The squiggle and the dx disappear after you've found the antiderivative.

Differentiation and antidifferentiation have the same relationship that x^3 and $\sqrt[3]{x}$ have. They're inverses or opposites.

Since differentiation and antidifferentiation are so connected, the formulas to memorize are nearly identical to the formulas from the previous section.

[3]Think of them as along the same lines as how you capitalize the start of a sentence and put a period at the end – the squiggle and dx are like math punctuation...sort of

$f(x)$	$\int f(x)\,dx$	Example
e^x	$e^x + C$	
$\frac{1}{x}$	$\ln x + C$	
k (*constant*)	$kx + C$	$5\,dx = 5x + C$
x^n	$\frac{x^{n+1}}{n+1} + C$	$\int x^2\,dx = \frac{x^3}{3} + C$
e^{kx} (*k is some number*)	$\frac{e^{kx}}{k} + C$	$\int e^{3x}\,dx = \frac{e^{3x}}{3} + C$
$\frac{1}{ax+b}$	$\frac{1}{a}\ln(ax+b)$ (*a and b are numbers*)	$\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln(3x+2) + C$

The last two formulas aren't mentioned explicitly in the textbook. But, they come up frequently enough that it's useful to memorize them.

If you have a common factor inside your integral, you can pull it out front: $\int 3e^x\,dx = 3\int e^x\,dx$.

Whenever integrating a function, you can check your answer by differentiating it. If you end up with your original function, then your answer is correct. If you're not sure whether $\int x^3\,dx$ is $\frac{x^4}{4} + C$ or not, then just differentiate your answer (pick C to be whatever number you want - I'm going to use 10). To differentiate $f(x) = \frac{x^4}{4} + 10$, we end up with $f'(x) = \frac{4x^3}{4} = x^3$.

It's easy to get confused about integration and differentiation. Here's one way to help keep them straight.

Differentiation will **d**ecrease the exponent

Integration will **i**ncrease the exponent

Whenever evaluating an integral consisting of the product of two terms – multiply them out before integrating!

$$\int (x+1)(x-2)\,dx = \int x^2 + x - 2x - 2\,dx = \int x^2 - x - 2\,dx = \frac{x^3}{3} - \frac{x^2}{2} - 2x + C$$

Whenever you're evaluating an integral where there's just a single term in the denominator – divide it through and integrate each of the terms separately.

$$\int \frac{x^4 + x^2}{x}\,dx = \int \frac{x^4}{x} + \frac{x^2}{x}\,dx = \int x^3 + x\,dx = \frac{x^4}{4} + \frac{x^2}{2} + C$$

If your integral contains a root, make sure to change the root into a numerical exponent before integrating. Likewise, whenever you have term like $\frac{1}{x}$ or $\frac{1}{x^6}$, you should bring the x up to the top and write it with a negative exponent – $\frac{1}{x} = x^{-1}$ and $\frac{1}{x^6} = x^{-6}$ – before integrating.

$$\int \sqrt{x} + \frac{4}{x^3}\,dx = \int x^{1/2} + 4x^{-3}\,dx = \frac{x^{3/2}}{\frac{3}{2}} + \frac{4x^{-2}}{-2} + C = \frac{2}{3}x^{3/2} - \frac{2}{x^2} + C$$

In general, always try to simplify the integral as much as possible before actually finding the antiderivative.

Definite Integrals

An integral with numbers on the top and bottom of the squiggle is called a *definite integral* – such as $\int_1^2 x^2\,dx$. In contrast with an *indefinite integral*, such as $\int x^2\,dx$. The numbers are called the "bounds of integration."

There are two main differences between definite integrals and indefinite integrals.

- After finding the antiderivative, leave out the "+C".
- After finding the antiderivative, substitute in the number at the top of the squiggle and subtract what you get after substituting the number at the bottom of the squiggle.

$$\int_1^5 x^2\,dx$$
$$= \left.\frac{x^3}{3}\right|_1^5$$

The vertical line indicates that we're about the plug in the top number for x and the subtract what we get when we plug in the bottom number. With definite integrals - it's always top minus bottom.

$$= \frac{5^3}{3} - \frac{1^3}{3}$$
$$= \frac{125}{3} - \frac{1}{3}$$
$$= \frac{124}{3}$$

Definite integrals should always end up with a number. Indefinite integrals should always end up with a function.

Substitution Rule

The Substitution Rule is the first of two main techniques to evaluate difficult integrals. The idea behind the substitution rule is that by defining a new variable (which we usually call u), we can turn a complicated integral into a much simpler one. (If it accidentally turns into a harder one, then you've done things horribly, horribly, horribly, horribly wrong.)

The Substitution Rule is best illustrated with an example:

Suppose we are given the integral

$$\int 3x^2 e^{x^3}\,dx$$

This is pretty hard to integrate using the formulas. And there's no real way to simplify it by multiplying things out or dividing, etc. One thing you may notice is that the integral contains a x^3 and it also contains a $3x^2$, and the derivative of x^3 happens to be $3x^2$. That's the key observation that tells us to use the Substitution Rule - one "part" of the integral will be the derivative of another "part."

So, we can define a new variable, u. We'll let $u = x^3$. As soon as you pick your u, you need to find du. du is equal to the derivative of u. In this case $du = 3x^2\,dx$.
(The dx always goes on the end.) Now we go back to our original problem.

$$\int 3x^2 e^{x^3}\,dx$$

Everywhere there's an x^3, replace it with u. And everywhere there's a $3x^2\,dx$, replace it with du. Our new integral is:

$$\int e^u\,du$$

This is *much* easier to work with! We can simply apply the basic formulas we know to get that

$$\int e^u \, du = e^u + C$$

Finally, for our last step, just replace u back with x^3.

$$e^u + C = e^{x^3} + C$$

All done!

One More Example

What if, instead of integrating $\int 3x^2 e^{x^3} \, dx$, we were integrating $\int x^2 e^{x^3} \, dx$? Although x^2 isn't the derivative of x^3, it's pretty close - it's a multiple of the derivative of x^3. And, it turns out, that's close enough. We can still use $u = x^3$. Then, we get

$$du = 3x^2 \, dx$$

But, we don't want $3x^2 \, dx$ here. If we look back at the integral, we only need to substitute in $x^2 \, dx$. That means we'll get rid of the 3 by cross multiplying it to the opposite side:

$$\frac{du}{3} = x^2 \, dx$$

Substituting, we get

$$\int e^u \frac{du}{3}$$

. We can factor out a $\frac{1}{3}$ and integrate to get

$$\frac{1}{3} \int e^u \, du = \frac{1}{3} e^u + C = \frac{1}{3} e^{x^3} + C$$

The substitution rule still works even if one part isn't exactly the derivative of the other part. As long as it's a multiple of the derivative, then the substitution rule will work. To summarize, the substitution rule for indefinite integrals:

- Recognize that one part of the integral is "close" to being the derivative of the other part.
- Pick u
- Differentiate u to get du
- Cross multiply the expression in du as necessary
- Find the antiderivative of the new integral
- Replace u with whatever you picked u to be in Step 2

Three Types

Since Math 1225 is an introductory Calculus course, there's a limited number of types of questions that can be asked about the substitution rule. When you're picking u, always keep in mind to look for which part needs to be differentiated to give the other part. Try to think ahead to what du is going to be. However, if you're stuck on a problem, but you're reasonably sure it's substitution, there's really only three different forms you'll encounter:

- $\int e^{\text{[pick this for } u\text{]}} \text{[other stuff]}\, dx$

  Some examples include: $\int xe^{x^2}, dx$, $\int 2x^3 e^{6x^4+10}\, dx$ and $\int x^2 e^{5x^3}\, dx$
- $\int \frac{\text{other stuff}}{\text{pick this for } u}\, dx$

  Some examples include: $\int \frac{x^2+1}{x^3+3x}\, dx$, $\int \frac{e^{3x}+2}{e^{3x}+6x}\, dx$ and $\int \frac{2x+1}{x^2+x+1}\, dx$
- $\int (\text{pick this for } u)^{\text{some number}} (\text{other stuff})\, dx$

  Some examples include: $\int (x^3+2x+1)^{10}(3x^2+2)\, dx$, $\int (e^{3x}+2x^3+3x)^{35}(e^{3x}+2x^2+x)\, dx$ and $\int (2x^2+4x+10)^{70}(x+1)\, dx$

Of course, these will work much of the time, but there's a few exceptions. The safest way is to get used to making the proper choice for u.

Substitution Rule for Definite Integrals

The steps change slightly with definite integrals.

Suppose that instead of solving $\displaystyle\int 3x^2 e^{x^3}\, dx$, we're solving $\displaystyle\int_1^2 3x^2 e^{x^3}\, dx$.

We'd start the same way - substituting $u = x^3$ and $du = 3x^2\, dx$. But, now, we have to do an extra step where we also substitute the bounds of integration: the 1 and 2. To do this, we plug in 1 and 2 into the formula $u = x^3$. The reasoning is this: 1 and 2 are both in terms of x, so we need to convert them to be in terms of u.[4] Substituting the 1 gives

$$u(1) = 1^3 = 1$$

and

$$u(2) = 2^3 = 8$$

Our new integral is

$$\int_1^8 e^u\, du$$

When we integrate, we still get

$$= e^u\big|_1^8$$

But, since we changed the 1 and 2 into 1 and 8, we can eliminate the step where we replace u with x^3. Instead, just evaluate the expression at 1 and at 8:

$$= e^8 - e^1$$

To summarize the substitution rule for definite integrals:

- Recognize that one part of the integral is "close" to being the derivative of the other part.
- Pick u
- Differentiate u to get du
- Cross multiply the expression in du as necessary
- Substitute the bounds of integration (i.e. the numbers on the integral) into the formula for u
- Find the antiderivative of the new integral
- Plug in the top number and subtract what you get when you plug in the bottom number

[4] There's also a way of doing definite integrals where you *don't* substitute the bounds of integration. But this involves an extra step at the end. If you're comfortable doing it that way, that's fine. If you're learning this for the first time, it's been my experience that students tend to make mistakes more often if they use this other method - it's best to stick with substituting the bounds of integration.

Integration by Parts

Integration by Parts is the second of two main techniques used to solve difficult integrals. Integration by Parts should be used when neither the Substitution Rule, nor any of the previous techniques are applicable. Integration by Parts is the last resort.

The formula associated with Integration by Parts is

$$\int u\,dv = uv - \int v\,du$$

To memorize, use the fact that "uv" sounds like uv rays (from the sun) and "vdu" sort of sounds like "video" or "voodoo."

The left side ($\int u\,dv$) corresponds to the integral you're trying to solve. So, our first step is always to decide which part of our integral will be u and which will be dv.

If we're trying to evaluate $\displaystyle\int xe^x\,dx$, then we need to decide whether $u = x$ and $dv = e^x\,dx$ OR $u = e^x$ and $dv = x\,dx$. (The dx always goes with dv)

To decide which part of our integral will be u, we can use the handy acronym LATE, which stands for the different function types you could encounter:

Logarithms (such as $\ln x$, $\log_5 x$, $\log_{10} x$, etc.)

Algebraic (such as x, x^5, $\frac{1}{x}$, $\sqrt{x}$, etc.)

Trigonometric (such as $\sin x$, $\cos x$, $\tan x$, etc.)

Exponential (such as e^x, 2^x, 10^x, etc.)

In order to us the LATE Rule, start at the top of list with Logarithms. Ask yourself - does our integral have any logarithms in it? If so, then that logarithm is what u should be. Whatever is leftover in the integral becomes dv. Our integral $\int xe^x\,dx$ has no logarithms. So, move one step down the list. Do we have any algebraic functions? (An algebraic function is one that *only* involves x.) Yes! We do. So, the rule says that we should make our x term into u and the $e^x\,dx$ becomes dv.

Always start from the top of LATE and work your way downwards. The first function type that you encounter in your integral is what u should become. Whatever is left over should be dv.

After picking u and dv, we can differentiate u to get du and antidifferentiate dv to get v.

Continuing with the example, we have $u = x$ and $dv = e^x\,dx$. Differentiating $u = x$ gives $du = 1\,dx$. Similarly, antidifferentiating $dv = e^x\,dx$ gives $v = e^x$. When finding v, the "+C" is always omitted.

Finally, we can plug everything into the formula $uv - \int v\,du$ and evaluate the new integral:

$$uv - \int v\,du = xe^x - \int e^x\,dx = xe^x - e^x + C$$

To summarize Integration by Parts for indefinite integrals:

1. Decide on which part of the integral will be u and which part will be dv. Remember, the entire integral must be divided between u and dv with nothing left over! To choose u, remember the acronym L-A-T-E.

2. Whatever didn't get chosen to be u ends up being set as dv

3. Differentiate u to get du and antidifferentiate dv to get v.

4. Plug everything into $uv - \int v\,du$ (if the integral $\int v\,du$ turns out to be more complicated than then the original integral, then you have made a horrible mistake. The new integral should always be easier to work with. Maybe try switching your choices of u and dv)

5. Antidifferentiate $\int v\,du$

6. All done!

When working with a definite integral, the only difference is that, at the final step, instead of a "+C", evaluate the whole expression by plugging in the top number minus what we get by plugging in the bottom number.

Since this is an introductory Calculus course, you'll be limited as to what sorts of questions you can be asked to solve. Many of the integration by parts questions will have a certain "look" to them. If you learn what the different types look like, you'll become more comfortable knowing when to use integration by parts. The integration by parts questions you'll encounter will usually fall into one of the categories below:

- The product of a power of x and $\ln x$

$$\int x \ln x\,dx \quad \int \sqrt{x} \ln x\,dx \quad \int x^5 \ln x\,dx$$

  The only exception to this is $\displaystyle\int \frac{\ln x}{x}\,dx$ which should be done with the substitution rule (with $u = \ln x$ and $du = \frac{1}{x}\,dx$

- The product of a power of x and e^{kx} (*k is some number*)

$$\int xe^x\,dx \quad \int x^2e^{5x}\,dx \quad \int xe^{-x}\,dx$$

Later, when trig is covered, we'll also be able to include:

- The product of a power of x and $\sin(kx)$

$$\int x \sin x\,dx \quad \int x^2 \sin 5x\,dx \quad \int x \sin 7x\,dx$$

- The product of a power of x and $\cos(kx)$

$$\int x^2 \cos 2x\,dx \quad \int x \cos 4x\,dx \quad \int x \cos x\,dx$$

For each of these cases, *usually* the power of x will be 1. Of course, this covers most, but not all, cases.

Summary of Integration Techniques

Here's a quick summary of what you should be thinking when you try to solve an integral:

1. **Can I evaluate the integral directly?** If you're lucky, the integral will be one of the formulas that you've (hopefully) memorized. If it is, then we can integrate the function right away.

$\int x^2 + e^x \, dx$	This integral can be evaluated by using the formulas directly
$= \frac{x^3}{3} + e^x + C$	

2. **Is the integral a product of a few terms that can be expanded out?** This is the first of two ways that an integral can be broken up into separate terms and each term integrated separately. Only expand this way when there are *two* polynomials being multiplied - any more than that and there's probably some shorter route to the answer.

$\int (x+2)(x-1) \, dx$	We can expand this using FOIL (First, Inside, Outside, Last).
$= \int x^2 + 2x - x - 2 \, dx$	Collect terms.
$= \int x^2 + x - 2 \, dx$	Integrate using the formulas.
$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + C$	

3. **Is the integral a quotient with a single term on the bottom that I can divide through?** This is the second way that we can try to break up the integral into separate terms. This method works well if the denominator is a single term like x or $2x^5$, but it doesn't work if the denominator is the sum of a few terms such as $x + 2$ or $x^2 + x + 4$.

$\int \frac{x^2 + 4x + 1}{x} \, dx$	Since there's just an x in the denominator, we can divide it through.
$= \int \frac{x^2}{x} + \frac{4x}{x} + \frac{1}{x} \, dx$	We've broken apart the fraction. Now, we can simplify.
$= \int x + 4 + \frac{1}{x} \, dx$	Finally, we can integrate.
$\frac{x^2}{2} + 4x + \ln x + C$	

4. **Does one part seem "close" to the derivative of the other part?** If part of the integral is the derivative of the other part, then the substitution rule can usually be used. Even if it's not quite the derivative, as long it's a *multiple* of the derivative, it's close enough to use the substitution rule.

$\int x^2 e^{x^3} \, dx$	Notice that the derivative of x^3 is *pretty close* to x^2. This tell us that the substitution rule is appropriate and $u = x^3$. Then, $du = 3x^2 \, dx$ and $\frac{du}{3} = x^2 \, dx$
$= \int e^u \frac{du}{3}$	Next, we can factor out the $\frac{1}{3}$.
$= \frac{1}{3} \int e^u \, du$	Now, we can evaluate the integral.
$= \frac{1}{3} e^u + C$	Finally, substitute back in $u = x^3$.
$= \frac{1}{3} e^{x^3} + C$	

5. **For a last resort, we can use Integration by Parts.** If nothing else works, the last thing you can try is Integration by Parts.

$\int x \ln x \, dx$

None of the previous techniques work here, so we're stuck with Integration by Parts. Using "LATE", we set $u = \ln x$ and $dv = x\,dx$. So, $du = \frac{1}{x}\,dx$ and $v = \frac{x^2}{2}$. Plug these into the formula, $uv - \int v\,du$.

$= (\ln x)\frac{x^2}{2} - \int \frac{x^2}{2}\frac{1}{x}\,dx$

Simplify the new integral.

$= (\ln x)\frac{x^2}{2} - \int \frac{x^2}{2x}\,dx$

$= (\ln x)\frac{x^2}{2} - \int \frac{x}{2}\,dx$

$= (\ln x)\frac{x^2}{2} - \frac{1}{2}\int x\,dx$

Integrate.

$= (\ln x)\frac{x^2}{2} - \frac{1}{2}\cdot\frac{x^2}{2} + C$

Simplify...more.

$= (\ln x)\frac{x^2}{2} - \frac{x^2}{4} + C$

Partial Fractions

Partial Fractions is a method to break up a single rational function into the sum of two separate functions.

Partial Fractions questions are usually phrased in the same way on a test:
Given that $\frac{5x+9}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+5}$, find A.

The denominator on the left will usually be factored for you and the question will always contain an A and a B. If you see a capital A and B in a question, it will always be a partial fractions question.

To solve, first we get a common denominator on the right side. The common denominator will be the product of $(x+3)$ and $(x+5)$. This gives us:

$$\frac{5x+9}{(x+3)(x+1)} = \frac{A(x+5)}{(x+3)(x+5)} + \frac{B(x+3)}{(x+3)(x+5)}$$

Next, notice that each term in our equation has the same denominator. This means we can rid of that denominator altogether and shift our focus towards the numerators:

$$5x+9 = A(x+5) + B(x+3)$$

Now, since we're solving for A, we want to find a value of x to make the B term disappear. In this case, we want $x = -3$[5]. Substituting that into the equation and solving for A, we get:

$$5(-3)+9 = A(-3+5) + B(-3+3)$$

$$-6 = 2A$$

$$-3 = A$$

Had we been trying to solve for B, we'd set $x = -5$ instead.

To solve a partial fractions question:

- Get a common denominator on the right side.

[5]Since $-3+3=0$

- Since both sides will now have the same denominator, it can be eliminated altogether and the numerators can be set equal.

- Pick an appropriate value to substitute for x that will make one of the terms disappear. If the question asks to find A, pick a value of x that will make the B term disappear. If the questions asks to find B, pick a value of x that will make the A term disappear.

- Solve for the remaining letter - either A or B.

A few more properties of Definite Integrals

A few final properties of definite integrals much be memorized.

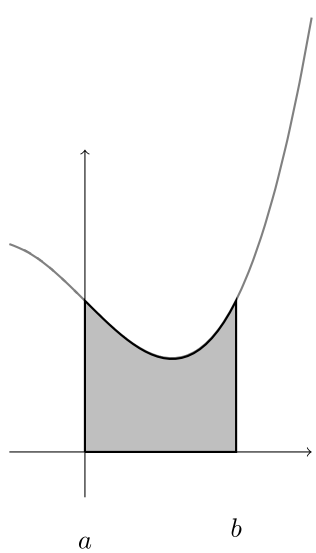
- $\int_a^a f(x)\,dx = 0$
  If the bounds of integration on the integral are equal, then the integral will be equal to zero. Regardless of the function being integrated.

- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
  If we have two integrals being added together, with the same function inside each and *the number on top of one integral is equal to the number on the bottom of the other*[6], then we can collapse the two integrals into a single integral that "skips over" the number they have in common. Instead of going from a to b and then b to c, we can just go from a to c.

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
  When flipping the bounded of integration, put a negative in front of the integral.

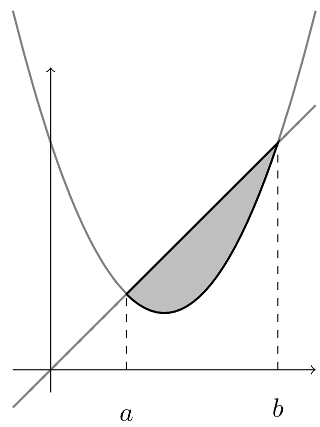
Area

The key idea to solving area problems is that definite integrals represent areas.



[6]in this case, the common number is "b"

To find the area shown above – between the curve $f(x)$ and the x-axis, between $x = a$ and $x = b$ – just find $\int_a^b f(x)\,dx$



When two curves are used, the area can be calculated by finding the area below the higher curve subtract the area below the lower curve: $\int_a^b f(x) - g(x)\,dx$. By subtracting the two, we'll be left with will be the area that they overlap.

If you don't have a graph, it helps to try to graph the curves. In fact, for many questions a picture is essential. Sometimes on the test, a picture will be given if the curves are particularly complicated. However, for simpler curves, you should attempt to graph the functions.

Finding the area between a curve and the x-axis is sometimes called vertical slicing. Finding the area between a curve and the y-axis is called horizontal slicing. Most questions will involve vertical slicing.

To find the area between two curves $f(x)$ and $g(x)$ with vertical slicing:

1. Figure out where the curves intersect. If you are given a diagram, this information may be given to you. If you aren't given the information, set the two curves equal ($f(x) = g(x)$) and solve for x. You should end up with two values of x. These are the bounds of integration - a and b in the integral $\int_a^b$. The bigger number should go on top.

2. Determine which curve is on top. If you are given a diagram, you can tell which is on top by looking at the picture. If you don't have a diagram, you can do the following:

   a) Pick a number between a and b

   b) Evaluate $f(x)$ and $g(x)$ at this point

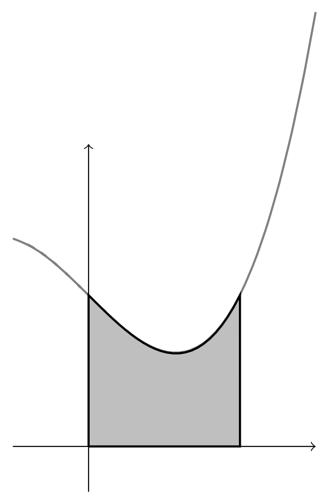
   c) If $f(x)$ is bigger than $g(x)$ at that point, then $f(x)$ is on top. If $g(x)$ is bigger than $f(x)$, then $g(x)$ is on top.

3. Set up and evaluate the integral $\int_a^b (\text{top curve}) - (\text{bottom curve})\,dx$

If you're finding the area through horizontal slicing, follow the exact same steps as above, except you'll need to isolate x before starting - your equation should be in the form $x = [\text{stuff involving } y]$.

Volume

Finding the volume by rotating a region about an axis is very similar to finding the area between the curve and that axis. The volume of a solid formed by rotating a single function around the x axis,



can be calculated by the integral $\pi \int_a^b [f(x)]^2 \, dx$.

When rotating around the x axis, be sure y is isolated. When rotating about the y axis, instead have x isolated.

As with area, you'll want to attempt a diagram if one isn't given.

Rotating a region bounded by two curves is slightly more complicated. To find the volume of the solid:

1. Isolate the proper variable. When rotating about the x axis, make sure that y is isolated. When rotating about the y axis, make sure that x is isolated.[7]

2. Figure out where the curves intersect. If you are given a diagram, this information may be given to you. If you aren't given the information, set the two equations equal ($f(x) = g(x)$) and solve the variable. You should end up with two values. These are the bounds of integration - a and b in the integral $\int_a^b$. The bigger number should go on top.

3. Determine which curve is on top. If you are given a diagram, you can tell which is on top by looking at the picture. If you don't have a diagram, you can do the following:

   a) Pick a number between a and b
   b) Evaluate $f(x)$ and $g(x)$ at this point
   c) If $f(x)$ is bigger than $g(x)$ at that point, then $f(x)$ is on top. If $g(x)$ is bigger than $f(x)$, then $g(x)$ is on top.

4. Set up and evaluate the integral $\pi \int_a^b (\text{top curve})^2 - (\text{bottom curve})^2 \, dx$

[7] In the steps that follow, I assumed y was isolated. If x is isolated, then replace the letter x with y. The same steps work.

Improper Integrals

Improper integrals are integrals where one of the numbers is ∞ or $-\infty$. Technically, the proper way to deal with these is through limits. So, many of the in-class and textbook examples involve limits. But, since the tests are entirely multiple choice and you never have to show your work, you can use a shorter method and skip the limits entirely.

Two important things to keep in mind:

- $\frac{1}{\infty} = 0$
- Always simplify your expression as much as possible before plugging in the values.

If you make sure to remember these, then improper integrals are about as hard as regular integrals. For example:

$$
\begin{aligned}
&\int_1^\infty x^{-3}\,dx \\
&= \left.\frac{x^{-2}}{-2}\right|_1^\infty \\
&= -\left.\frac{1}{2x^2}\right|_1^\infty && \text{Simplify the integral before substituting in} \\
&= \left(-\frac{1}{2\left(\infty\right)^2}\right) - \left(-\frac{1}{2(1)^2}\right) \\
&= 0 + \frac{1}{2} && \text{Using the fact that } \tfrac{1}{\infty} = 1 \\
&= \frac{1}{2} && \text{The integral converges to } \tfrac{1}{2}
\end{aligned}
$$

After evaluating the integral, if you end up with a single number, then the integral converges. If you end up with either ∞ or $-\infty$, then the integral diverges.

Two other shortcuts for evaluating improper integrals:

- $\displaystyle\int_a^\infty x^n\,dx$ always diverges when $n \geq -1$
- $\displaystyle\int_a^\infty e^{kx}\,dx$ always diverges when $k \geq 0$

Partial Derivatives

When we have a function with several variables, the function will have several derivatives - one associated with each variable. In this case, the derivatives are called partial derivatives. The derivative of f with respect to x is written as f_x or $\dfrac{\partial f}{\partial x}$.

To find the partial derivatives of a function, every variable except the one being differentiated is treated as though it was a constant. When finding f_x, only x is being differentiated - everything else is staying constant.

Suppose we have the function $f(x, y, z) = x^2 + 2xy + e^{yz}$ and we're finding f_x. To do this, imagine all the y and z terms are held constant. Sometimes, it can help to physically replace y and z with actual numbers and think about how you'd differentiate *that* function:

$f(x) = x^2 + 2x(3) + e^{3\cdot 4} \qquad f'(x) = 2x + 2(3)$

$g(x, y, z) = x^2 + 2xy + e^{yz} \qquad g_x(x, y, z) = 2x + 2y$

Optimization

Optimization involves finding the maximum or minimum value of an equation. To do this:

1. Find all the first partial derivatives
2. Set each of the first partial derivatives equal to zero and solve for x and y
3. The points (x, y) you found in the previous step are called Critical Points
4. Find the second partial derivatives f_{xx}, f_{xy} and f_{yy} and set up the function $T(x, y) = f_{xx}f_{yy} - (f_{xy})^2$. This is used to test whether the Critical Points are maximums, minimums or saddle points[8].
5. Substitute each of the Critical Points into the $T(x, y)$ function as well as into f_{xx}
6. Use the table below to determine whether each point is a maximum, minimum or saddle point

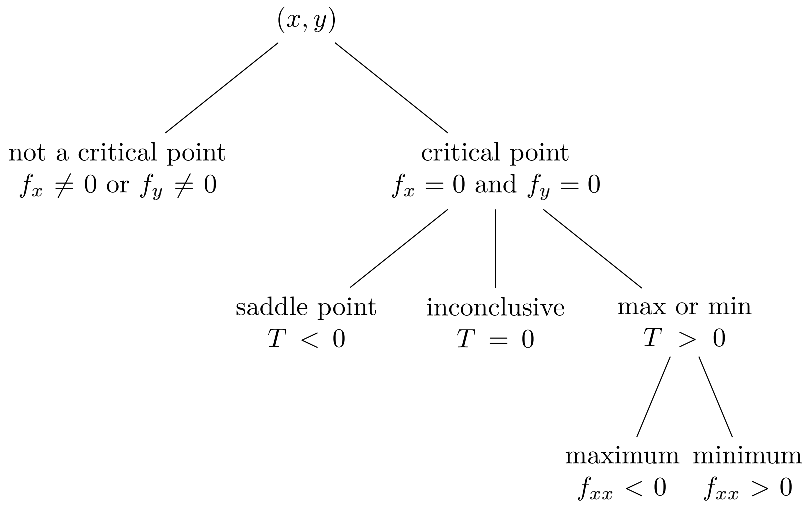
- If $T(x, y) > 0$ and $f_{xx}(x, y) > 0$, then (x, y) is a minimum
- If $T(x, y) > 0$ and $f_{xx}(x, y) < 0$, then (x, y) is a maximum
- If $T(x, y) < 0$, then (x, y) is a saddle point
- If $T(x, y) = 0$, then the test is inconclusive and we can't say anything about (x, y)

In order to be a maximum or minimum, a point *must* be a critical point. In many test questions, you're given all of the first and second partial derivatives. Then, you're presented with a point and asked to determine what's going on with it. For these questions, follow these steps:

1. First, check if it's actually a critical point. Plug the point into each of the first partial derivatives and make sure that each of the first partial derivatives is equal to zero. If any of the equations is not equal to zero, then the point is not a critical point. If all are equal to zero, we have to keep going and see if it's a maximum, minimum or a saddle point.
2. Plug the point into $T(x, y) = f_{xx}f_{yy} - (f_{xy})^2$ and into $f_{xx}(x, y)$ and use the list of four cases to determine if it's a maximum, minimum or a saddle point.

This can be summarized further as

[8] A saddle point is a point that's in between being a maximum and a minimum



Lagrange Multipliers

Lagrange Multipliers are used to find the maximum or minimum of a function subject to some constraint. For example, you may want to build a fence enclosing the largest possible area, but you are limited in how much wood you're able to use. In each question, you'll be given the function you're trying to maximize or minimize (this is always called $f(x, y)$) and the constraint (this is always called $g(x, y)$).

If the constraint is given as an equation such as $x + y = 5$, then you need to move everything to one side before calling it $g(x, y)$. Here we'd get $g(x, y) = x + y - 5$.

After finding $f(x, y)$ and $g(x, y)$, to solve the Lagrange Multiplier question:

1. Set up the Lagrange function: $L(x, y) = f(x, y) + \lambda g(x, y)$
2. Find the first partial derivatives of $L(x, y)$ with respect to each variable - x, y, λ (and z if applicable)
3. Set each first partial derivative equal to zero
4. Solve the equations for x and y (and z, if applicable).
5. If you have several values of (x, y), then substitute each into $f(x, y)$ to determine which gives the maximum or minimum values

Trigonometry

Some basic facts about trig functions that you should know:

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$

These can be memorized with the mnemonic SOH CAH TOA.

Additionally, three more trig functions - secant, cosecant and cotangent - are defined in terms of sine, cosine and tangent:

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

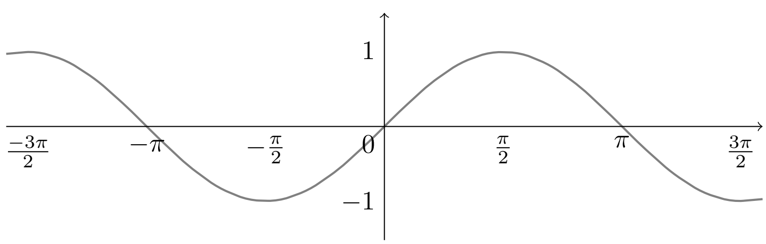
$\cot x = \frac{1}{\tan x}$

The tangent and cotangent functions can both be defined in terms of sine and cosine:

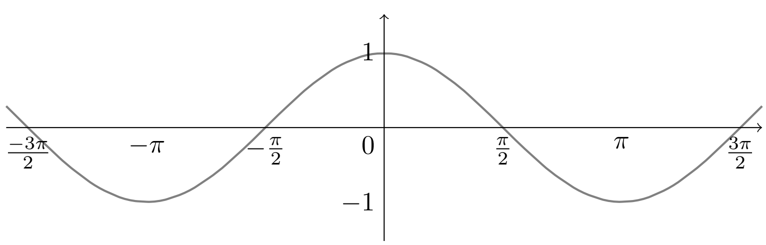
$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

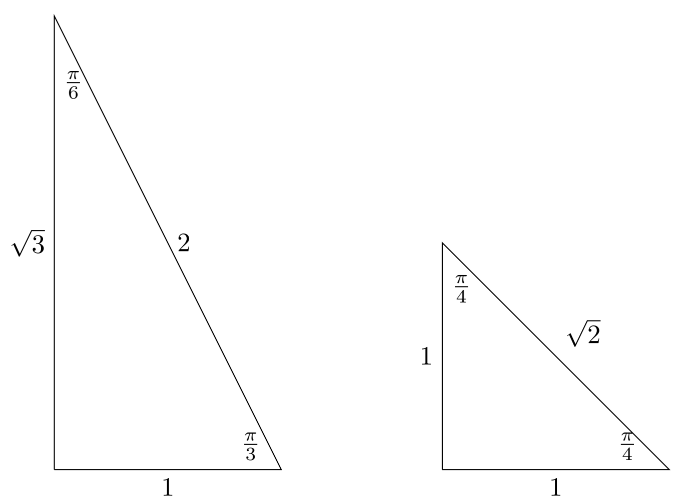
You'll need to be familiar with the values of sine, cosine, tangent, secant, cosecant and cotangent at multiples of $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$, $\frac{\pi}{6}$ and π. You can use the formulas above combined with the curves below and the special triangles.



$y = \sin(x)$



$y = \cos(x)$



Derivatives

The following derivatives must be memorized

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

Remember, if the function starts with a "c", then the derivative is negative. Also, the derivatives can be memorized in pairs: sin/cos, tan/cot, sec/csc.

Antiderivatives

The following antiderivatives must be memorized

$f(x)$	$\int f(x)\,dx$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$

If you have an antiderivative with both sin and cos terms OR with both tan and sec terms, then you probably want to use the Substitution Rule.

Differential Equations

Differential equations are types of equations that relate a function y and it's derivative $\frac{dy}{dx}$. Two types of differential equations are covered.

Separable Differential Equations

A separable differential equation is a type of differential equation where the x terms and y terms can be separated and moved to opposite sides of the equal sign. Of the two types of differential equations, this is the easiest to deal with.

Usually, if the terms in the equation are clustered together next to the equal sign – such as $\frac{dy}{dx} = xy^2$ – then the differential equation is separable.

To solve a separable differential equation:

- Cross multiply to separate the x terms to one side and the y terms to the opposite side. Make sure that dx and dy aren't in the denominator!
- Integrate both sides
- Isolate y if necessary

Linear Differential Equations

The second type of differential equation is the Linear Differential Equation. Linear Differential Equations can be identified because they'll have three terms and can be put in the form:

$$\frac{dy}{dx} + p(x) \cdot y = q(x)$$

To solve a linear differential equation:

- If the equation isn't already in the form $\frac{dy}{dx} + p(x) \cdot y = q(x)$, start by putting it in this form
- Find the integrating factor $I(x)$ by using the formula $I(x) = e^{\int p(x)}$
- The solution to the differential equation will be $y = \dfrac{\int I(x) \cdot q(x)}{I(x)}$

Applications of Differential Equations

For many questions involving Applications of Differential Equations, you'll be asked to simply set up the equations and not solve anything. For others, you'll actually need to solve the equations. Each application has a particular set of equations associated with it.

Exponential Growth

If $\frac{dy}{dt} = ky$, then the solution is $y = Ae^{kt}$, where A is the value of y when $t = 0$.

Mixing Problems

$$\frac{dy}{dx} = \text{input} - \text{output}$$

$$\frac{dy}{dx} = (\text{flow rate}) \times (\text{input concentration}) - (\text{flow rate}) \times \frac{y}{\text{capacity of tank}}$$

Newton's Law of Cooling

If t is time, T represents the temperature of an object and M is the constant temperature of the room the object has been placed in, then the change in the temperature of the object is given as:

$$\frac{dT}{dt} = k(T - M)$$

Falling Objects

$$\text{height} = s(t)$$
$$\text{velocity} = v(t) = s'(t)$$
$$\text{acceleration} = a(t) = v'(t) = s''(t)$$

Slope

$$\frac{dy}{dx} = \text{slope of the curve}$$

Practice Problems

Exponential and Logarithms

1. Simplify $2\log_6 3 + \log_6 4$.

A: 36	B: 2	C: 0	D: 24	E: 6

2. Solve for x: $e^{3x} = 2$.

A: $\frac{2}{3}$	B: $3\ln 2$	C: $\frac{1}{3}\ln 2$	D: $\log_2 3$	E: $\log_3 2$

3. Simplify $e^{\ln 2 + \ln 5}$

A: 7	B: 10	C: $\ln 7$	D: $\ln 10$	E: 2^5

4. Simplify $\log_5 45 - \log_5 9$.

A: 0	B: 1	C: 3	D: 5	E: 15

5. Find the value of $\log_{100} 10$.

A: 2	B: $\frac{1}{2}$	C: 1	D: 10	E: $\frac{1}{10}$

6. Simplify $e^{\ln 2 + \ln 3}$

A: 5	B: 6	C: 8	D: 9	E: 0

7. $e^{-\ln 3} =$

A: -3	B: $-\frac{1}{3}$	C: 0	D: $\frac{1}{3}$	E: 3

8. $3^{2x+5} = 81$, find x.

A: -1	B: $-\frac{1}{2}$	C: $\frac{5}{2}$	D: $\frac{2}{5}$	E: 0

9. Simplify as much as possible $\frac{\ln x + \ln y}{\ln(x^2y^2)}$.

A: $\frac{x+y}{x^2y^2}$	B: $\frac{1}{xy}$	C: $\frac{1}{2}$	D: $\ln\left(\frac{1}{xy}\right)$	E: $\ln\left(\frac{x+y}{x^2y^2}\right)$

10. Simplify $\frac{\log_2 36 - \log_2 4}{\log_2 27}$.

A: $\frac{32}{27}$	B: $\frac{2}{3}$	C: $\frac{1}{3}$	D: $\log_2\left(\frac{1}{3}\right)$	E: $\log_2\left(\frac{32}{27}\right)$

11. Express $\dfrac{\log_2 8 - \log_3 9}{\log_4 2}$ as simply as possible.

A: $\frac{1}{2}$	B: $\frac{3}{4}$	C: 2	D: 3	E: $\log_2\left(\frac{2}{9}\right)$

12. Find the value of $5^{2\log_5 10}$

A: 5	B: 10	C: 20	D: 40	E: 100

13. If $2^{3x-1} = 16$, what is the value of x?

A: 5	B: 1	C: 4	D: $\frac{4}{3}$	E: $\frac{5}{3}$

14. $\log_3 36 - \log_3 4 =$

A: 32	B: 2	C: 12	D: 9	E: 144

15. Find the value of $\log_3 2 - \log_3 18$

A: -36	B: -16	C: -2	D: $\frac{1}{9}$	E: 2

16. Simplified, $e^{5\ln 2}$ is equal to

A: 0	B: 10	C: 25	D: 32	E: e^{10}

17. Simplified, $\log_6 8 + \log_6 27$ is equal to

A: 0	B: 1	C: 2	D: 3	E: 6

18. Simplified as much as possible, $e^{2\ln 7} =$

A: 128	B: 14	C: 49	D: $\frac{2}{7}$	E: $\frac{7}{2}$

19. Simplified as much as possible, $\dfrac{\log_5 1 - \log_5 27}{\log_5 9} =$

A: -3	B: $-\frac{3}{2}$	C: $\frac{1}{243}$	D: $\log_5 3$	E: 15

20. Find the value of $e^{-2\ln 3}$

A: -6	B: -9	C: $\frac{1}{9}$	D: $\frac{1}{6}$	E: $\frac{2}{3}$

21. Simplified, $e^{3\ln 5} =$

A: 0	B: $\frac{3}{5}$	C: 15	D: 125	E: 243

22. Find the value of $5^{\log_5 7}$

A: 5	B: 7	C: 35	D: 7^5	E: 5^7

23. $\log_5 2 - \log_5 50 =$

A: -100	B: -48	C: -2	D: $\frac{1}{25}$	E: 25

24. Simplify $\dfrac{\log_3 4 + \log_3 8}{\log_3 8}$.

A: $\log_3 4$	B: $\log_3 5$	C: $\frac{5}{3}$	D: 3	E: 4

Differentiation

25. Find $\lim\limits_{h\to 0} \dfrac{\ln(4+h) - \ln 4}{h}$ by recognizing it as a derivative.

A: $\frac{1}{4}$	B: 4	C: $\ln 4$	D: 0	E: not defined

26. Evaluate $\lim\limits_{h\to 0} \dfrac{\ln\left[(2+h)^2+3\right] - \ln 7}{h}$ by recognizing it as the value of a certain derivative.

A: $\frac{4}{7}$	B: 0	C: $\frac{7}{4}$	D: $\frac{1}{4}$	E: $\frac{1}{7}$

27. Evaluate $\lim\limits_{h\to 0} \dfrac{5^{2+h} - 25}{h}$ by recognizing it as the value of a certain derivative.

A: 1	B: 25	C: 0	D: $25\ln 5$	E: $\frac{25}{\ln 5}$

28. Evaluate $\lim\limits_{h\to 0} \dfrac{2^{3+h} - 2^3}{h}$ by recognizing it as the value of a certain derivative.

A: 2	B: $8\ln 2$	C: 8	D: $\frac{8}{\ln 2}$	E: $\ln 3$

29. Find $f'(x)$ where $f(x) = e^x \ln x$

A: $\frac{e^x}{x}$	B: $e^x \ln x$	C: $e^x(1+\ln x)$
D: xe^x	E: $e^x\left(\frac{1}{x} + \ln x\right)$	

30. If $f(x) = \log_3(x^2+1)$, find $f'(2)$.

A: $\frac{4}{5\ln 3}$	B: $\frac{4}{5}$	C: $\frac{4\ln 3}{5}$	D: $\frac{1}{5\ln 3}$	E: $\log_3 5$

31. If $g(x) = \ln(x^2)$, then $g'(e) =$

A: $\frac{2}{e}$	B: $\frac{1}{e}$	C: $\frac{1}{e^2}$	D: 2	E: 0

32. If $f(x) = (\ln x)^2$, find $f'(x)$.

A: $\frac{2}{x}$	B: $2\ln x$	C: $\ln(2x)$	D: $\frac{2\ln x}{x}$	E: $\frac{1}{x^2}$

33. If $f(x) = x + e^x$, find $f'(0)$.

A: 2	B: 1	C: $\ln 2$	D: $\frac{2}{\ln 2}$	E: undefined

34. If $f(x) = e^{x^2+3x}$, find $f'(1)$

A: e^4	B: $2e^4$	C: $3e^4$	D: $5e^4$	E: $6e^4$

35. If $f(x) = \log_3 x$, find $f'(x)$.

A: $\frac{1}{x\ln 3}$	B: $\frac{\ln 3}{x}$	C: $\frac{x}{\ln 3}$	D: $x\ln 3$	E: 3^x

36. If $f(x) = \log_5 x$, find $f'(x)$.

A: $\frac{1}{x}$	B: $\frac{5}{x}$	C: $\frac{1}{5x}$	D: $\frac{1}{x\ln 5}$	E: $\frac{\ln 5}{x}$

37. If $f(x) = e^{3x}$, find $f''(\ln 2)$.

A: 6	B: 8	C: $9e^6$	D: 54	E: 72

38. If $f(x) = \frac{\ln x}{x^2}$, find $f'(e)$.

A: $-\frac{1}{e}$	B: $-\frac{1}{e^2}$	C: $-\frac{1}{e^3}$	D: $-\frac{1}{e^4}$	E: $-\frac{2}{e^4}$

39. Find $g''(2)$, where $g(x) = e^{3x}$.

A: e^6	B: $3e^9$	C: $9e^9$	D: $3e^6$	E: $9e^6$

40. Find $f'(e)$ if $f(x) = x^3 \ln x$

A: $3e^2$	B: $3e$	C: $4e$	D: $4e^2$	E: e^3

41. If $f(x) = (x^2 - 2)^{10}$, find $f'(-1)$.

A: -20	B: -10	C: 0	D: 10	E: 20

42. If $f(x) = \ln(\ln x)$, find $f'(e)$.

A: 0	B: $\frac{1}{e}$	C: 1	D: e	E: $2e$

43. If $f(x) = e^3$, the $f'(x)$ is

A: e^3	B: $3e^2$	C: e^2	D: 0	E: undefined

44. If $f(x) = xe^x$, find $f'(2)$.

A: 0	B: e^2	C: $2e^2$	D: $3e^2$	E: $4e^2$

45. If $f(x) = \dfrac{\ln x}{x^2}$, find $f'(x)$.

A: $\frac{1}{2x^2}$	B: $\frac{1}{x^4}$	C: $\frac{2x \ln x - x}{x^4}$	D: $\frac{2 \ln x - 1}{x^4}$	E: $\frac{1 - 2 \ln x}{x^3}$

46. If $f(x) = \dfrac{\ln x}{x^4}$, find $f'(e)$.

A: $-\frac{5}{e}$	B: $-\frac{2}{e^3}$	C: $-\frac{1}{e^4}$	D: $-\frac{3}{e^5}$	E: $-\frac{4}{e^8}$

47. If $f(x) = 3e^x - 2 \ln x + \frac{1}{4x}$, what is $f'(x)$?

A: $3e^x - \frac{2}{x} - \frac{1}{4x^2}$	B: $3e^x - \frac{2}{x} + \frac{1}{4}$	C: $e^{3x} - \frac{2}{x} - 2 \ln x + \frac{1}{2x^2}$
D: $3e^x - \frac{1}{x^2} + \ln(4x)$	E: $3e^x - \frac{2}{x} - \frac{1}{16x^2}$	

48. If $f(x) = \dfrac{e^x}{2} - x^2$, find $f'(\ln 4)$.

A: $2 - \ln 8$	B: $2 - \ln 16$	C: $2 - (\ln 4)^2$	D: $-3 \ln 2$	E: $\frac{3}{2} \ln 4$

49. If $f(x) = \log_5 x$, find $f'(x) =$

A: $\frac{\ln x}{\ln 5}$	B: $\frac{1}{x \ln 5}$	C: $\frac{1}{x}$	D: $\frac{1}{5x}$	E: $\frac{1}{\ln 5}$

50. If $f(x) = \dfrac{e^x}{x^2}$, find $f'(1)$.

A: $-e$	B: $\frac{1}{2}e$	C: e	D: $2e$	E: $3e$

51. If $f(x) = [\ln(3x + e)]^{2e}$, find $f'(x)$.

A: $\left(\frac{1}{3x + e}\right)^{2e}$	B: $\left(\frac{3}{3x + e}\right)^{2}$	C: $\frac{6e}{3x + e} [\ln(3x + e)]^2$
D: $\frac{6e}{3x + e} [\ln(3x + e)]^{2e-1}$	E: $\frac{8e}{3x + e} [\ln(3x + e)]^{2e}$	

52. Find $\frac{d}{dx}(\log_5 x^2)$.

A: $\frac{2}{x}$	B: $\frac{2}{x \ln 5}$	C: $x^2 \ln 5$	D: $\frac{1}{x^2 \ln 5}$	E: $2x^3 \ln 5$

53. If $f(x) = 5^{2x}$, then $f'(1) =$

A: 10	B: 25	C: 50	D: $50 \ln 5$	E: $100 \ln 5$

54. If $y = 10^{2x+1}$, then $\frac{dy}{dx} =$

A: $(2x+1)10^{2x}$	B: $(2 \ln 10)10^{2x+1}$	C: $\left(\frac{2}{\ln 10}\right) 10^{2x+1}$
D: 10^{2x+1}	E: $(10^{2x+1}) \cdot 2$	

55. If $f(x) = x^2 e^{x^2}$, then $f'(x) =$

A: $x^4 e^{x^2-1}$	B: $4x^2 e^{x^2}$	C: $e^{x^2}(1+2x)$	D: $e^{x^2}(x^2+2x)$	E: $e^{x^2}(2x^3+2x)$

56. If $f(x) = \frac{x^3}{\ln x}$, then $f'(e) =$

A: $2e^2$	B: $-2e^2$	C: 0	D: $3e^3$	E: $-3e^3$

57. Find $f'(x)$, where $f(x) = 2^x$

A: 2	B: 2^x	C: $2^x \ln x$	D: $2^x \ln 2$	E: $\frac{1}{2^x \ln 2}$

58. The slope of the tangent line to the curve $y = (3 + \ln x)^2$ at the point where $x = \frac{1}{e}$ is

A: $\frac{4}{e}$	B: $6e$	C: 6	D: $4e$	E: 4

59. Find the slope of the tangent line to the graph of $y = 7^x$ at the point where $x = 1$.

A: 0	B: 7	C: $\ln 7$	D: $\frac{7}{\ln 7}$	E: $7 \ln 7$

60. At what point on the curve $y = x + \ln x$ is the tangent line parallel to the line $y = 3x - 5$?

A: $x = 3$	B: $x = \frac{1}{2}$	C: $x = \frac{1}{3}$	D: $x = \frac{1}{4}$	E: $x = \frac{1}{15}$

61. Determine the x-coordinate of the point on the curve $y = 7^x$ where the slope of the tangent line is equal to $\ln 49$.

A: 1	B: 2	C: 7	D: $\frac{\ln 2}{\ln 7}$	E: $\ln 14$

62. At what point on the curve $y = 3x + e^x$ is the tangent line parallel to the line $y = 7x + 10$.

A: $x = 0$	B: $x = 1$	C: $x = 3$	D: $x = \ln 3$	E: $x = \ln 4$

63. If the tangent line at the point (a, b) on the curve $y = 4 + 3\ln x$ is parallel to the line $y = \frac{1}{4}x + 2$, then a is

A: $\frac{1}{2}$	B: 2	C: $\frac{3}{x}$	D: $\frac{3}{4}$	E: 12

64. Find the slope of the tangent line to the graph of $y = 3^x$ at the point where $x = 0$.

A: $\frac{1}{\ln 3}$	B: $\ln 3$	C: 3	D: 1	E: 0

65. Determine the x-coordinate of the point on the curve $y = 8\ln(x + 1)$ where the tangent line is parallel to the line $y = 2x + 5$.

A: -1	B: 0	C: 1	D: 2	E: 3

66. If $f(x) = xe^{x^2}$, find $f'(2)$.

A: $9e^4$	B: e^4	C: $2e^4$	D: $3e^4$	E: $5e^4$

67. If $f(x) = \log_2 x$, find $f'(1)$.

A: $\ln 2$	B: 0	C: 1	D: $\frac{1}{\ln 2}$	E: 2

68. If $f(x) = x\ln x$, then $f'(e) =$

A: 0	B: 1	C: 2	D: $\frac{1}{e}$	E: e

69. If $f(x) = 3^{2x}$, find $f'(x)$.

A: $(2x)3^{2x-1}$	B: 3^{2x}	C: $2(3^{2x})$	D: $2(\ln 3)3^{2x}$	E: $(\ln 3)3^{2x}$

70. If $f(x) = 3^{x^2}$, find $f'(x)$.

A: $(2\ln 3)x3^{x^2}$	B: $(\ln 3)3^{x^2}$	C: $2xe^{x^2}$
D: $(\ln 3)x^2 3^{x^2-1}$	E: $(2\ln 3)x^3 3^{x^2-1}$	

71. Find $g'(1)$, where $g(x) = x^2e^{3x}$.

A: e^3	B: $2e^3$	C: $5e^3$	D: $6e^3$	E: 0

72. If $f(x) = e^2 \ln x$, then $f'(e) =$

A: e^2	B: e	C: $2e$	D: $3e$	E: 0

73. If $f(x) = x^3 e^x$, then $f'(1)$ is

A: $4e$	B: 4	C: $3e$	D: 3	E: 2

Intervals of Increase, Decrease and Concavity

74. The function $f(x) = x^4 e^x$ is decreasing on the interval

A: $(-\infty, -4)$	B: $(-4, 0)$	C: $(0, \infty)$	D: $(0, 4)$	E: $(-\infty, \infty)$

75. Given that $f(x) = xe^{-x}$, $f'(x) = (1-x)e^{-x}$ and $f''(x) = (2-x)e^{-x}$, find the interval on which $f(x)$ is increasing.

A: $(1, \infty)$	B: $(2, \infty)$	C: $(1, 2)$	D: $(-\infty, 1)$	E: $(-\infty, 2)$

76. Given that $f(x) = xe^x$, $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$, find the interval on which $f(x)$ is increasing.

A: $(-\infty, -1)$ only	B: $(-1, \infty)$ only	C: $(-\infty, -2)$ only
D: $(-2, \infty)$ only	E: $(-2, -1)$ only	

77. Given that $f(x) = xe^x$, $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$, find the interval on which $f(x)$ is concave up.

A: $(-\infty, -1)$ only	B: $(-1, \infty)$ only	C: $(-\infty, -2)$ only
D: $(-2, \infty)$ only	E: $(-2, -1)$ only	

78. Given that $f(x) = (x-4)e^x$, $f'(x) = (x-3)e^x$ and $f''(x) = (x-2)e^x$, find the interval on which $f(x)$ is increasing.

A: $(-\infty, 3)$ only	B: $(3, \infty)$ only	C: $(-\infty, 2)$ only
D: $(2, \infty)$ only	E: $(2, 3)$ only	

79. You are given $f(x) = (2-x)e^x$, $f'(x) = (1-x)e^x$ and $f''(x) = -xe^x$. Then $f(x)$ decreases on the interval

A: $(-\infty, 0)$	B: $(-\infty, 1)$	C: $(0, \infty)$
D: $(1, \infty)$	E: $(0, 1)$	

80. You are given that $f(x) = xe^x$, $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$. Select the statement which is false.

A: $f(x)$ decreases on $(-\infty, -1)$	B: $f(x)$ increases on $(-1, \infty)$
C: $f(x)$ decreases on $(\infty, -2)$	D: $f(x)$ increases on $(-2, \infty)$
E: the minimum value of $f(x)$ is $-\frac{1}{e}$	

81. The function $f(x) = x \ln x$ is increasing on the interval

A: $\left(0, \dfrac{1}{e}\right)$	B: $(0, 1)$	C: $(0, \infty)$	D: $(1, \infty)$	E: $\left(\dfrac{1}{e}, \infty\right)$

Logarithmic Differentiation

82. If $y = (x+2)^{\ln x}$, find $\dfrac{dy}{dx}$ at $x = 1$.

A: 0	B: 3	C: 1	D: $\ln 3$	E: $3 \ln 3$

83. If $f(x) = x^{3x}$, then $f'(x) =$

A: 3	B: x^{3x}	C: $3x^{3x}$	D: $x^{3x}(1 + \ln x)$	E: $3x^{3x}(1 + \ln x)$

84. Find $f'(2)$ where $f(x) = x^x$.

A: $4 + 4\ln 2$	B: $4 \ln 2$	C: $4 + \ln 2$	D: 4	E: $\ln 2$

85. If $f(x) = x^{\ln x}$, find $f'(x) =$

A: $(\ln x)(x^{\ln x - 1})$	B: $(x^{\ln x})(\ln x)$	C: $(x^{\ln x})\left(\dfrac{1}{x}\right)$
D: $(2x^{\ln x})\left(\dfrac{\ln x}{x}\right)$	E: $\dfrac{1}{x}$	

86. If $y = (4x+3)^{x+1}$, find $\frac{dy}{dx}$ at $x = 0$.

A: 1	B: 4	C: $3\ln 3 + 4$	D: $1 + \ln 3$	E: $4 \ln 3$

87. If $f(x) = (3x+2)^x$, then $f'(x) =$

A: $(3x+2)^x$	B: $(3x+2)^x \ln(3x+2)$
C: $(3x+2)^x \left[\frac{x}{3x+2} + \ln(3x+2)\right]$	D: $x(3x+2)^{x-1}$
E: $(3x+2)^x \left[\frac{3x}{3x+2} + \ln(3x+2)\right]$	

88. If $f(x) = (x+1)^{\sqrt{x}}$, then $f'(x)$ is

A: $(x+1)^{\sqrt{x}}$	B: $\frac{1}{2}x^{-1/2}(x+1)^{\sqrt{x}}$
C: $\sqrt{x}\ln(x+1)$	D: $\sqrt{x}(x+1)^{\sqrt{x}-1}$
E: $(x+1)^{\sqrt{x}} \left(\frac{\sqrt{x}}{x+1} + \frac{\ln(x+1)}{2\sqrt{x}}\right)$	

89. If $f(x) = x^{2x}$, find $f'(2)$

A: $32\ln 2$	B: $32(1+\ln 2)$	C: $16(1+\ln 2)$	D: 32	E: 16

90. Find $\frac{dy}{dx}$ if $y = (2x+5)^{4x}$

A: $4x(2x+5)^{4x-1}$	B: $4x(2x+5)^{4x-1}(2)$
C: $(2x+5)^{4x}\left(\frac{8}{2x+5}\right)$	D: $(2x+5)^{4x}(4\ln(4x+5))$
E: $(2x+5)^{4x}\left(4\ln(2x+5) + \frac{8x}{2x+5}\right)$	

91. Find $\frac{dy}{dx}$ where $y = (x+1)^{3x}$

A: $3x(x+1)^{3x-1}$	B: $(x+1)^{3x}3\ln(x+1)$
C: $\frac{3(x+1)^{3x}}{x+1}$	D: $(x+1)^{3x}\left[3\ln(x+1) + \frac{3x}{x+1}\right]$
E: $(x+1)^{3x}\left[\ln(x+1) + \frac{x}{x+1}\right]$	

92. If $f(x) = (x+2)^x$, find $f'(1)$.

A: $\ln 3$	B: 3	C: $3\ln 3+1$
D: $\ln 3+\dfrac{1}{3}$	E: None of A,B,C,D	

93. If $f(x)=(2x+3)^x$, find $f'(1)$.

A: $5\ln 5+2$	B: 10	C: $\ln 5+\dfrac{2}{5}$	D: 2	E: 1

94. If $y=(x+1)^x$, find $\frac{dy}{dx}$ at $x=1$.

A: 0	B: 1	C: $\dfrac{1}{2}+\ln 2$	D: $1+\ln 2$	E: $1+2\ln 2$

Basic Antiderivatives

95. Evaluate $\displaystyle\int \frac{\sqrt{x}+3}{x}\,dx$

A: $\dfrac{\frac{2}{3}x^{3/2}+3x}{x^2}+C$	B: $\dfrac{2}{3}x^{3/2}+3x-\dfrac{x^2}{2}+C$	C: $2x^{1/2}+3x+C$
D: $\left(\dfrac{2}{3}x^{3/2}+3x\right)\ln\lvert x\rvert+C$	E: $2x^{1/2}+3\ln\lvert x\rvert+C$	

96. Evaluate $\displaystyle\int x(x+2)\,dx$

A: $\dfrac{x^2}{2}\left(\dfrac{x^2}{2}+2\right)+C$	B: x^2+2x+C	C: $\dfrac{x^2}{2}+2x+C$
D: $\dfrac{x^3}{3}+x^2+C$	E: $\dfrac{x^3}{3}+\dfrac{x^2}{2}+C$	

97. Find $\displaystyle\int \frac{3x^2+1}{3x^2}\,dx$

A: $x-\dfrac{1}{3}x^{-1}+C$	B: $\dfrac{x^3+x}{x^3}+C$	C: $1-\dfrac{1}{3}x^{-2}+C$
D: $(x^3+x)x^3+C$	E: $(x^3+x)(-3x^{-1})+C$	

98. Evaluate $\displaystyle\int \frac{x^3+x}{x^2}\,dx$

A: $\dfrac{\frac{x^4}{4}+\frac{x^2}{2}}{\frac{x^3}{3}}+C$	B: $\dfrac{x^2}{2}+\ln\lvert x\rvert+C$	C: $\dfrac{x^4}{4}+\ln\lvert x\rvert+C$
D: $\left(\dfrac{x^4}{4}+\dfrac{x^2}{2}\right)\ln\lvert x^2\rvert+C$	E: x^2+C	

99. If $f'(x) = 2e^x + 2x + \frac{2}{x}$, what is $f(x)$?

A: $2e^x + 2 - \dfrac{2}{x^2}$	B: $e^{2x} + x^2 + 2 + C$	C: $2e^x + x^2 + 2\ln\lvert x\rvert + C$
D: $2e^x + x^2 + \ln\left\lvert \dfrac{x}{x} \right\rvert + C$	E: $2(e^x + x^2 + \ln\lvert x\rvert) + C$	

100. $\displaystyle\int x(x+1)\,dx =$

A: $\dfrac{x^2}{2}\left(\dfrac{x^2}{2} + x\right) + C$	B: $x^2 + x + C$	C: $\dfrac{x^3}{3} + \dfrac{x^2}{2} + C$
D: $2x + 1$	E: 1	

101. Find $\displaystyle\int \frac{2x+1}{2x}\,dx$

A: $\dfrac{x^2 + x}{x^2} + C$	B: $\dfrac{(2x+1)^2}{2}\ln\lvert 2x\rvert + C$	C: $\ln\lvert 2x+1\rvert - \ln\lvert 2x\rvert + C$
D: $x + \ln\lvert 2x\rvert + C$	E: $x + \dfrac{1}{2}\ln\lvert x\rvert + C$	

102. If $f'(x) = e^x + \frac{1}{x}$ and $f(e) = e^e$, find $f(1)$.

A: e	B: $e - 1$	C: $e + 1$	D: $2e - 1$	E: $2e$

103. If $F'(x) = 3x^2 + 3$ and $F(0) = 4$, then $F(1) =$

A: 8	B: 7	C: 6	D: 5	E: 4

104. If $f'(x) = 4x^3 - 3x^2 + 2$ and $f(0) = 5$, then $f(1)$ is

A: 3	B: 4	C: 5	D: 6	E: 7

105. If $f'(t) = e^t$ and $f(0) = 2$, then $f(t)$ is

A: e^t	B: $e^t + C$	C: $e^t + 2$	D: $e^t + 1$	E: $2e^t$

106. Evaluate $\displaystyle\int_1^e \frac{1}{x}\,dx$

A: 1	B: e	C: -1	D: $\dfrac{1}{e} - 1$	E: $e - 1$

107. Evaluate $\displaystyle\int_1^2 \frac{x+1}{x}\,dx$

A: $\ln 3$	B: $1 + \ln 3$	C: $2 + \ln 3$	D: $2 + \ln 2$	E: $1 + \ln 2$

108. $\int_0^1 (x+1)^2\,dx =$

A: $\frac{7}{3}$	B: $-\frac{7}{3}$	C: $\frac{8}{3}$	D: $\frac{1}{3}$	E: 3

109. Evaluate $\int_0^{\ln 2} e^{3x+\ln 2}\,dx$

A: $\frac{10}{3}$	B: $\frac{14}{3}$	C: $\frac{16}{3}$	D: 6	E: 14

110. Find $\int \frac{x^3+x^2-1}{x+1}\,dx$

A: $\frac{\frac{x^4}{4}+\frac{x^3}{3}-x}{x^2+x}+C$	B: $\frac{x^3}{3}-\frac{1}{(x+1)^2}+C$	C: $2x-(x+1)^{-2}+C$
D: $\frac{x^3}{3}-\ln\lvert x+1\rvert+C$	E: None of A, B, C or D	

111. Evaluate $\int \frac{x+1}{x+2}\,dx$

A: $\frac{\frac{x^2}{2}+x}{\frac{x^2}{2}+2x}+C$	B: $x-\ln\lvert x+2\rvert+C$	C: $(x+1)\ln\lvert x+2\rvert+C$
D: $\frac{1}{(x+2)^2}+C$	E: $\ln\left\lvert\frac{x+1}{x+2}\right\rvert+C$	

Properties of Antiderivatives

112. Find $\int_1^2 \frac{d}{dx}x^x\,dx$

A: 4	B: 0	C: 2	D: 1	E: 3

113. If $F(x)=x\sqrt{x^3+1}$, find $\int_0^2 F'(x)\,dx$

A: 6	B: $\ln 3$	C: $2\sqrt{3}$	D: 7	E: 2

114. If $f(x)=\ln(x^2+1)$, evaluate $\int_{-1}^2 f'(x)\,dx$

A: $\ln 10+\ln 2$	B: $\ln\left(\frac{5}{2}\right)$	C: $\ln 3$	D: $\ln 7$	E: $\frac{9}{5}$

115. If $F(x)=\frac{1}{1+x^2}$, then $\int_0^1 F'(x)\,dx =$

A: $\frac{1}{2}$	B: $-\frac{1}{2}$	C: 0	D: 1	E: -1

116. Use the following information for parts a and b below.

$$\int_{-1}^{3} f(x)\,dx = -2 \quad \int_{3}^{5} f(x)\,dx = 6 \quad \int_{3}^{5} g(x)\,dx = -3$$

a) Find $\int_{-1}^{5} f(x)\,dx$

A: -2	B: 6	C: 4
D: -3	E: cannot be determined	

b) Find $\int_{3}^{5} [2f(x) - 3g(x)]\,dx$

A: 3	B: 12	C: 9
D: 21	E: cannot be determined	

117. $\int_{0}^{2} \frac{d}{dx}(xe^x)\,dx$ is

A: $2e^2$	B: e^2	C: $3e^2 - 1$	D: $e^2 + 1$	E: -1

Substitution Rule

118. Evaluate $\int 2x\sqrt{x^2+3}\,dx$

A: $x^2\sqrt{\frac{x^3}{3} + x} + C$	B: $\sqrt{x^2+3} + C$	C: $\frac{2}{3}x^2(x^2+3)^{3/2} + C$
D: $\frac{2}{3}(x^2+3)^{3/2} + C$	E: $\frac{2}{3}x^3 + \sqrt{3}x^2 + C$	

119. Find $\int x\sqrt{x^2+1}\,dx$

A: $\sqrt{2x} + C$	B: $\frac{(x^2+1)^{3/2}}{3} + C$	C: $\frac{x^2(x^2+1)^{3/2}}{3} + C$
D: $\frac{1}{2\sqrt{x^2+1}} + C$	E: $\frac{x^2}{4\sqrt{x^2+1}} + C$	

120. Evaluate $\int_{1}^{2} \frac{e^{1/x}}{x^2}\,dx$

A: $\sqrt{e}-e$	B: $e-e^2$	C: $\ln 2$	D: e^2-2	E: $e-\sqrt{e}$

121. If $f(x)=\dfrac{3x^2-1}{x^3-x+5}$, what is $\int f(x)\,dx$?

A: $\ln\lvert x^3-x+5\rvert+C$	B: $(3x^2-1)\ln\lvert x^3-x+5\rvert+C$	C: $(x^3-x)\ln\lvert x^3-x+5\rvert+C$
D: $\dfrac{3x^2-1}{x^3-x+5}+C$	E: $\dfrac{x^3-x}{x^3-x+5}+C$	

122. Find $\int xe^{x^2}\,dx$

A: $e^{x^2}+C$	B: $\dfrac{1}{2}e^{x^2}+C$	C: $2e^{x^2}+C$
D: $\dfrac{x^2}{2}e^{x^2}+C$	E: $\dfrac{x^2}{2}\left(\dfrac{e^{x^2}}{2x}\right)+C$	

123. $\int \dfrac{\ln x}{x}\,dx=$

A: $(\ln x)^2-1+C$	B: $\dfrac{1}{2}\ln(x^2)+C$	C: $\dfrac{1}{2}(\ln x)^2+C$	D: $\ln\lvert x\rvert+C$	E: $\dfrac{(\ln x)^2}{x^2}+C$

124. Find $\int \dfrac{1}{(x+1)^2}\,dx$

A: $\ln(x+1)^2+C$	B: $\dfrac{3}{(x+1)^3}+C$	C: $\dfrac{(x+1)^3}{3}+C$	D: $-\dfrac{1}{x+1}+C$	E: $-\ln\lvert x+1\rvert+C$

125. Find $\int \dfrac{4x}{x^2+1}\,dx$

A: $4\ln(x^2+1)+C$	B: $2\ln(x^2+1)+C$	C: $4\ln\lvert x\rvert+2x^2+C$
D: $\dfrac{2x^2}{x^3+x}+C$	E: $\dfrac{4x^2}{(x^2+1)^2}+C$	

126. Find $\int \dfrac{4(1+\ln x)^3}{x}\,dx$

A: $16(1+\ln x)^4+C$	B: $4(1+\ln x)^4+C$	C: $(1+\ln x)^4+C$
D: $12(1+\ln x)^2+C$	E: None of A, B, C, D	

127. Find $\int x^2e^{x^3}\,dx$

A: $\dfrac{1}{3}e^{x^3}+C$	B: $3e^{x^3}+C$	C: $e^{x^3}+C$	D: $\left(\dfrac{x^3}{3}\right)e^{x^3}+C$	E: $2xe^{x^3}+C$

128. Evaluate $\int \frac{x \ln(x^2+1)}{x^2+1}\,dx$

A: $\frac{\frac{x^3}{3}\ln\left(\frac{x^3}{3}+x\right)}{\frac{x^3}{3}+x}+C$	B: $\frac{1}{2}\left(\ln(x^2+1)\right)^2+C$	C: $\frac{1}{4}\left(\ln(x^2+1)\right)^2+C$
D: $\frac{x^2}{2}+C$	E: $\frac{1}{2}x^{x^2+1}+C$	

129. If $\int (x^5+10x+1)^9(x^4+2)\,dx = k(x^5+10x+1)^{10}+C$, then $k =$

A: 10	B: $\frac{1}{10}$	C: 50	D: $\frac{1}{50}$	E: 5

130. If $\int_1^2 \frac{x^2+1}{x^3+3x}\,dx = k\int_a^b \frac{1}{u}\,du$, then

A: $a=-1, b=2$	B: $a=4, b=14$	C: $a=6, b=7$	D: $a=6, b=9$	E: $a=1, b=4$

131. If $\int 2x^3 e^{x^4}\,dx = k\int e^u\,du$, then $k =$

A: $\frac{1}{8}$	B: $\frac{1}{4}$	C: $\frac{1}{2}$	D: 4	E: 8

132. In evaluating, $\int_1^{e^2} \frac{(3+\ln x)^2}{x}\,dx$, we obtain $\int_1^{e^2} \frac{(3+\ln x)^2}{x}\,dx = \int_a^b u^2\,du$, where

A: $a=1, b=e^4$	B: $a=3, b=5$	C: $a=3, b=7$	D: $a=4, b=7$	E: $a=9, b=49$

133. If $\int \left(x^4+12x-5\right)^6\left(x^3+3\right)\,dx = k\left(x^4+12x-5\right)^7+C$, find k.

A: $\frac{1}{7}$	B: 7	C: $\frac{1}{28}$	D: 28	E: 4

134. Evaluate $\int_0^2 \frac{3x^2}{(x^3+1)^{1/2}}\,dx$

A: $2\sqrt{2}$	B: $\frac{52}{3}$	C: 1	D: 4	E: 6

135. Evaluate $\int_1^2 \frac{2t+1}{t^2+t}\,dt$

A: $\ln 2$	B: $\ln 3$	C: $\ln 4$	D: $\ln 5$	E: $\ln 6$

136. Evaluate $\int_0^1 \frac{x^2}{(1+x^3)^2}\,dx$

A: $\frac{1}{2}$	B: $-\frac{1}{2}$	C: $-\frac{1}{6}$	D: $\frac{1}{6}$	E: $-\frac{1}{3}$

137. Find $\int_0^1 \frac{1}{2x+1}\,dx$

A: $\ln 3$	B: $2\ln 3$	C: $\frac{1}{2}\ln 3$	D: $\frac{4}{9}$	E: $\frac{16}{9}$

138. Evaluate $\int_0^1 (1-3x)^5\,dx$

A: $\frac{1}{6}$	B: $\frac{63}{18}$	C: $-\frac{63}{18}$	D: $\frac{63}{6}$	E: $-\frac{63}{6}$

139. Evaluate $\int_0^4 \frac{(\sqrt{x}+1)^3}{\sqrt{x}}\,dx$

A: 40	B: 80	C: 10	D: 20	E: 41

140. Evaluate $\int_0^1 \frac{x^2}{x^3+1}\,dx$

A: $\frac{e^3}{3}$	B: $\frac{\ln 2}{3}$	C: $\frac{1}{3}$
D: 0	E: None of the above	

141. Find $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

A: e^2+e	B: e^4-e	C: e^2-e	D: $2(e^2-e)$	E: $2(e^4-e)$

142. Evaluate $\int_0^2 x^2(1+x^3)^{-1/2}\,dx$

A: 4	B: $\frac{2}{3}$	C: $\frac{1}{3}$	D: $\frac{4}{3}$	E: $\frac{1}{2}$

143. Evaluate $\int_1^e \frac{(\ln x)^3}{x}\,dx$

A: $\frac{1}{4}\left(e^4-1\right)$	B: $\frac{1}{4}\left(1-e^4\right)$	C: $\frac{1}{e}$	D: $\frac{1}{4}$	E: $-\frac{1}{4}$

144. Evaluate $\int_0^1 \frac{x}{\sqrt{8x^2+1}}\,dx$

A: $\frac{1}{2}$	B: $\frac{1}{4}$	C: $\frac{1}{8}$	D: 1	E: 2

Integration by Parts

145. $\int x^2 e^{3x}\,dx =$

A: $x^2e^{3x} - \int x^3e^{3x}\,dx$	B: $\dfrac{2xe^{3x}}{3} + \dfrac{1}{3}\int x^2e^{3x}\,dx$	C: $\dfrac{x^2e^{3x}}{3} - \dfrac{2}{3}\int xe^{3x}\,dx$
D: $\dfrac{2}{3}xe^{3x} - \dfrac{1}{3}\int x^2e^{3x}\,dx$	E: $\dfrac{x^2e^{3x}}{3} + \dfrac{2}{3}\int xe^{3x}\,dx$	

146. Find $\int \ln(2x)\,dx$

A: $(2x)\ln(2x) - 2x + C$	B: $x\ln x - x + C$	C: $\dfrac{1}{2x} + C$
D: $\dfrac{1}{x} + C$	E: $x\ln(2x) - x + C$	

147. What is $\int xe^{2x}\,dx$

A: $3xe^{2x} + C$	B: $x^2e^{2x} + C$	C: $e^{2x}(x-1) + C$
D: $\dfrac{x^2e^{2x}}{4} + C$	E: $\dfrac{e^{2x}}{2}\left(x - \dfrac{1}{2}\right) + C$	

148. $\int 5x^4 \ln x\,dx =$

A: $x^5\ln x - \int 5x^4\,dx$	B: $x^5\ln x + \int 5x^4\,dx$	C: $x^5\ln x + \int x^4\,dx$
D: $x^5\ln x - \int x^4\,dx$	E: None of A, B, C, D	

149. $\int xe^{5x}\,dx =$

A: $\dfrac{1}{5}xe^{5x} - \int \dfrac{1}{5}e^{5x}\,dx$	B: $5xe^{5x} - \int 5e^{5x}\,dx$	C: $xe^{5x} - \int e^{5x}\,dx$
D: $\dfrac{x^2}{2}e^{5x} + C$	E: $\dfrac{x^2}{2} \cdot \dfrac{e^{5x+1}}{(5x+1)} + C$	

150. $\int x^5 \ln x\,dx =$

A: $\dfrac{x^5}{5}\ln x - \int \dfrac{x^6}{5}\,dx$	B: $\dfrac{x^6}{6}\ln x - \int \dfrac{x^5}{6}\,dx$	C: $x^4 - \int 5x^3\,dx$
D: $x^5 - \int 5x^4\ln x\,dx$	E: $x^5\ln x - \int x^4\ln x\,dx$	

151. $\int x^2 \ln dx =$

A: $\frac{x^3}{3} \cdot \frac{(\ln x)^2}{2} + C$	B: $\frac{x^2}{2} \ln x + \frac{1}{2} \int x^3\, dx$	C: $\frac{x^2}{2} \ln x - \frac{1}{2} \int x^3\, dx$
D: $\frac{x^3}{3} \ln x + \frac{1}{3} \int x^2\, dx$	E: $\frac{x^3}{3} \ln x - \frac{1}{3} \int x^2\, dx$	

152. $\int (\ln x)^2 \; dx =$

A: $\frac{2 \ln x}{x} + C$	B: $\frac{(\ln x)^3}{3} + C$	C: $(x \ln x - x)^2 + C$
D: $x (\ln x)^2 - 2 \int \ln x\, dx$	E: $x^2 \ln x - \frac{1}{2} \int \ln x\, dx$	

153. In determining $\int x^{3/2} \ln x\, dx$, we use the formula $\int u\, dx = uv - \int v\, du$ for integrating by parts with $u = \ln x$ and we obtain $\int x^{3/2} \ln x\, dx = \frac{2}{5} x^{5/2} \ln x - \frac{2}{5} \int x^{3/2}\, dx$.

A: True	B: False	C: Cannot be determined

154. Evaluate $\int_1^2 3x^2 \ln x\, dx$

A: $8 \ln 2$	B: $8 \ln 2 - 4$	C: $8 \ln 2 - \frac{7}{3}$	D: $\ln 16 - 7$	E: $12 \ln 2 + 3$

155. Evaluate $\int_0^2 x e^x\, dx$

A: $e^2 + 1$	B: $e^2 - 1$	C: e^2	D: $3e^2 + 1$	E: $3e^2 - 1$

156. Find $\int_1^e 16x^3 \ln x\, dx$

A: $5e^4 - 1$	B: $3e^4 + 1$	C: $16e^3$	D: $48e^2$	E: $4e^4$

157. Evaluate $\int_1^e x \ln x\, dx$

A: $e^2 + 1$	B: $e^2 - 1$	C: $\frac{1}{2} e^2$	D: $\frac{1}{4}(e^2 - 1)$	E: $\frac{1}{4}(e^2 + 1)$

158. Find $\int_1^e \ln x\, dx$

A: e	B: $2e - 1$	C: $2e$	D: 1	E: $\frac{1}{2}$

Partial Fractions

159. In the partial fraction decomposition, $\dfrac{1}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$, find the value of A.

A: $-\dfrac{1}{2}$	B: $\dfrac{1}{2}$	C: 1
D: -1	E: None of the above	

160. In the partial fraction decomposition, $\dfrac{x-2}{(x+2)(x+4)} = \dfrac{A}{x+2} + \dfrac{B}{x+4}$, find the value of A.

A: -2	B: -1	C: 1	D: 2	E: 3

161. In the partial fraction decomposition, $\dfrac{2x-17}{(x-5)(x+2)} = \dfrac{A}{x+2} + \dfrac{B}{x-5}$, find the value of A.

A: 5	B: 4	C: 3	D: -1	E: -2

162. If $\dfrac{x-22}{(x-2)(x+3)} = \dfrac{A}{x-2} + \dfrac{B}{x+3}$, find the value of A.

A: 2	B: -2	C: -3	D: -4	E: 5

163. If $\dfrac{9x+11}{(x+3)(x-5)} = \dfrac{A}{x+3} + \dfrac{B}{x-5}$, find B.

A: -6	B: -3	C: 2	D: 5	E: 7

164. If $\dfrac{5x+6}{(x+2)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{x+3}$ find the value of A.

A: -2	B: -3	C: -4	D: 6	E: 9

165. If $\dfrac{6x-25}{(x-3)(x+4)} = \dfrac{A}{x-3} + \dfrac{B}{x+4}$, find A.

A: 1	B: -1	C: 7	D: -7	E: 3

166. If $\dfrac{3x-1}{x^2-3x+2} = \dfrac{A}{x-2} + \dfrac{B}{x-1}$, find the value of A.

A: 5	B: 2	C: -5	D: -2	E: 1

167. If $\dfrac{8x-11}{(x-2)(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{x-1}$, find the value of A.

A: 1	B: 2	C: 3	D: 4	E: 5

168. If $\dfrac{4x+17}{(x-2)(x+3)} = \dfrac{A}{x-2} + \dfrac{B}{x+3}$, find A.

A: -1	B: 1	C: 0	D: -5	E: 5

169. If $\dfrac{A}{x-2} + \dfrac{B}{(x+3)} = \dfrac{3x+14}{(x-2)(x+3)}$, then the constant B is

A: 2	B: -3	C: 4	D: -1	E: 1

170. If $\dfrac{9-2x}{(x-2)(x+3)} = \dfrac{A}{x-2} + \dfrac{B}{x+3}$, then

A: $A=-3$	B: $B=1$	C: $B=-1$	D: $A=-1$	E: $B=-3$

171. If $\dfrac{2x-26}{(x+5)(x-1)} = \dfrac{A}{x+5} + \dfrac{B}{x-1}$, find the value of A.

A: 6	B: -5	C: -4	D: 3	E: 1

172. If $\displaystyle\int \frac{9x-2}{(x+2)(x-3)}\,dx = A\ln|x+2| + B\ln|x-3| + C$, find A.

A: 5	B: 9	C: -2	D: 3	E: 4

173. If $\displaystyle\int \frac{12x-11}{(x-5)(x+2)}\,dx = A\ln|x-5| + B\ln|x+2| + C$, find the value of A.

A: 5	B: 12	C: 11	D: -2	E: 7

174. If $\displaystyle\int \frac{x+8}{x^2+x-2}\,dx = \int \left(\frac{A}{x-1} + \frac{B}{x+2}\right) dx$, then A is

A: 1	B: 2	C: 3	D: -2	E: -3

175. If $\displaystyle\int \frac{x+8}{(x-1)(x+2)}\,dx = \int \left(\frac{A}{x-1} + \frac{B}{x+2}\right) dx$, then

A: $A=-3$	B: $A=-2$	C: $A=3$	D: $B=2$	E: $B=3$

176. Find A if $\int \frac{2x-5}{x^2+x-2}\,dx = \int \left(\frac{A}{x+2} + \frac{B}{x-1}\right)\,dx$

A: 1	B: -1	C: -2	D: 2	E: 3

Area & Volume

177. Determine an integral that represents the area of the region bounded by the curves $y = 6 - x^2$ and $y = x$.

A: $\int_{-3}^{2} (x - (6 - x^2))\,dx$	B: $\int_{-3}^{2} \sqrt{6-y}\,dx$	C: $\int_{0}^{2} (6 - x^2)\,dx$
D: $\int_{-3}^{0} (x - (6 - x^2))\,dx$	E: $\int_{-3}^{2} (6 - x^2 - x)\,dx$	

178. Determine an integral that represents the area of the region bounded by the curves $y = x$, $y = -x + 2$ and $y = 0$.

A: $\int_{0}^{2} (2 - y)\,dy$	B: $\int_{0}^{2} (x - (-x + 2))\,dx$	C: $\int_{0}^{2} (2 - 2y)\,dy$
D: $\int_{0}^{1} (2 - 2y)\,dy$	E: $\int_{0}^{2} (2y - 2)\,dy$	

179. The area bounded by the curves $y = -x$ and $y = -x^2 + 2$ is represented by the definite integral

A: $\int_{-1}^{2} (-x^2 + 2 + x)\,dx$	B: $\int_{-1}^{2} (-x + x^2 - 2)\,dx$	C: $\int_{-1}^{2} (-x - x^2 + 2)\,dx$
D: $\int_{1}^{2} (-x + x^2 - 2)\,dx$	E: $\int_{1}^{2} (-x^2 + 2 + x)\,dx$	

180. The area bounded by the curves $y = x^2 + 1$ and $y = x + 3$ is represented by the definite integral

A: $\int_{-2}^{1} \big[(x+3) - (x^2+1)\big]\,dx$	B: $\int_{-1}^{2} \big[(x+3) - (x^2+1)\big]\,dx$	C: $\int_{-1}^{2} \big[(x^2+1) - (x+3)\big]\,dx$
D: $\int_{-2}^{1} \big[(x^2+1) - (x+3)\big]\,dx$	E: None of the above	

181. The area of the region bounded by $y = x^2 - 4$ and $y = 3x$ is represented by

A: $\int_{0}^{3} \big(3x - x^2 + 4\big)\,dx$	B: $\int_{-1}^{4} \big(3x - x^2 + 4\big)\,dx$	C: $\int_{0}^{3} \big(x^2 - 4 - 3x\big)\,dx$
D: $\int_{-1}^{4} \big(x^2 - 4 - 3x\big)\,dx$	E: None of A, B, C, D	

182. The area of the region bounded by $x = 2y^2 - 1$ and $x = y^2$ is represented by

A: $\int_{-2}^{2} \left[(2y^2-1) - y^2\right]\, dy$	B: $\int_{-2}^{2} \left[y^2 - (2y^2-1)\right]\, dy$	C: $\int_{-1}^{1} \left[y^2 - (2y^2-1)\right]\, dy$
D: $\int_{-1}^{1} \left[(2y^2-1) - y^2\right]\, dy$	E: None of A, B, C, D	

183. Determine an integral that represents the area of the region bounded by $x = y^2$ and $x = 2y + 3$.

A: $\int_{0}^{9} \left(2y + 3 - y^2\right)\, dy$	B: $\int_{0}^{9} \left(y^2 - 2y - 3\right)\, dy$	C: $\int_{0}^{9} \left(\sqrt{x} - \frac{x-3}{2}\right)\, dx$
D: $\int_{-1}^{3} \left(2y + 3 - y^2\right)\, dy$	E: $\int_{-1}^{3} \left(y^2 - 2y - 3\right)\, dy$	

184. Determine the area of the region bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and $x = 1$.

A: $1 - e$	B: e	C: $e - 1$	D: 1	E: $\sqrt{e} - 1$

185. Find the area of the region bounded by the curves $y = x^4$ and $y = x$.

A: $\frac{3}{10}$	B: $\frac{1}{2}$	C: $\frac{1}{5}$	D: $\frac{2}{5}$	E: $\frac{7}{10}$

186. Find the area of the region bounded by the curves $y = 3x^2$ and $y = x^3$.

A: 54	B: $\frac{54}{4}$	C: $\frac{27}{4}$	D: $\frac{3}{4}$	E: 0

187. Find the area of the region bounded by the curves $y = x^4 + 2$ and $y = 0$ between $x = 0$ and $x = 1$.

A: 4	B: 2	C: $\frac{12}{5}$	D: $\frac{11}{5}$	E: $\frac{5}{11}$

188. Find the area of the region bounded by $y = x^2$ and the x-axis between $x = 1$ and $x = 2$.

A: $\frac{1}{3}$	B: 2	C: $\frac{7}{3}$	D: $\frac{8}{3}$	E: 3

189. Find the area of the region bounded by the curves $y = e^{2x}$ and $y = 0$ between $x = \frac{1}{2}$ and $x = 1$.

A: $e - e^{1/2}$	B: $\frac{e}{2} - \frac{e^{1/2}}{2}$	C: $e - e^{1/4}$	D: $\frac{e^2}{2} - \frac{e}{2}$	E: $e^2 - e$

190. Find the area of the region bounded by the curves $y = x^2$ and the x-axis between $x = 1$ and $x = 3$.

A: $\frac{28}{3}$	B: 9	C: $\frac{26}{3}$	D: 8	E: 10

191. Find the area of the region bounded by $y = \frac{1}{x}$ and the x-axis between $x = 2$ and $x = 5$.

A: $\frac{\ln 5}{\ln 2}$	B: $\ln 3$	C: $\ln 10$	D: $\ln\left(\frac{5}{2}\right)$	E: $\ln\left(\frac{2}{5}\right)$

192. Find the area of the region bounded by $y = x^2$ and $y = 4$.

A: $\frac{8}{3}$	B: $\frac{11}{3}$	C: $\frac{13}{3}$	D: $\frac{16}{3}$	E: $\frac{32}{3}$

193. The region bounded by $x = 0$, $y = 8$ and $y = x^3$ is rotated about the y-axis. The volume of the resulting solid is represented by

A: $\pi \int_0^2 x^6 \, dx$	B: $\pi \int_0^8 (2 - y)^2 \, dy$	C: $\pi \int_0^2 (8 - x^3)^2 \, dx$
D: $\pi \int_0^8 y^{2/3} \, dy$	E: $\pi \int_0^2 (8^2 - x^6) \, dx$	

194. The region bounded by $x = 0$, $y = 8$ and $y = x^3$ is rotated about the x-axis. The volume of the resulting solid is represented by

A: $\pi \int_0^2 x^6 \, dx$	B: $\pi \int_0^8 (2 - y)^2 \, dy$	C: $\pi \int_0^2 (8 - x^3)^2 \, dx$
D: $\pi \int_0^8 y^{2/3} \, dy$	E: $\pi \int_0^2 (8^2 - x^6) \, dx$	

195. Determine an integral that represents the volume generated when the region bounded by $y = x^2$, $y = 0$ and $x = 2$ is rotated about the y-axis.

A: $\pi \int_0^4 (4 - y) \, dy$	B: $\pi \int_0^2 (4 - y) \, dy$	C: $\pi \int_0^2 \left(2 - \sqrt{y}\right)^2 \, dy$
D: $\pi \int_0^4 \left(2 - \sqrt{y}\right)^2 \, dy$	E: $\pi \int_0^4 y^4 \, dy$	

196. Find the volume generated when the region bounded by the curves $y = x^3$, $y = 0$ and $x = 1$ is rotated about the x-axis.

A: $\frac{\pi}{7}$	B: $\frac{\pi}{6}$	C: $\frac{\pi}{5}$	D: $\frac{\pi}{4}$	E: $\frac{\pi}{3}$

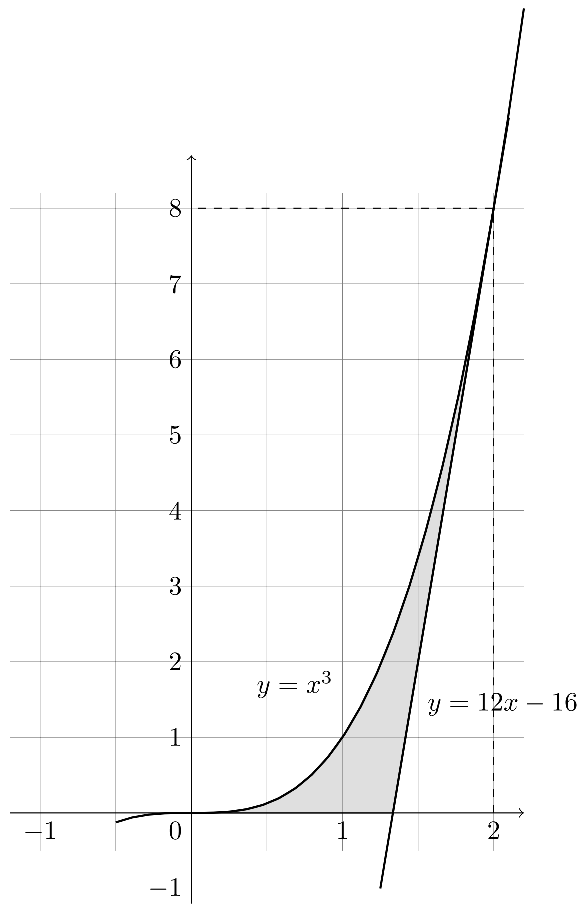
197. Find the volume generated when the region bounded by the curves $y = x^2$, $y = 0$ and $x = 1$ is rotated about the x-axis.

A: $\frac{1}{3}\pi$	B: π	C: $\frac{4}{5}\pi$	D: $\frac{2}{3}\pi$	E: $\frac{1}{5}\pi$

198. Find the volume obtained when the region bounded by $y = x^2$, $y = 0$ and $x = 2$ is rotated about the x-axis.

A: $\frac{4}{3}\pi$	B: $\frac{32}{5}\pi$	C: 6π	D: 8π	E: $\frac{1024}{5}\pi$

199. The shaded region is bounded by the curves $y = 0$, $y = 12x - 16$ and $y = x^3$.



a) The area of the region is represented by the definite integral $\int_0^2 \left[x^3 - (12x - 16)\right]\, dx$.

A: True	B: False	C: Cannot be determined

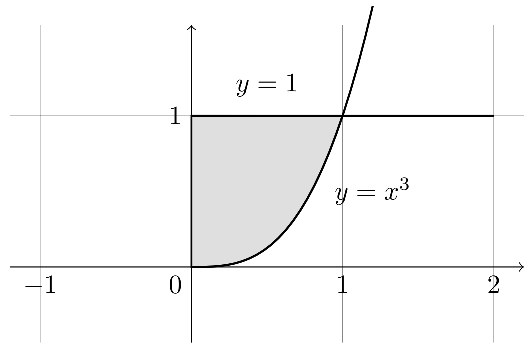
b) The area of the region is represented by the definite integral $\int_0^8 \left[\left(\frac{1}{12}y + \frac{4}{3}\right) - y^{1/3}\right]\, dy$.

A: True	B: False	C: Cannot be determined

c) If the region is revolved about the x-axis, the volume of the resulting solid of revolution is represented by $\pi \int_0^2 x^6\, dx - \pi \int_{\frac{4}{3}}^2 (12x - 16)^2\, dx$.

A: True	B: False	C: Cannot be determined

200. The region below is bounded by the curves $y =$, $y = x^3$ and the y-axis.



a) Find the area of the shaded region.

A: $\frac{2}{3}$	B: $\frac{1}{4}$	C: $\frac{1}{2}$	D: $\frac{3}{4}$	E: $\frac{7}{8}$

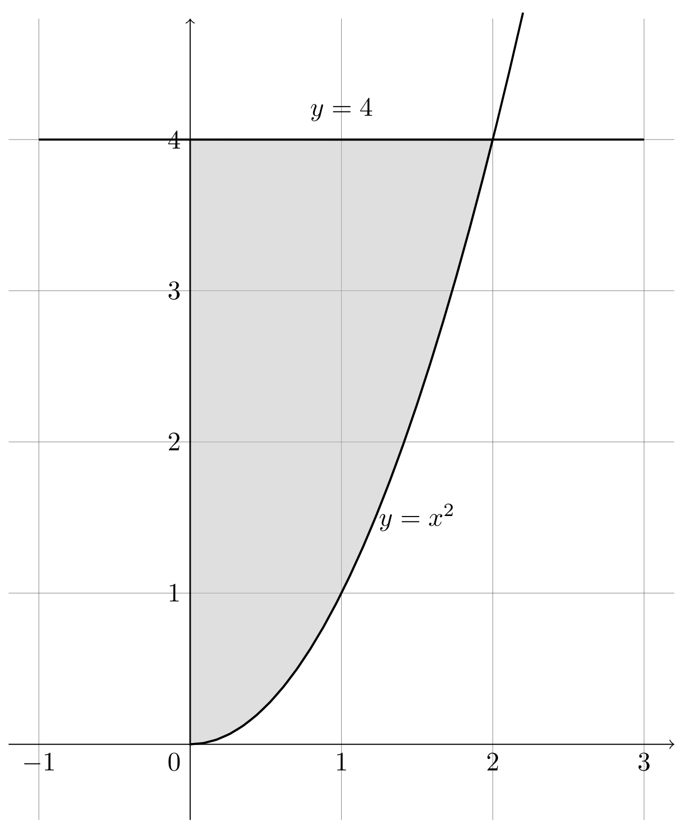
b) Find the volume of the solid of revolution obtained by revolving the shaded region about the x-axis.

A: $\frac{3}{5}\pi$	B: $\frac{2}{5}\pi$	C: π	D: $\frac{1}{7}\pi$	E: $\frac{6}{7}\pi$

c) Find the volume of the solid of revolution obtained by revolving the shaded region about the y-axis.

A: $\frac{3}{5}\pi$	B: $\frac{2}{5}\pi$	C: π	D: $\frac{1}{7}\pi$	E: $\frac{6}{7}\pi$

201. The region below is bounded by the curves $y = 4$ and $y = x^2$ and the y-axis.



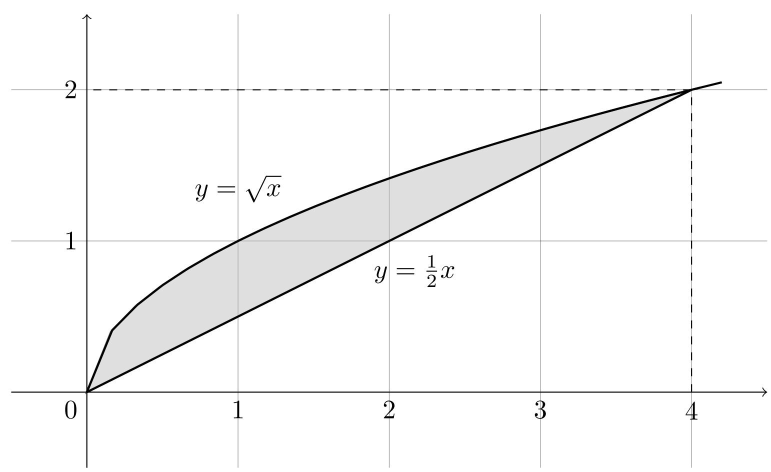
a) The area of the shaded region is represented by which definite integral?

A: $\int_0^{16} \left(4 - x^2\right)\, dx$	B: $\int_0^{4} \left(4 - x^2\right)\, dx$	C: $\int_0^{2} \left(x^2 - 4\right)\, dx$
D: $\int_0^{4} \sqrt{y}\, dy$	E: None of the above	

b) If the region is revolved about the x-axis, the volume of the resulting solid of revolution is represented by the definite integral

A: $\pi \int_0^{4} y\, dy$	B: $\pi \int_0^{2} (4 - x^2)^2\, dx$	C: $\pi \int_0^{4} (16 - x^4)\, dx$
D: $\pi \int_0^{2} (16 - x^2)\, dx$	E: None of the above	

202. The region below is bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$.



a) The area of the region is given by the integral

A: $\int_0^{2} \left(\sqrt{x} - \frac{1}{2}x\right)\, dx$	B: $\int_0^{4} \left(\frac{1}{2}x - \sqrt{x}\right)\, dx$	C: $\int_0^{2} \left(2y - y^2\right)\, dy$
D: $\int_0^{2} \left(y^2 - 2y\right)\, dy$	E: $\int_0^{4} \left(2y - y^2\right)\, dy$	

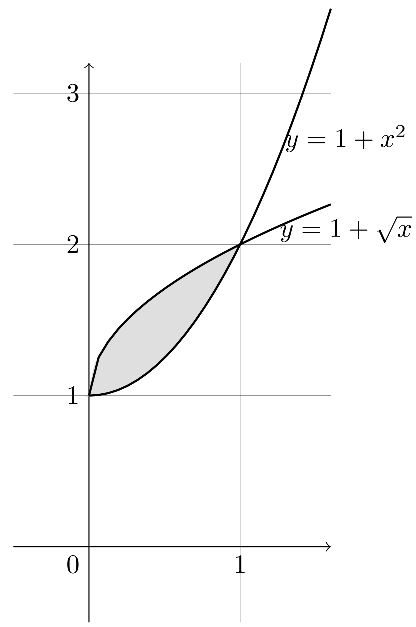
b) If the region is revolved about the x-axis, the volume of the resulting solid is give by the integral

A: $\int_0^{4} \pi \left(x - \frac{1}{4}x^2\right)\, dx$	B: $\int_0^{4} \pi \left(\sqrt{x} - \frac{1}{2}x\right)^2\, dx$	C: $\int_0^{2} \pi \left(x - \frac{1}{4}x^2\right)\, dx$
D: $\int_0^{2} \pi(y^4 - 4y^2)\, dy$	E: $\int_0^{4} \pi(y^4 - 4y^2)\, dy$	

c) If the region is revolved about the y-axis, the volume of the resulting solid is give by the integral

A: $\int_0^2 \pi \left(y^2 - 2y\right)^2 \, dy$	B: $\int_0^4 \pi \left(y^2 - 2y\right)^2 \, dy$	C: $\int_0^4 \pi \left(4y^2 - y^4\right) \, dy$
D: $\int_0^2 \pi \left(x - \frac{1}{4}x2\right) \, dx$	E: $\int_0^2 \pi \left(4y^2 - y^4\right) \, dy$	

203. The shaded region in the diagram below is bounded by the graphs of $y = 1 + x^2$ and $y = 1 + \sqrt{x}$.



a) Which integral represents the area of the region?

A: $\int_0^1 \left[\left(1+\sqrt{x}\right) - \left(1+x^2\right)\right] \, dx$	B: $\int_0^1 \left[\left(1+x^2\right) - \left(1+\sqrt{x}\right)\right] \, dx$
C: $\int_1^2 \left[\left(1+x^2\right) - \left(1+\sqrt{x}\right)\right] \, dx$	D: $\int_1^2 \sqrt{y-1} \, dy$
E: $\int_1^2 \left[(y-1)^2 - (y-1)^{1/2}\right] \, dy$	

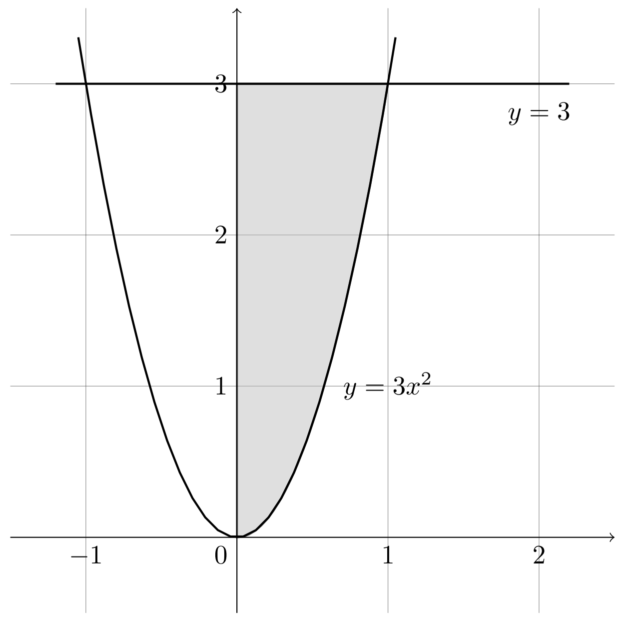
b) If the shaded region is revolved about the x-axis, the volume of the resulting solid of revolution is represented by

A: $\pi \int_0^1 \left[\left(1+\sqrt{x}\right) - \left(1+x^2\right)\right]^2 \, dx$	B: $\pi \int_0^1 \left[\left(1+\sqrt{x}\right)^2 - \left(1+x^2\right)^2\right] \, dx$
C: $\pi \int_0^1 \left[\left(1+x^2\right)^2 - \left(1+\sqrt{x}\right)^2\right] \, dx$	D: $\pi \int_1^2 \left[(y-1) - (y-1)^4\right] \, dy$
E: $\pi \int_1^2 \left[(y-1)^4 - (y-1)\right] \, dy$	

c) If the shaded region is revolved about the y-axis, the volume of the resulting solid of revolution is represented by

A: $\pi \int_0^1 \left[\left(1+\sqrt{x}\right) - \left(1+x^2\right)\right]^2 \, dx$	B: $\pi \int_0^1 \left[\left(1+\sqrt{x}\right)^2 - \left(1+x^2\right)^2\right] \, dx$
C: $\pi \int_0^1 \left[\left(1+x^2\right)^2 - \left(1+\sqrt{x}\right)^2\right] \, dx$	D: $\pi \int_1^2 \left[(y-1) - (y-1)^4\right] \, dy$
E: $\pi \int_1^2 \left[(y-1)^4 - (y-1)\right] \, dy$	

204. The region below is bounded by the curves $y = 3$, $y = 3x^2$ and the y-axis.



a) Which of the stated integrals gives the area of the region?

A: $\int_0^1 3x^2 \, dx$	B: $\int_0^9 \left(3-3x^2\right) \, dx$	C: $\int_0^3 \left(3-3x^2\right) \, dx$
D: $\int_0^3 \sqrt{\frac{y}{3}} \, dy$	E: $\int_0^3 \frac{\sqrt{y}}{3} \, dy$	

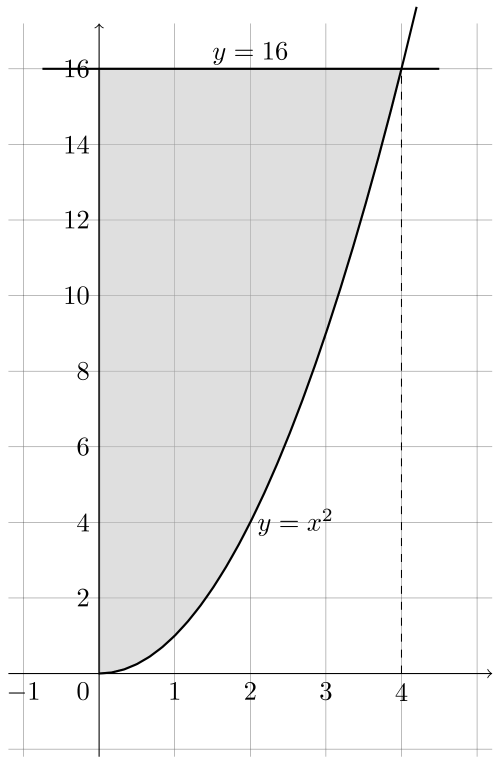
b) Which of the stated integrals gives the volume of the solid generated when the shaded region is rotated about the x-axis?

A: $\pi \int_0^3 \frac{y}{3} \, dy$	B: $\pi \int_0^3 \sqrt{\frac{y}{3}} \, dy$	C: $\pi \int_0^1 9x^4 \, dx$
D: $\pi \int_0^1 (3-3x^2)^2 \, dx$	E: $\pi \int_0^1 (9-9x^4) \, dx$	

c) Which of the stated integrals gives the volume of the solid generated when the shaded region is rotated about the y-axis?

A: $\pi \int_0^3 \frac{y}{3} \, dy$	B: $\pi \int_0^3 \sqrt{\frac{y}{3}} \, dy$	C: $\pi \int_0^1 9x^4 \, dx$
D: $\pi \int_0^1 (3-3x^2)^2 \, dx$	E: $\pi \int_0^1 (9-9x^4) \, dx$	

205. The region below is bounded by the curves $y = x^2$ and $y = 16$.



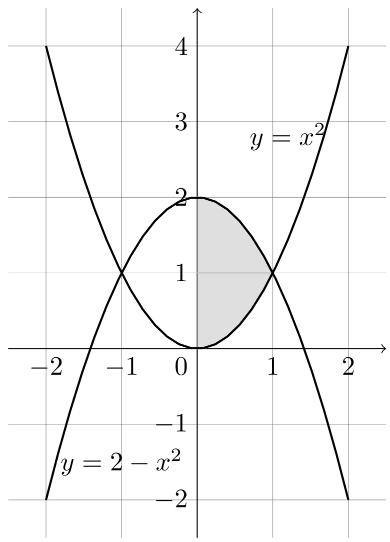
a) Find the volume of the solid obtained by revolving the shaded region about the y-axis.

A: $\frac{\pi}{5}(4)^2$	B: $\pi(16)^2$	C: $\frac{\pi}{2}(4)^2$	D: $\frac{\pi}{2}(16)^2$	E: $\pi(4)^5$

b) If the same region is revolved about the x-axis, the volume is represented by the integral

A: $\pi \int_0^4 x^4\,dx$	B: $\pi \int_0^4 x^2\,dx$	C: $\pi \int_0^4 (16 - x^2)\,dx$
D: $\pi \int_0^4 \left[(16)^2 - x^4\right]\,dx$	E: None of the above	

206. The region below is bounded by the curves $y = x^2$ and $y = 2 - x^2$.



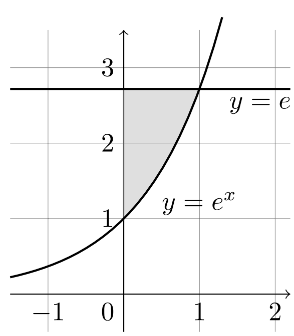
a) The volume of the solid of revolution obtained by revolving the shaded region about the x-axis is represented by

A: $\pi \int_0^1 \left[(2-x^2)^2\right]\,dx$	B: $\pi \int_0^1 \left[(x^2)^2 - (2-x^2)^2\right]\,dx$
C: $\pi \int_0^1 \left[(2-x^2)^2 - (x^2)^2\right]\,dx$	D: $\pi \int_0^1 \left[\left(\sqrt{2-y}\right)^2 - (\sqrt{y})^2\right]\,dx$
E: $\pi \int_0^1 (\sqrt{y})^2\,dy + \pi \int_1^2 \left(\sqrt{2-y}\right)^2\,dy$	

b) The volume of the solid of revolution obtained by revolving the shaded region about the y-axis is represented by

A: $\pi \int_0^1 y\,dy + \pi \int_1^2 (2-y)\,dy$	B: $\pi \int_0^2 \left[(\sqrt{y})^2 - \left(\sqrt{2-y}\right)^2\right]\,dy$
C: $\pi \int_0^1 \left[\left(2-x^2\right)^2 - x^4\right]\,dx$	D: $\pi \int_0^1 (2-y)\,dy + \pi \int_1^2 y\,dy$
E: $\pi \int_0^1 x^4\,dx + \pi \int_1^2 (2-x^2)^2\,dx$	

207. The region below is bounded by the curves $y = e^x$, $y = e$ and the y-axis.



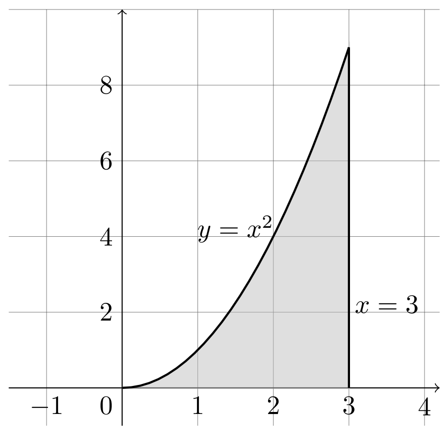
a) Determine an integral that represents the volume generated when the shaded region is revolved about the x-axis.

A: $\pi\int_0^1 (e-e^x)\,dx$	B: $\pi\int_0^1 (e^2-e^{2x})\,dx$	C: $\pi\int_0^1 (e^{2x}-e^2)\,dx$
D: $\pi\int_0^1 (e^x-e)\,dx$	E: $\pi\int_0^1 (e-e^{2x})\,dx$	

b) Determine an integral that represents the volume generated when the shaded region is revolved about the y-axis.

A: $\pi\int_0^1 e^{2y}\,dy$	B: $\pi\int_1^e e^{2y}\,dy$	C: $\pi\int_0^1 (\ln y)^2\,dy$
D: $\pi\int_1^e (\ln y)^2\,dy$	E: $\pi\int_1^e \ln y\,dy$	

208. The region below is bounded by the curves $x = 3$, $y = x^2$ and the y axis.



a) Find the volume of the solid of revolution obtained by revolving the shaded region about the x-axis.

A: 27π	B: 9π	C: 108π	D: $\frac{243}{5}\pi$	E: 81π

b) If the shaded region is revolved about the y-axis, then the volume of the resulting solid of revolution is represented by

A: $\pi\int_0^3 x^4\,dx$	B: $\pi\int_0^{\sqrt{3}} (3-\sqrt{y})^2\ dy$	C: $\pi\int_0^9 (9-y)\ dy$
D: $\pi\int_0^9 (3-y)\ dy$	E: $\int_0^{\sqrt{3}} (\sqrt{y}-3)\ dy$	

Improper Integrals

209. Evaluate the improper integral $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$

A: 3	B: −3	C: 2	D: −2	E: diverges

210. Evaluate the improper integral $\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$

A: 3	B: -3	C: 2	D: -2	E: diverges

211. Evaluate the improper integral $\int_0^\infty e^{-5x}\,dx$

A: 5	B: 0	C: $\frac{1}{5}$	D: 1	E: diverges

212. Evaluate the improper integral $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

A: $\frac{1}{2}$	B: 2	C: $-\frac{1}{2}$	D: -2	E: diverges

213. Evaluate the improper integral $\int_2^\infty e^{-2x}\,dx$

A: $-\frac{1}{2e^4}$	B: $\frac{1}{2e^4}$	C: $-\frac{1}{e^4}$	D: $\frac{1}{e^4}$	E: diverges

214. Evaluate the integral $\int_1^\infty \frac{dt}{t}$

A: $\frac{1}{4}$	B: -2	C: $-\frac{1}{2}$	D: $\frac{1}{2}$	E: ∞

215. Find the value of the integral $\int_0^\infty e^{-4x}\,dx$

A: 0	B: 1	C: $\frac{1}{4}$
D: $-\frac{1}{4}$	E: not convergent	

216. Find $\int_0^\infty -e^{-x}\,dx$

A: 0	B: 1	C: -1	D: e	E: diverges

217. The value of the improper integral $\int_0^\infty te^{-t^2}\,dt$ is

A: 0	B: $\frac{1}{2}$	C: ∞	D: $-\frac{1}{2}$	E: -1

218. Evaluate the improper integral $\int_0^\infty e^{3x}\,dx$

A: -3	B: $-\frac{1}{3}$	C: $\frac{1}{3}$	D: 3	E: divergent

219. Evaluate the improper integral $\int_1^\infty \frac{1}{x^4}\,dx$

A: -3	B: $-\frac{1}{3}$	C: $\frac{1}{3}$	D: 3	E: divergent

Multivariable Functions & Partial Derivatives

220. If $f(x,y,z) = ze^{xy}\ln y$, find $f\left(-1,\sqrt{e},5\right)$

A: $\frac{5}{\sqrt{e}}$	B: $\frac{5\sqrt{e}}{e^{\sqrt{e}}}$	C: $-\frac{5e^{\sqrt{e}}}{2}$	D: $\frac{5}{2e^{\sqrt{e}}}$	E: $-\frac{\ln 5}{e^{\sqrt{e}}}$

221. If $f(x,y) = \frac{x^2}{y}$, then $f_{xy}(x,y) =$

A: $-\frac{2x}{y^2}$	B: $\frac{2x}{y}$	C: $-\frac{x^2}{y^2}$	D: 0	E: 2

222. If $f(x,y) = x^3e^{xy}$, then $f_x(x,y) =$

A: x^4e^{xy}	B: $3x^3e^{xy}$	C: $3x^2e^{xy} + x^3e^{xy}$
D: $3x^2e^{xy} + x^3ye^{xy}$	E: $3x^2e^{xy} + x^4e^{xy}$	

223. If $f(x,y,z) = ze^{xy}\ln y$, find $f_x(x,y,z)$

A: ye^{xy}	B: $ze^{x}\ln y$	C: $ze^{xy}\ln y$	D: $yze^{xy}\ln y$	E: $\frac{ze^{xy}}{y}$

224. Find $\frac{\partial w}{\partial y}$ where $w = xe^{yz}$

A: xe^{yz}	B: xe^{z}	C: e^{z}	D: xze^{yz}	E: e^{yz}

225. Find $\frac{\partial z}{\partial x}$ if $z = \ln(x^2+y^2)$

A: $\frac{1}{x^2+y^2}$	B: $2x\ln(x^2+y^2)$	C: 0	D: $\frac{2x}{x^2+y^2}$	E: $\frac{2y}{x^2+y^2}$

226. If $w = xe^{yz^2} + z$, find $\frac{\partial w}{\partial y}$

A: $2xze^{yz^2} + z$	B: $xz^2e^{yz^2}$	C: $xz^2e^{yz^2} + z$
D: $e^{yz^2} + xz^2e^{yz^2}$	E: $e^{yz^2} + xz^2e^{yz^2} + 1$	

227. Find $\dfrac{\partial w}{\partial z}$ where $w = x\ln(yz+1)$

A: $\dfrac{1}{yz+1}$	B: $\dfrac{x}{yz+1}$	C: $\dfrac{xy}{yz+1}$
D: $\dfrac{x}{yz+1} + x\ln(yz+1)$	E: $\dfrac{x}{yz+1} + \ln(yz+1)$	

228. Find $f_y(1,2)$ where $f(x,y) = x^3 + y^3 + x + y$

A: 4	B: 12	C: 17	D: 9	E: 13

229. If $f(x,y) = xy^3 - \dfrac{x^2}{y}$, find $f_y(2,1)$

A: 7	B: -3	C: 10	D: 2	E: -4

230. If $f(x,y) = \ln\left(3x^2y + y^2\right)$, then $f_y(1,2) =$

A: $\dfrac{7}{10}$	B: $\dfrac{14}{13}$	C: $\dfrac{1}{10}$	D: $\dfrac{1}{13}$	E: $\dfrac{13}{17}$

231. If $f(x,y) = \dfrac{x + \ln y}{y}$, then the partial derivative $f_x(1,-2)$ is

A: -1	B: $-\dfrac{1}{2}$	C: $-\dfrac{1}{4}$	D: $\dfrac{1}{4}$	E: 1

232. If $f(x,y) = x^2 - 3xy + y^3$, then $f_y(3,1)$ is

A: 1	B: 3	C: 24	D: -6	E: -7

233. Find $f_y(1,2)$ where $f(x,y) = x^2y^3 - 3x^3y$.

A: -12	B: -2	C: 2	D: 9	E: 15

234. If $f(x,y) = \ln\left(x^2 + y^2\right)$, then $f_x(2,0) =$

A: $\sqrt{3}$	B: -1	C: 0	D: 2	E: 1

235. If $f(x,y) = x^3 - 3xy + y^2$, then $f_x(2,3) =$

A: 0	B: 3	C: 6	D: 21	E: -5

236. If $f(x,y) = x^3 - 3xy + y^2$, then $f_y(2,3) =$

A: 0	B: 3	C: 6	D: 21	E: -5

237. Find $f_x(3,-2)$ where $f(x,y) = x^2y + 2x + 3y$.

A: -10	B: 6	C: 8	D: -8	E: -18

238. If $f(x,y) = e^{2xy^2}$, then the partial derivative $f_x(0,-2) =$

A: 2	B: -2	C: -8	D: 0	E: 8

239. Find $f_x(1,1)$ where $f(x,y) = x^3 - y^3 + x - y$.

A: 12	B: 3	C: 4	D: 0	E: 2

240. Find f_{xy} if $f(x,y) = xe^y$

A: e^y	B: x	C: xe^y	D: ye^x	E: xy

241. Find f_{xx} where $f(x,y) = \ln(xy+1)$.

A: $\dfrac{y}{xy+1}$	B: $-\dfrac{y^2}{(xy+1)^2}$	C: $-\dfrac{1}{(xy+1)^2}$	D: $\dfrac{1}{xy+1}$	E: $-\dfrac{1}{x^2}$

242. Find $f_{xy}(x,y)$ where $f(x,y) = x^2y^3 - x^3y$.

A: $6x$	B: $6xy^2$	C: $6xy^2 - y^3$	D: $6xy^2 - 3x^2$	E: $6x^2y - 3x^2$

243. If $f(x,y) = x^2\sqrt{y} + \ln(xy)$, find $f_{xy}(x,y)$

A: $2x$	B: $\dfrac{x}{\sqrt{y}}$	C: $2x\sqrt{y} + \dfrac{1}{x}$	D: $\dfrac{x}{\sqrt{y}} - \dfrac{1}{xy^2}$	E: $\dfrac{x^2}{2\sqrt{y}} + \dfrac{1}{y}$

244. Find $f_{xy}(2,3)$ where $f(x,y) = e^{xy}$

A: e^6	B: $2e^6$	C: $3e^6$	D: $4e^6$	E: $7e^6$

Optimization

245. The only critical points of $f(x,y) = x^2 + xy - \frac{1}{12}y^3$ are

A: $(0,0)$	B: $(-1,2)$	C: $(0,0),(1,-2)$	D: $(-2,1),(0,0)$	E: $(0,0),(-1,2)$

246. Find all critical points of the function $f(x,y) = x^3 - 3xy^2 + 3y^2$

A: $(0,0)$	B: $(1,-1),(1,1)$
C: $(0,0),(1,-1),(1,1)$	D: $(1,-1),(1,1),(-1,1),(-1,-1)$
E: $(0,0),(1,-1),(1,1),(-1,1),(-1,-1)$	

247. Find <u>all</u> the critical points of $f(x,y)=xy-x^2-\frac{1}{8}y^4$.

A: $(0,0)$	B: $(0,0),(2,4),(-2,-4)$	C: $(0,0),\left(\frac{1}{2},1\right),\left(-\frac{1}{2},-1\right)$
D: $(0,0),\left(\frac{1}{2},1\right)$	E: $(0,0),(2,4)$	

248. The critical points of $f(x,y)=x^2+y^3-2xy$ are

A: $(0,0),\left(\frac{3}{2},\frac{3}{2}\right)$	B: $(0,0),\left(\frac{2}{3},\frac{2}{3}\right)$	C: $(0,0),(1,1)$
D: $(1,1),\left(\frac{2}{3},\frac{2}{3}\right)$	E: $(1,1),\left(\frac{3}{2},\frac{3}{2}\right)$	

249. Use the following information for parts a, b and c.

$$\begin{aligned} f(x,y) &= xy-x^2y-xy^2 \\ f_x &= y-2xy-y^2 \\ f_y &= x-x^2-2xy \\ f_{xy} &= 1-2x-2y \\ f_{xx} &= -2y \\ f_{yy} &= -2x \end{aligned}$$

a) How many critical points does f have?

A: 1	B: 2	C: 3	D: 4	E: 0

b) Which one of the following is true?

A: $(0,0)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(0,0)$
C: There is a local minimum at $(0,0)$
D: There is a saddle point at $(0,0)$
E: The status of (0,0) cannot be determined

c) What function $T(x, y)$ is used to help test the critical points of f?

A: $T(x, y) = 1 - 2x - 2y$	B: $T(x, y) = (1 - 2x - 2y)^2 - 4xy$
C: $T(x, y) = 4xy - (1 - 2x - 2y)^2$	D: $T(x, y) = x - x^2 - 2xy$
E: $T(x, y) = y - 2xy - y^2$	

250. Use the following information for parts a, b and c.

$$
\begin{aligned}
f(x, y) &= xy - x^3y - xy^3 \\
f_x &= y - 3x^2y - y^3 \\
f_y &= x - x^3 - 3xy^2 \\
f_{xx} &= -6xy \\
f_{xy} &= 1 - 3x^2 - 3y^2 \\
f_{yy} &= -6xy
\end{aligned}
$$

a) Which one of the following is true for the point (1,0)?

A: $(1, 0)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(1, 0)$
C: There is a local minimum at $(1, 0)$
D: There is a saddle point at $(1, 0)$
E: Cannot be determined

b) Which one of the following is true for the point $\left(\frac{1}{2}, 1\right)$?

A: $\left(\dfrac{1}{2}, 1\right)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $\left(\frac{1}{2}, 1\right)$
C: There is a local minimum at $\left(\frac{1}{2}, 1\right)$
D: There is a saddle point at $\left(\frac{1}{2}, 1\right)$
E: Cannot be determined

c) Which one of the following is true for the point $\left(\frac{1}{2}, \frac{1}{2}\right)$?

A: $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $\left(\frac{1}{2}, \frac{1}{2}\right)$
C: There is a local minimum at $\left(\frac{1}{2}, \frac{1}{2}\right)$
D: There is a saddle point at $\left(\frac{1}{2}, \frac{1}{2}\right)$
E: Cannot be determined

251. Use the following information for parts a, b and c.

$$
\begin{aligned}
f(x, y) &= x^3 + y^3 - 9xy \\
f_x &= 3x^2 - 9y \\
f_y &= 3y^2 - 9x \\
f_{xx} &= 6x \\
f_{xy} &= -9 \\
f_{yy} &= 6y
\end{aligned}
$$

a) Which one of the following is true for the point (0,0)?

A: $(0, 0)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(0, 0)$
C: There is a local minimum at $(0, 0)$
D: There is a saddle point at $(0, 0)$
E: Cannot be determined

b) Which one of the following is true for the point $\left(1, \frac{1}{3}\right)$?

A: $\left(1, \frac{1}{3}\right)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $\left(1, \frac{1}{3}\right)$
C: There is a local minimum at $\left(1, \frac{1}{3}\right)$
D: There is a saddle point at $\left(1, \frac{1}{3}\right)$
E: Cannot be determined

c) Which one of the following is true for the point (3,3)?

A: $(3,3)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(3,3)$
C: There is a local minimum at $(3,3)$
D: There is a saddle point at $(3,3)$
E: Cannot be determined

252. Use the following information for parts a, b and c.

$$f(x,y) = 8x^2 + y^2 + 2x^2y + 3$$
$$f_x = 16x + 4xy$$
$$f_y = 2y + 2x^2$$
$$f_{xx} = 16 + 4y$$
$$f_{xy} = 4x$$
$$f_{yy} = 2$$

a) Which one of the following is true for the point (2,4)?

A: $(2,4)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(2,4)$
C: There is a local minimum at $(2,4)$
D: There is a saddle point at $(2,4)$
E: Cannot be determined

b) Which one of the following is true for the point (2,-4)?

A: $(2,-4)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(2,-4)$
C: There is a local minimum at $(2,-4)$
D: There is a saddle point at $(2,-4)$
E: Cannot be determined

c) Which one of the following is true for the point (0,0)?

A: $(0,0)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(0,0)$
C: There is a local minimum at $(0,0)$
D: There is a saddle point at $(0,0)$
E: Cannot be determined

253. Use the following information for parts a, b and c.

$$f(x,y) = x^3 - 3x + 3xy^2$$
$$f_x = 3x^2 - 3 + 3y^2$$
$$f_y = 6xy$$
$$f_{xx} = 6x$$
$$f_{xy} = 6y$$
$$f_{yy} = 6x$$

The critical points of $f(x,y)$ are $(1,0), (-1,0), (0,1), (0,-1)$.

a) Find the point(s) where $f(x,y)$ has a local maximum.

A: $(1,0)$	B: $(-1,0)$	C: $(1,0)$ and $(-1,0)$
D: $(0,1)$ and $(0,-1)$	E: There are none	

b) Find the point(s) where $f(x,y)$ has a local minimum.

A: $(1,0)$	B: $(-1,0)$	C: $(1,0)$ and $(-1,0)$
D: $(0,1)$ and $(0,-1)$	E: There are none	

c) Find the location of any saddle points of $f(x,y)$.

A: $(1,0)$	B: $(-1,0)$	C: $(1,0)$ and $(-1,0)$
D: $(0,1)$ and $(0,-1)$	E: There are none	

254. A function $f(x,y)$ has second partial derivatives $f_{xx}(x,y) = \frac{12}{x^3}$,$f_{xy}(x,y) = 1$ and $f_{yy}(x,y) = \frac{6}{y^3}$ and critical point $(2,1)$. Select the statement which is true.

A: $f(x, y)$ has a local minimum at $(2, 1)$
B: $f(x, y)$ has a local maximum at $(2, 1)$
C: $f(x, y)$ has a saddle point at $(2, 1)$
D: $(2, 1)$ is not a critical point of $f(x, y)$
E: The second partials test yields no information

255. A function $f(x, y)$ has second partial derivatives $f_{xx}(x, y) = 6x$,$f_{xy}(x, y) = -12$ and $f_{yy}(x, y) = 6y$ and critical point $(0, 0)$. Select the statement which is true.

A: $f(x, y)$ has a local minimum at $(0, 0)$
B: $f(x, y)$ has a local maximum at $(0, 0)$
C: $f(x, y)$ has a saddle point at $(0, 0)$
D: $(0, 0)$ is not a critical point of $f(x, y)$
E: The second partials test yields no information

256. Use the following information for parts a and b.

$$f(x, y) = 6xy - x^3 - y^2 + 4$$
$$f_x = 6y - 3x^2$$
$$f_y = 6x - 2y$$
$$f_{xx} = -6x$$
$$f_{yy} = -2$$
$$f_{xy} = 6$$

a) The critical points of $f(x, y)$ are

A: $(0, 0)$ and $(18, 6)$	B: $(0, 0)$ and $(6, 3)$	C: $(0, 0)$ and $(6, 2)$
D: $(0, 0)$ and $(6, 18)$	E: $(0, 0)$ and $(2, 6)$	

b) $f(x, y)$ has a saddle point at

A: $(18, 6)$	B: $(6, 3)$	C: $(0, 0)$
D: $(6, 18)$	E: $(2, 6)$	

257. You are given the following information:

$$f(x,y) = x^2 + 4xy + 4y^2 + 33$$
$$f_x(x,y) = 2x + 4y$$
$$f_y(x,y) = 4x + 8y$$

Select the true statement

A: $(2,1)$ is a critical point of $f(x,y)$
B: $(0,0),(2,-1)$ and $(-2,1)$ are the only critical points of $f(x,y)$
C: $(2b,b)$ is a critical point of $f(x,y)$ for every real number b
D: $(2b,-b)$ is a critical point of $f(x,y)$ for every real number b
E: $f(x,y)$ has no critical points

258. Use the following information to solve parts a, b and c.

$$f(x,y) = x^3 - y^3 - 27x + 3y$$
$$f_x(x,y) = 3x^2 - 27$$
$$f_y(x,y) = -3y^2 + 3$$
$$f_{xx}(x,y) = 6x$$
$$f_{xy}(x,y) = 0$$
$$f_{yy}(x,y) = -6y$$

a) Which one of the following is true for the point (3,1)?

A: $(3,1)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(3,1)$
C: There is a local minimum at $(3,1)$
D: There is a saddle point at $(3,1)$
E: Cannot be determined

b) Which one of the following is true for the point (-3,1)?

A: $(-3,1)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(-3,1)$
C: There is a local minimum at $(-3,1)$
D: There is a saddle point at $(-3,1)$
E: Cannot be determined

c) Which one of the following is true for the point (1,-3)?

A: $(1, -3)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(1, -3)$
C: There is a local minimum at $(1, -3)$
D: There is a saddle point at $(1, -3)$
E: Cannot be determined

259. Use the following equations to solve parts a, b and c.

$$f(x,y) = \frac{1}{3}x^3 - x + xy^2$$
$$f_x = x^2 - 1 + y^2$$
$$f_y = 2xy$$
$$f_{xx} = 2x$$
$$f_{xy} = 2y$$
$$f_{yy} = 2x$$

a) Which one of the following is true for the point (1,0)?

A: $(1, 0)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(1, 0)$
C: There is a local minimum at $(1, 0)$
D: There is a saddle point at $(1, 0)$
E: Cannot be determined

b) Which one of the following is true for the point (0,1)?

A: $(0, 1)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(0, 1)$
C: There is a local minimum at $(0, 1)$
D: There is a saddle point at $(0, 1)$
E: Cannot be determined

c) Which one of the following is true for the point (-1,0)?

A: $(-1, 0)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(-1, 0)$
C: There is a local minimum at $(-1, 0)$
D: There is a saddle point at $(-1, 0)$
E: Cannot be determined

260. Use the following information for parts a and b.

$$f(x, y) = x^3 + y^3 - 12xy - 10$$
$$f_x(x, y) = 3x^2 - 12y$$
$$f_y(x, y) = 3y^2 - 12x$$
$$f_{xx}(x, y) = 6x$$
$$f_{xy}(x, y) = 6y$$
$$f_{yy}(x, y) = -12$$

a) Consider the point (0,0).

A: $(0, 0)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(0, 0)$
C: There is a local minimum at $(0, 0)$
D: There is a saddle point at $(0, 0)$
E: Cannot be determined

b) Consider the point (4,4).

A: $(4, 4)$ is not a critical point of $f(x, y)$
B: There is a local maximum at $(4, 4)$
C: There is a local minimum at $(4, 4)$
D: There is a saddle point at $(4, 4)$
E: Cannot be determined

261. Use the following information for parts a, b and c.

$$f(x,y) = x^3 - 3xy - y^3$$
$$f_x = 3x^2 - 3y$$
$$f_y = -3x - 3y^2$$
$$f_{xx} = 6x$$
$$f_{xy} = -3$$
$$f_{yy} = -6y$$

a) Consider the point (0,0).

A: $(0,0)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(0,0)$
C: There is a local minimum at $(0,0)$
D: There is a saddle point at $(0,0)$
E: Cannot be determined

b) Consider the point (-1,1).

A: $(-1,1)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(-1,1)$
C: There is a local minimum at $(-1,1)$
D: There is a saddle point at $(-1,1)$
E: Cannot be determined

c) Consider the point (1,-1).

A: $(1,-1)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(1,-1)$
C: There is a local minimum at $(1,-1)$
D: There is a saddle point at $(1,-1)$
E: Cannot be determined

262. Use the following information for parts a and b.

$$f(x,y) = x^3 - y^3 - 3x + 3y$$
$$f_x(x,y) = 3x^2 - 3$$
$$f_y(x,y) = -3y^2 + 3$$
$$f_{xx}(x,y) = 6x$$
$$f_{xy}(x,y) = 0$$
$$f_{yy}(x,y) = -6y$$

a) Select the statement which is true.

A: $f(x,y)$ has no critical points
B: $(a,0)$ is a critical point for all a
C: $f(x,y)$ has 4 critical points
D: $(1,0)$ and $(0,1)$ are critical points
E: $(1,0),(-1,0)$ and $(2,0)$ are critical points

b) Consider the point (1,-1).

A: $(1,-1)$ is not a critical point of $f(x,y)$
B: There is a local maximum at $(1,-1)$
C: There is a local minimum at $(1,-1)$
D: There is a saddle point at $(1,-1)$
E: Cannot be determined

263. Consider the problem of finding the maximum of the function $f(x,y) = x + 3y$ subject to the constraint that $x^2 + y^2 = 10$. At what point (x,y) does the maximum value occur?

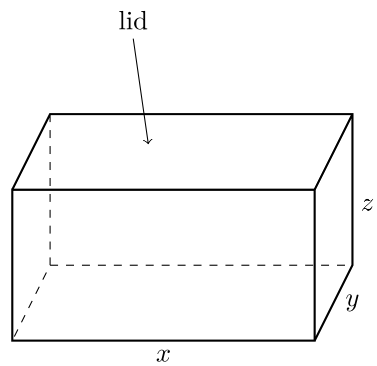
A: $(1,3)$	B: $(-1,-3)$	C: $(-1,3)$	D: $(1,-3)$	E: $(0,0)$

264. A box with an open top is to be constructed from cardboard. Assume x =length, y =width and z =height. We wish to minimize the material used, assuming the volume is 4. What is an equation for the amount (surface area) of material S used?

A: $S = xy + 8y^{-1} + 8x^{-1}$	B: $S = xy$	C: $S = 2xy + 2xz + 2yz$
D: $S = xy + xz + yz$	E: $S = xyz$	

265. A rectangular box is to have a volume of 50 cubic meters. The cost of construction is \$2 per square meter for the lid, \$4 per square meter for the sides and \$7 per square meter for the bottom. In order to find the dimensions of the box which minimize the cost of construction, what is the

mathematical problem that must be solved? (Let x, y and z be the indicated lengths in meters shown in the picture below.



A: Minimize $C = 9xy + 4xz + 4yz$ where $xyz = 50$
B: Minimize $C = 2xy + 2xz + 2yz$ where $xyz = 50$
C: Minimize $C = xyz$ where $2xy + 2xz + 2yz = 50$
D: Minimize $C = 2xy + 4xz + 4yz$ where $xyz = 50$
E: Minimize $C = 9xy + 8xz + 8yz$ where $xyz = 50$

Lagrange Multipliers

266. Suppose we use Lagrange's method to minimize $f(x, y) = x - y$ subject to the constraint $4x^2 + 9y^2 = 36$. What system of equations must we solve?

A: $x - y + \lambda\left(4x^2 + 9y^2 - 36\right) = 0$	B: $4x^2 + 9y^2 - 36 + \lambda(x - y) = 0$
C: $8 + \lambda x = 0$ $-18 + \lambda y = 0$ $4x^2 + 9y^2 - 36 = 0$	D: $1 + 8\lambda x = 0$ $-1 + 18\lambda y = 0$ $4x^2 + 9y^2 - 36 = 0$
E: $x - y = 0$ $4x^2 + 9y^2 - 36 = 0$	

267. Suppose

$$2 + \lambda x = 0$$
$$2 - \lambda y = 0$$
$$x^2 + y^2 = 8$$

The possible solutions for (x, y) are:

A: $(2,-2),(-2,2)$	B: no solutions	C: $(4,-4),(-4,4)$
D: $(2,-2)$	E: $(4,-4)$	

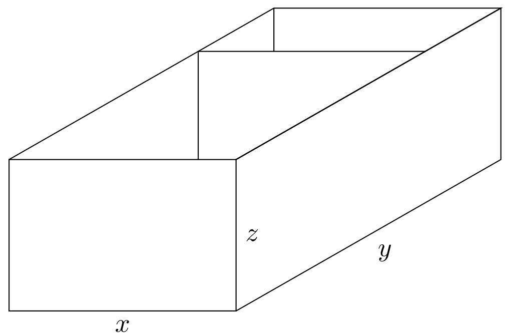
268. Suppose we wish to maximize $f(x,y,z) = x+y+2z$ subject to the constraint $x^2+y^2+z^2 = 24$. Using Lagrange's Method we must solve which system of equations?

A: $x+y+2z+\lambda\left(x^2+y^2+z^2-24\right)=0$	B: $x+2\lambda x=0$ $y+2\lambda y=0$ $2z+2\lambda z=0$ $x^2+y^2+z^2-24=0$
C: $1+2\lambda x=0$ $1+2\lambda y=0$ $2+2\lambda z=0$ $x^2+y^2+z^2-24=0$	D: $x^2+y^2+z^2-24+\lambda(x+y+2z)$
E: $2x+\lambda=0$ $2y+\lambda=0$ $2z+2\lambda=0$ $x^2+y^2+z^2-24=0$	

269. What is the maximum value of $f(x,y,z) = x+y+2z$ subject to the constraint $x^2+y^2+z^2 = 24$?

A: 2	B: 4	C: 8	D: 10	E: 12

270. An open rectangular bin with one partition (shown below) is built using 20 square feet of material. In order to use the method of Lagrange multipliers to find the dimensions x, y and z (in feet) of the bin with the largest volume which can be so constructed, what is the constraint equation?



A: $xyz = 20$	B: $2xy + 2yz + 2xz = 20$	C: $xy + 2yz + 2xz = 20$
D: $xy + 2yz + 3xz = 20$	E: $xy + yz + xz = 20$	

271. In using the method of Lagrange multipliers to find the maximum value of $f(x, y) = xy + 1$ subject to the constraint $x^2 + y^2 = 4$, we must solve the following system of equations

A: $y + 2\lambda x = 0$ $x + 2\lambda y = 0$	B: $x + 2\lambda x = 0$ $y + 2\lambda y = 0$ $x^2 + y^2 = 4$
C: $y + 2\lambda x = 0$ $x + 2\lambda y = 0$ $x^2 + y^2 = 4$	D: $y + 2\lambda(x + y) = 0$ $x + 2\lambda(x + y) = 0$ $x^2 + y^2 = 4$
E: $xy + 1 = 0$ $x^2 + y^2 = 4$	

272. Using the method of Lagrange multipliers to find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 - 5x^2y^2 + y^4 + 7 = 0$, one must solve the system of equations

A: $2x + \lambda\left(4x^3 - 10xy^2\right) = 0$ $2y + \lambda\left(-10x^2y + 4y^3\right) = 0$	B: $2x + \lambda\left(4x^3 - 10\right) = 0$ $2y + \lambda\left(-10x^2y\right) = 0$ $x^4 - 5x^2y^2 + y^4 + 7 = 0$
C: $x^2 + y^2 = 0$ $x^4 - 5x^2y^2 + y^4 + 7 = 0$	D: $x + \lambda\left(x^2 + y^2\right) = 0$ $y + \lambda\left(x^2 + y^2\right) = 0$ $x^4 - 5x^2y^2 + y^4 + 7 = 0$
E: $2x + \lambda\left(4x^3 - 10xy^2\right) = 0$ $2y + \lambda\left(-10x^2y + 4y^3\right) = 0$ $x^4 - 5x^2y^2 + y^4 + 7 = 0$	

273. Using the method of Lagrange multipliers to find the maximum value of $f(x, y) = xy - 10$ subject to the constraint $2x^2 + 4y^2 = 9$, we must solve the system of equations

A: $1 + 4\lambda x = 0$ $1 + 8\lambda y = 0$ $2x^2 + 4y^2 - 9 = 0$	B: $x + 4\lambda x = 0$ $y + 8\lambda y = 0$ $2x^2 + 4y^2 - 9 = 0$	C: $xy - 10 = 0$ $2x^2 + 4y^2 - 9 = 0$
D: $y + 4\lambda x = 0$ $x + 8\lambda y = 0$	E: $y + 4\lambda x = 0$ $x + 8\lambda y = 0$ $2x^2 + 4y^2 - 9 = 0$	

274. A rectangular box with top, bottom and sides, which has 8 internal compartments (3 dividers widthwise and 1 divider lengthwise) is to be constructed. If there are 100 square centimeters of material available and the volume of the box is to be maximized, then the problem is to maximize $f(x, y, z) = xyz$ subject to the constraint $2xy + 5xz + 3yz = 100$. To solve this problem using Lagrange's method, what system of equations must be solved?

A: $xyz + \lambda(2xy + 5xz + 3yz - 100) = 0$	B: $xyz + \lambda(2y + 5z) = 0$ $xyz + \lambda(2x + 3z) = 0$ $xyz + \lambda(5x + 3y) = 0$ $2xy + 5xz + 3yz = 100$
C: $yz + 2y\lambda = 0$ $xz + 2x\lambda = 0$ $xy + 5x\lambda = 0$ $2xy + 5xz + 3yz = 100$	D: $yz + \lambda(2y + 5z) = 0$ $xz + \lambda(2x + 3z) = 0$ $xy + \lambda(5x + 3y) = 0$ $2xy + 5xz + 3yz = 100$
E: $yz + \lambda(2xy + 5xz + 3yz - 100) = 0$ $xz + \lambda(2xy + 5xz + 3yz - 100) = 0$ $xy + \lambda(2xy + 5xz + 3yz - 100) = 0$	

275. Suppose we use Lagrange's method to minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + 2y + 2z = 18$.

a) What system of equations must we solve?

A: $x^2 + \lambda x = 0$ $y^2 + 2\lambda y = 0$ $z^2 + 2\lambda z = 0$ $x + 2y + 2z = 18$	B: $2x + \lambda = 0$ $2y + 2\lambda = 0$ $2z + 2\lambda = 0$ $x + 2y + 2z = 18$
C: $2x + \lambda x = 0$ $2y + 2\lambda y = 0$ $2z + 2\lambda z = 0$ $x + 2y + 2z = 18$	D: $x + \lambda = 0$ $y + \lambda = 0$ $z + \lambda = 0$ $x + 2y + 2z = 18$
E: $x^2 + y^2 + z^2 + \lambda (x + 2y + 2z - 18) = 0$	

b) What is the minimum value of $f(x, y, z) = x^2+y^2+z^2$ subject to the constraint $x+2y+2z = 18$?

A: 10	B: 0	C: 18	D: 36	E: 12

276. Use the following information for parts a, b and c below.

The method of Lagrange multipliers is used to find the maximum value of $f(x, y) = xy + 4$ subject to the constraint $x^2 + 4y^2 = 8$.

a) The system of equations to be solved is

A: $1 + 2\lambda x = 0$ $1 + 8\lambda y = 0$ $x^2 + 4y^2 - 8 = 0$	B: $x + 2\lambda x = 0$ $y + 8\lambda y = 0$ $x^2 + 4y^2 - 8 = 0$	C: $y + 2\lambda x = 0$ $x + 8\lambda y = 0$ $x^2 + 4y^2 - 8 = 0$
D: $xy + 4 = 0$ $x^2 + 4y^2 - 8 = 0$	E: $y + 2\lambda x = 0$ $x + 8\lambda y = 0$	

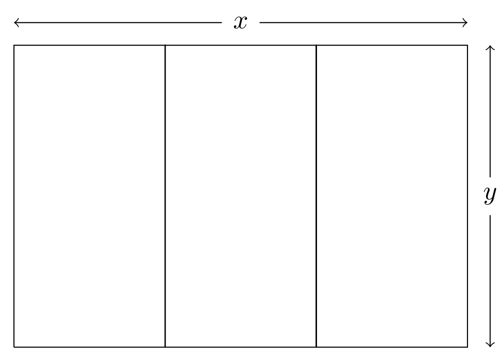
b) All the candidate points (x, y) that maximize $f(x, y)$ subject to the constraint are

A: $(0,0),(2,1),(2,-1),(2,2)$
B: $(2,1),(2,-1),(-2,1),(-2,-1)$
C: $(1,2),(1,-2),(-1,2),(-1,-2)$
D: $\left(\sqrt{2},1\right),\left(\sqrt{2},-1\right),\left(-\sqrt{2},1\right),\left(-\sqrt{2},-1\right)$
E: $\left(1,\sqrt{2}\right),\left(1,-\sqrt{2}\right),\left(-1,\sqrt{2}\right),\left(-1,-\sqrt{2}\right)$

c) The maximum value of $f(x,y)$ subject to the constraint it

A: 2	B: 4	C: $4+\sqrt{2}$	D: 6	E: 8

277. A farmer has 300 meters of fence with which to make a rectangular, three-pen enclosure as shown in the diagram below.



We are to find the dimensions of the enclosure for which the total area is a maximum. In using the Method of Lagrange Multipliers to solve this problem, we must maximize $f(x,y)=xy$ subject to the constraint $g(x,y)=0$, where $g(x,y)$ is

A: $2x+2y$	B: $2x+2y-300$	C: $2x+4y$	D: $2x+4y-300$	E: $6x+4y-300$

278. Using the method of Lagrange multipliers to maximize $f(x,y)=xy+10$ subject to the constraint $x^2+9y^2=18$, we must solve the system of equations

$$\begin{aligned} y+2\lambda x &= 0 \\ x+18\lambda y &= 0 \\ x^2+9y^2-18 &= 0 \end{aligned}$$

Select the set of <u>all</u> the candidates for the point (x,y) that maximizes $f(x,y)$ subject to the constraint.

A: $(0,0),(3,1),(3,-1),(3,3)$	B: $\left(3\sqrt{2},0\right),\left(-3\sqrt{2},0\right),\left(0,\sqrt{2}\right),\left(0,-\sqrt{2}\right)$
C: $(3,1),(3,-1),(-3,1),(-3,-1)$	D: $(1,3),(-1,3),(1,-3),(-1,-3)$
E: $\left(\sqrt{3},1\right),\left(\sqrt{3},-1\right),\left(-\sqrt{3},1\right),\left(-\sqrt{3},-1\right)$	

279. Suppose you are to design a rectangular box with no top, of length l, width w and height h, with the minimum surface area S. Assume also that the volume of the box must be 1000 cm^3. Set this problem up as an optimization problem.

A: Minimize $S = lw + 2lh + 2wh$ where $lwh = 1000$
B: Minimize $S = lw + 2lh + 2wh$ where $l + w + h = 1000$
C: Minimize $V = lwh$ where $lw + 2lh + 2wh = 1000$
D: Minimize $V = lwh$ where $2lw + 2lh + 2wh = 1000$
E: Minimize $S = 2lw + 2lh + 2wh$ where $lwh = 1000$

Trigonometric Derivatives

280. If $f(x) = \sin^3(5x)$, then $f'(x) =$

A: $5\cos^3(5x)\sin(5x)$	B: $30\sin^3(5x)\cos(5x)$	C: $3\sin^2(5x)\cos(5x)$
D: $3\sin(5x)\cos(5x)$	E: $15\sin^2(5x)\cos(5x)$	

281. If $f(x) = x\cos x + \tan\left(\frac{\pi}{4}\right)$, then $f'(x)$ is

A: $-x\sin x$	B: $-x\sin x + \sec^2\left(\frac{\pi}{4}\right)$	C: $\cos x - x\sin x + \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}\right)$
D: $\cos x + x\sin x$	E: $\cos x - x\sin x$	

282. If $f(x) = \sec(5x)$, find $f'(x)$

A: $5\sec x\tan x$	B: $5\sec(5x)\tan(5x)$	C: $25\sec(5x)\tan(5x)$
D: $-5\sec(x)\tan(x)$	E: $\sec(5x)$	

283. If $f(x) = 3\sec^2 x$, what is $f'(x)$?

A: $\sec^3 x$	B: $3\tan^2 x$	C: $6\sec x \tan x$
D: $6\sec^2 x \tan x$	E: $6\sec\left(x^2\right)\tan(x)$	

284. Find $f'(x)$, where $f(x) = \ln\left[\sin\left(x^2\right)\right]$.

A: $\sec(2x)$	B: $\csc\left(x^2\right)$	C: $\cot\left(x^2\right)$	D: $2x\tan\left(x^2\right)$	E: $2x\cot\left(x^2\right)$

285. If $f(x) = (2+\cos x)^x$, then $f'(x) =$

A: $x(2+\cos x)^{x-1}(\sin x)$	B: $-x(2+\cos x)^{x-1}(\sin x)$
C: $-2(2+\cos x)^x \sin x$	D: $(2+\cos x)^x\left(\ln(2+\cos x) + \dfrac{x\sin x}{2+\cos x}\right)$
E: $(2+\cos x)^x\left(\ln(2+\cos x) - \dfrac{x\sin x}{2+\cos x}\right)$	

286. If $f(x) = \ln\left[\cos\left(x^2\right)\right]$, then $f'(x) =$

A: $\dfrac{2x}{\cos\left(x^2\right)}$	B: $\dfrac{1}{\cos\left(x^2\right)}$	C: $-\dfrac{2x\sin\left(x^2\right)}{\cos\left(x^2\right)}$	D: $-\dfrac{\sin\left(x^2\right)}{\cos\left(x^2\right)}$	E: $-\dfrac{2x}{\sin\left(x^2\right)}$

287. If $f(x) = \cos\left(x^3\right)$, then $f'(x)$ is

A: $3\cos\left(x^2\right)(-\sin x)$	B: $3(\cos x)^2(-\sin x)$	C: $-3x^2\sin\left(3x^2\right)$
D: $3x^2\sin\left(x^3\right)$	E: $-3x^2\sin\left(x^3\right)$	

288. If $f(x) = (\cos x)^x$, find $f'(x)$

A: $(\cos x)^x\left[\ln(\cos x) + x\sec x\right]$	B: $(\cos x)^x \ln(\cos x)$
C: $-x(\cos x)^{x-1}\sin x$	D: $-(\cos x)^x \sin x$
E: $(\cos x)^x\left[\ln(\cos x) - x\tan x\right]$	

289. If $f(x) = \tan^3\left(x^2\right)$, find $f'(x)$

A: $6x\tan^2\left(x^2\right)$	B: $3\tan^2(2x)$	C: $6x\tan^2\left(x^2\right)\sec^2\left(x^2\right)$
D: $6x\tan^3\left(x^2\right)\sec\left(x^2\right)$	E: $2x\sec^6\left(x^2\right)$	

290. If $f(x) = \cos^2 x$, find $f'(x)$.

A: $-\sin^2 x$	B: $\sin^2 x$	C: $2\sin x\cos x$	D: $-2\sin x\cos x$	E: $-2\sin x$

291. If $f(x) = e^{\sin x}$, find $f'(x)$.

A: $e^{\cos x}$	B: $e^{-\cos x}$	C: $e^{\cos x}(-\sin x)$
D: $e^{\sin x}(-\cos x)$	E: $e^{\sin x}\cos x$	

292. Let $f(x) = \sin(\cos x)$. Find $f'(x)$.

A: $\cos(\cos x)$	B: $\cos(-\sin x)$	C: $\cos(\cos x) + \sin(-\sin x)$
D: $(-\sin x)\cdot\cos(\cos x)$	E: $(\sin x)\cdot\cos(\sin x)$	

293. Let $f(t) = \sin\left(\frac{\pi t^2}{4}\right)$. Then $f'(1) =$

A: $\frac{\pi}{2}$	B: $\frac{\sqrt{2}}{2}$	C: $\frac{\pi}{2\sqrt{2}}$	D: 0	E: 1

294. The derivative of $f(x) = \tan\left(x^2 + \pi + x\right)$ at $x = 0$ is

A: 1	B: π	C: -1	D: $\pi + 1$	E: $\pi - 1$

295. If $f(x) = x\cos x$, then $f'(\pi) =$

A: π	B: $-\pi$	C: 1	D: -1	E: 0

296. If $f(x) = \sin(\pi x)$, then $f'\left(\frac{1}{2}\right) =$

A: -1	B: 0	C: 1	D: π^2	E: $-\pi^2$

297. If $f(x) = \cos^2 x$, find the slope of the tangent line to the graph of $y = f(x)$ at $x = \frac{\pi}{6}$

A: $\frac{1}{4}$	B: $-\frac{1}{4}$	C: $\frac{3}{4}$	D: $\frac{\sqrt{3}}{2}$	E: $-\frac{\sqrt{3}}{2}$

298. If $f(x) = 1 + 2\cos x$, find $f'\left(\frac{\pi}{6}\right)$

A: $-\sqrt{3}$	B: $\sqrt{3}$	C: $-\sqrt{2}$	D: 1	E: -1

299. Find the slope of the tangent line to the curve $y = \tan x$ at the point $\left(\frac{\pi}{3}, \sqrt{3}\right)$.

A: $\frac{1}{4}$	B: 4	C: $\frac{4}{3}$	D: $\sqrt{3}$	E: $-\sqrt{3}$

300. Find $f'\left(\frac{\pi}{3}\right)$ if $f(x) = \sec x$.

A: $2\sqrt{3}$	B: $\frac{\sqrt{3}}{2}$	C: $\frac{2}{3}$	D: $\frac{2}{\sqrt{3}}$	E: $\frac{1}{2}$

301. If $f(x) = e^x \sin x$, then $f'(0) =$

A: 1	B: -1	C: 0	D: e	E: 2

302. Find $f''(\frac{\pi}{2})$ if $f(x) = \sin^3 x$.

A: 6	B: 0	C: 3	D: -3	E: -2

303. If $f(x) = \tan(2x)$, find $f'\left(\frac{\pi}{6}\right)$.

A: 16	B: 8	C: 4	D: 2	E: $\sqrt{3}$

304. If $f(x) = x \sin x$, find the slope of the tangent line to the graph of $y = f(x)$ at $x = \frac{\pi}{2}$.

A: 0	B: 1	C: -1	D: $\frac{\pi}{2}$	E: $-\frac{\pi}{2}$

Trigonometric Integrals

305. Evaluate $\int_0^{\pi/4} \sin x \, dx$

A: $\frac{1}{2}$	B: $\frac{1}{\sqrt{2}}$	C: $-\frac{1}{\sqrt{2}}$	D: $1 - \frac{1}{\sqrt{2}}$	E: $\frac{1}{\sqrt{2}} - 1$

306. Evaluate $\int \cos x \, dx$.

A: $\cos x + C$	B: $-\sin x + C$	C: $-\cos x + C$	D: $\frac{\cos^2 x}{2} + C$	E: $\sin x + C$

307. Evaluate $\int 2 \sin x \, dx$

A: $\sin^2 x \cos x + C$	B: $-2\cos x + C$	C: $2\cos x + C$	D: $\cos^2 x + C$	E: $2\sin x + C$

308. Evaluate $\int \sec x \tan x \, dx$

A: $\sec x + C$	B: $\tan x + C$	C: $\cot x + C$	D: $\sec^2 x + C$	E: $\sec^3 x + C$

309. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\cos x\, dx$

A: $\sqrt{3}-1$	B: $1-\sqrt{3}$	C: $\frac{1}{4}\left(\sqrt{3}-1\right)$	D: $\frac{1}{4}\left(1-\sqrt{3}\right)$	E: 0

310. Evaluate $\int_{0}^{\frac{\pi}{3}} \sin(3x)\, dx$.

A: $\frac{2}{3}$	B: $\frac{1}{3}$	C: 2	D: $-\frac{2}{3}$	E: -2

311. Find $\int \cos(\sin x)\cos x\, dx$

A: $\sin(\sin x)+C$	B: $-\sin(\sin x)+C$	C: $\sin(\cos x)+C$
D: $-\cos(\cos x)+C$	E: $-\sin(\cos x)\sin x+C$	

312. Find $f'(x)$ if $f(x)=\sin(3x)$.

A: $\cos(3x)$	B: $-3\cos(3x)$	C: $3\cos x$	D: $3\cos(3x)$	E: $-\cos(3x)$

313. If $g'(x)=\sec(x+\pi)\tan(x+\pi)$ and $g(-\pi)=2$, find $g(x)$

A: $\sec(x+\pi)+1$	B: $\sec(x+\pi)+2$	C: $\tan(x+\pi)+2$	D: $\sec(x+\pi+1)$	E: undefined

314. Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}\, dx$

A: $\ln\left(\frac{2}{3}\right)$	B: $\ln\left(\frac{3}{2}\right)$	C: -1	D: 0	E: 1

315. Evaluate $\int_{0}^{\frac{\pi}{4}} \tan x \sec^2 x\, dx$

A: 1	B: $\frac{1}{2}$	C: $\frac{1}{3}$	D: $\frac{1}{4}$	E: $\frac{1}{\sqrt{2}}$

316. Evaluate $\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}}{2\sqrt{x}}\, dx$

A: 0	B: $\frac{1}{2}$	C: 1
D: -1	E: $\cos\left(\frac{\pi^2}{4}\right)-\cos\left(\pi^2\right)$	

317. Evaluate the definite integral $\int_{0}^{\pi/2} \frac{\cos x}{1+\sin x}\, dx$

A: -1	B: $-\ln 2$	C: $\frac{1}{2}$	D: $\ln 2$	E: 1

318. $\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\,dx =$

A: $\frac{\pi}{4}$	B: $\ln 2$	C: $\ln\left(\frac{\pi}{4}\right)$	D: 0	E: 1

319. Evaluate $\int_0^{\pi/2} \sin^2 x \cos x\,dx$

A: $\frac{\pi^2}{24}$	B: $\frac{1}{3}$	C: 0	D: $-\frac{1}{3}$	E: -1

320. $\int_0^{\pi/2} \sin x \cos x\,dx$

A: -1	B: $-\frac{1}{2}$	C: 1	D: 0	E: $\frac{1}{2}$

321. Evaluate $\int_0^{\frac{\pi}{8}} \sin(8x)\,dx$

A: 4	B: 2	C: 1	D: $\frac{1}{2}$	E: $\frac{1}{4}$

322. $\int \frac{\sec^2 x}{1+\tan x}\,dx =$

A: $-\ln\|1+\tan x\|+C$	B: $\ln\|1+\tan x\|+C$	C: $\frac{1}{2}(1+\tan x)^2+C$
D: $\sec x \tan x + C$	E: $\tan^2 x + C$	

323. $\int_0^{\pi/3} \tan^3 t \sec^2 t\,dt =$

A: 9	B: $\frac{1}{36}$	C: $\frac{9}{4}$	D: $\frac{1}{9}$	E: 36

324. Evaluate $\int_0^{\frac{\pi}{6}} \sin^3 x \cos x\,dx$.

A: $\frac{1}{32}$	B: $\frac{9}{64}$	C: $\frac{1}{64}$	D: $\frac{1}{16}$	E: $\frac{9}{16}$

325. Evaluate $\int_0^{\pi/4} \frac{\sec^2 x}{2+\tan x}\,dx$

A: 1	B: $\ln 2$	C: $\ln 3$	D: $\ln\left(\frac{2}{3}\right)$	E: $\ln\left(\frac{3}{2}\right)$

326. $\int x \sin x \, dx =$

A: $-x\cos x + \int \cos x \, dx$	B: $x\cos x - \int \cos x \, dx$	C: $-x\cos x - \int \cos x \, dx$
D: $x\sin x - \int \sin x \, dx$	E: $x\sin x + \int \sin x \, dx$	

327. Evaluate $\int x \cos x \, dx$.

A: $\cos x + x\sin x + C$	B: $-\cos x + x\sin x + C$	C: $\sin x + x\cos x + C$
D: $-\sin x + x\cos x + C$	E: $\sin x + x\sin x + C$	

328. The integral $\int x^3 \cos x \, dx$ can be written as

A: $x^3\cos x - 3\int x^2 \sin x \, dx$	B: $x^3\sin x - 3\int x^2 \sin x \, dx$	C: $x^3\sin x - 3\int x^2 \cos x \, dx$
D: $x^2\sin x - 3\int x^2 \sin \, dx$	E: $x^2\sin x - 3\int x^2 \cos x \, dx$	

329. $\int \frac{d}{dx}(\sin x) \, dx =$

A: $\sin x + C$	B: $-\sin x + C$	C: $\cos x + C$	D: $-\cos x + C$	E: $\frac{1}{2}(\sin x)^2 + C$

330. If $F(x) = 4\sin x \cos x$, then $\int_0^{\pi/4} F'(x) \, dx$ is

A: 2	B: -2
C: 4	D: 1
E: the value cannot be determined	

331. $\int \frac{d}{dx}(\tan x) \, dx =$

A: $\sec^2 x + C$	B: $\frac{1}{2}(\tan x)^2 + C$	C: $-\ln|\cos x| + C$
D: $-\ln|\sec x| + C$	E: $\tan x + C$	

Basic Differential Equations

332. If $\dfrac{dy}{dt} = 4t$ and $y(0) = 3$, find $y(1)$.

A: $3e^4$	B: $4e^3$	C: 5	D: 7	E: 12

333. If $\dfrac{dy}{dx} = \cos x$ and $y(0) = 2$, find $y\left(\frac{\pi}{2}\right)$.

A: 0	B: 1	C: $\dfrac{\sqrt{2}}{2}$	D: 3	E: -1

334. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2x}{e^y}$

A: $y = e^{x^2} + C$	B: $y = 2\ln x + C$	C: $y = Ce^{x^2}$
D: $y = x^2 + C$	E: $y = \ln\left(x^2 + C\right)$	

335. If $\dfrac{dy}{dx} = x^2y^4$ and $y(2) = 1$, then

A: $y = \dfrac{1}{3x-5}$	B: $y = \dfrac{1}{x^3-7}$	C: $y = \dfrac{1}{9-x^3}$	D: $y = \dfrac{1}{\sqrt[3]{x^3-7}}$	E: $y = \dfrac{1}{\sqrt[3]{9-x^3}}$

336. Suppose $\dfrac{dy}{dx} = 4x^3y^2$ and $y(0) = 1$. Then

A: $y = 1 + x^4$	B: $y = \dfrac{1}{1-x^4}$	C: $y = \dfrac{1}{1+x^4}$
D: $y = \left(1 + x^4\right)^2$	E: $y = \ln(e + x^4)$	

337. Find the general solution to the first order differential equation $\dfrac{dy}{dx} = \dfrac{\cos x}{3y^2}$

A: $y = (\sin x)^{1/3} + C$	B: $y = (\sin x + C)^{1/3}$	C: $y = \dfrac{1}{3}(\sin x)^{1/3} + C$
D: $y = (-\sin x + C)^{1/3}$	E: $y = -\dfrac{1}{3}(\sin x)^{1/3} + C$	

338. Solve for y if $\dfrac{dy}{dx} = \dfrac{e^x}{e^y}$

A: $y = x + C$	B: $y = e^x + C$	C: $y = \ln\left(e^x + C\right)$
D: $y = Ce^x$	E: $y = \ln x + C$	

339. Find the solution $y = y(x)$ of the differential equation $\dfrac{dy}{dx} = y^2 \sin x$ subject to the condition that $y(0) = \frac{1}{2}$.

A: $y = \dfrac{\cos x}{\cos x + 1}$	B: $y = \dfrac{1}{\sin x + 2}$	C: $y = \dfrac{1}{2} \sec x$
D: $y = \dfrac{1}{3 - \cos x}$	E: $y = \dfrac{1}{1 + \cos x}$	

340. Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{3x-y}$.

A: $y = \dfrac{1}{3} \ln \left(e^{3x} + C\right)$	B: $y = x + C$	C: $y = \ln x + C$
D: $y = \ln (x + C)$	E: $y = \ln \left(\dfrac{e^{3x}}{3} + C\right)$	

341. If $\dfrac{dy}{dx} = x + e^x$ and $y(0) = 2$, find $y(4)$.

A: $4 + e^4$	B: $8 + e^4$	C: $9 + e^4$	D: $16 + e^4$	E: $17 + e^4$

342. If $\dfrac{dy}{dx} = xy$ and $y(1) = 1$, then $y(0)$ is

A: $-\dfrac{1}{2}$	B: 1	C: $e^{-1/2}$	D: $e^{1/2}$	E: e

343. Find y, where $\dfrac{dy}{dx} = e^x y^2$ and it is known that $y(\ln 2) = 1$.

A: $y = \dfrac{1}{e^x - 1}$	B: $y = -\dfrac{1}{e^x - 3}$	C: $y = -\dfrac{1}{e^x} + \dfrac{3}{2}$	D: $y = -\dfrac{1}{e^x} + \dfrac{2}{3}$	E: $y = \dfrac{1}{\ln 2}$

344. If $\dfrac{dy}{dt} = 3t$ and $y(0) = 5$, find $y(1)$.

A: $\dfrac{3}{2}$	B: $\dfrac{13}{2}$	C: $5e^3$	D: $3e^5$	E: 15

345. An integrating factor that can be used to solve $\dfrac{dy}{dx} - \dfrac{1}{x^2} y = x$ is

A: e^x	B: $\dfrac{1}{x}$	C: $e^{1/x}$	D: x	E: $\dfrac{1}{x^2}$

346. An integrating factor that can be used to solve $\dfrac{dy}{dx} - \left(\dfrac{3}{x}\right) y = x^2$ is

A: $-\dfrac{3}{x}$	B: $\dfrac{1}{x^3}$	C: $\dfrac{3}{x}$	D: e^{-3x}	E: $-3 \ln x$

347. An integrating factor used to solve the first order differential equation $\dfrac{dy}{dx} + \left(\dfrac{\cos x}{\sin x}\right) y = \cos x$ is

A: $\dfrac{1}{\sin x}$	B: $\sin x$	C: $e^{\sin x}$	D: $e^{-\sin x}$	E: $e^{\cos x}$

348. An integrating factor used to solve the differential equation $\dfrac{dy}{dx} + \dfrac{1}{x}y = x \sin x$ is

A: $\dfrac{1}{x}$	B: x	C: $e^{-1/x}$	D: e^{x}	E: $-\ln x$

349. An integrating factor used to solve the differential equation $\dfrac{dy}{dx} + \dfrac{2y}{x} = 4$ is

A: $2x$	B: e^{2x}	C: x^2	D: $\dfrac{1}{x}$	E: $\dfrac{1}{x^2}$

350. An integrating factor used to solve $\dfrac{dy}{dx} - \dfrac{2y}{x} = x^3$ is

A: $e^{-2/x}$	B: $-\dfrac{2}{x}$	C: e^{-2x}	D: $2\ln x$	E: $\dfrac{1}{x^2}$

351. An integrating factor for the differential equation $\dfrac{dy}{dx} + (\sin x)\, y = \cos x$ is

A: $e^{\sin x}$	B: $e^{-\cos x}$	C: $e^{-\sin x}$	D: $-\sin x$	E: $-\cos x$

352. Which of the following is an integrating factor that can be used to solve $\dfrac{dy}{dx} + x^2 y = e^x$

A: e^{x}	B: e^{x^2}	C: e^{e^x}	D: $\dfrac{x^3}{3}$	E: $e^{\frac{x^3}{3}}$

353. To solve $\dfrac{dy}{dx} - \dfrac{y}{x+1} = x^2$, one first needs to calculate the integrating factor $I(x)$. What is $I(x)$?

A: x^2	B: $\dfrac{1}{x-1}$	C: $\dfrac{1}{x+1}$	D: $e^{-\frac{1}{x+1}}$	E: $\ln(x+1)$

354. The differential equation $\dfrac{dy}{dx} - \dfrac{2y}{x} = \dfrac{1}{x^2}$ can be written in the form $\dfrac{d}{dx}\left(I(x) \cdot y\right) = \dfrac{I(x)}{x^2}$ for a suitable integrating factor $I(x)$. Find such a function $I(x)$.

A: $e^{-2/x}$	B: $-\dfrac{2}{x}$	C: $-2x$	D: $\dfrac{1}{x^2}$	E: x^2

355. The differential equation $\dfrac{dy}{dx} + \dfrac{x}{x^2+1}y = x$ can be written in the form $\dfrac{d}{dx}\left(I(x) \cdot y\right) = xI(x)$ for a suitable integrating factor $I(x)$. Find such a function $I(x)$.

A: $\dfrac{1}{2}\ln\left(x^2+1\right)$	B: $e^{\frac{x^2+1}{2}}$	C: $e^{1/2}\left(x^2+1\right)$	D: $\sqrt{x^2+1}$	E: $\dfrac{1}{2}\left(x^2+1\right)$

356. To solve $\dfrac{dy}{dx} + \dfrac{3y}{x} = x^2$, we need an integrating factor $I(x)$. Find $I(x)$.

A: $\dfrac{3}{x}$	B: $e^{3/x}$	C: e^{x^3}	D: x^3	E: e^{3x}

357. Which of the following is an integrating factor that can be used to solve $\dfrac{dy}{dx} + \dfrac{3}{x}y = \sin x$?

A: $3\ln x$	B: $3x$	C: $e^{3/x}$	D: $\dfrac{3}{x}$	E: x^3

358. Find $y(1)$ where $\dfrac{dy}{dx} + y = e^{-x}$, $y(0) = 3$ and an integrating factor for the differential equation is e^x.

A: $4e^{-1}$	B: $3e^{-1}$	C: $4e$	D: $3e$	E: e^{-1}

359. Find $y(1)$ where $\dfrac{dy}{dx} - y = xe^{2x}$ and $y(0) = 1$, using the integrating factor $I(x) = e^{-x}$.

A: $2e$	B: 2	C: $\dfrac{e^2}{2} + e$	D: $\dfrac{e^3+1}{4}$	E: $\dfrac{2e^2+1}{9}$

360. Find the solution of the first order linear differential equation $\dfrac{dy}{dx} + \dfrac{1}{x}y = 3x$ subject to the condition that $y(1) = 2$, given that the integrating factor $I(x) = x$.

A: $y = x^2 + 1$	B: $y = x^2 - \dfrac{1}{x}$	C: $y = x^2 + \dfrac{1}{x}$	D: $y = 6 - \dfrac{4}{x}$	E: $y = 3x - \dfrac{1}{x}$

361. Find $y\left(\frac{\pi}{4}\right)$ where $\dfrac{dy}{dx} + (\tan x)\,y = \dfrac{1}{\sec x}$ and $y(0) = 1$, given that an integrating factor for the differential equation is $\sec x$.

A: $\dfrac{\pi}{4\sqrt{2}}$	B: $\sqrt{2}$	C: $\sqrt{2}\left(\dfrac{\pi}{4}+1\right)$	D: $\dfrac{1}{\sqrt{2}}\left(\dfrac{\pi}{4}+1\right)$	E: $\dfrac{\sqrt{2}}{4}\pi$

362. Solve for y if $\dfrac{dy}{dx} - y = e^x$ and $y(0) = 1$.

A: $y = xe^x + e^x$	B: $y = xe^x - e^x$	C: $y = xe^x$	D: $y = e^x$	E: $y = e^x - xe^x$

363. Find the solution of $\dfrac{dy}{dx} + \dfrac{y}{x} = 3x$ subject to the condition $y(1) = 2$ given that an integrating factor for the differential equation is x.

A: $y = 3x - \dfrac{1}{x}$	B: $y = 6 - \dfrac{4}{x}$	C: $y = x^2 + 1$	D: $y = x^2 - \dfrac{1}{x}$	E: $y = x^2 + \dfrac{1}{x}$

Exponential Growth

364. If $\dfrac{dy}{dt} = 4y$ and $y(0) = 3$, find $y(1)$.

A: $3e^4$	B: $4e^3$	C: 5	D: 7	E: 12

365. If $\frac{dy}{dt} = -2y$ and $y(0) = 6$, then find y.

A: $y = -2e^{6t}$	B: $y = e^{-2t} + 6$	C: $y = 6e^{-2t}$	D: $y = -2e^{t} + 6$	E: $y = 6$

366. If $\frac{dy}{dt} = 3y$ and $y(0) = 5$, find $y(1)$

A: $\frac{3}{2}$	B: $\frac{13}{2}$	C: $5e^{3}$	D: $3e^{5}$	E: 15

367. Find $y(3)$ if $\frac{dy}{dx} = 5y$ and $y(0) = 2$.

A: $6e^{15}$	B: $15e^{2}$	C: $2e^{15}$	D: $5e^{6}$	E: $6e^{5}$

368. If $\frac{dy}{dt} = ky$ and it is known that $y(1) = 2y(0)$, what is the value of k?

A: $\frac{1}{2}$	B: $\ln\left(\frac{1}{2}\right)$	C: 2
D: $\ln 2$	E: Cannot be determined	

369. If $\frac{dy}{dt} = 8y$ and $y(0) = 4$, find $y(1)$.

A: $4e^{8}$	B: $e^{8} + 4$	C: $9 + e^{4}$	D: $16 + e^{4}$	E: $17 + e^{4}$

370. Find k where $\frac{dy}{dx} = ky$ if $y(0) = 4$ and $y(1) = 12$.

A: 3	B: $\ln 3$	C: 4	D: $\ln 4$	E: $\ln 12$

371. If $y = y(t)$ is the solution to the differential equation $\frac{dy}{dx} = ky$ with $y(0) = 4$ and $y(1) = 12$, find the value of t such that $y(t) = 108$.

A: 3	B: 4	C: 9	D: 27	E: $\ln 27$

372. If $y = y(x)$ is a solution of the differential equation $\frac{dy}{dx} = 5y$ subject to the condition $y(0) = 4$, find $y(1)$.

A: 2	B: $4e^{5}$	C: $-\frac{3}{4}$	D: $5 + \ln 3$	E: $e^{5} + 3$

373. If $\frac{dy}{dt} = ky$ with $y(1) = 30$ and $y(0) = 10$, then k is

A: 3	B: $\frac{1}{3}$	C: $\ln 3$	D: $-\ln 3$	E: 20

374. Find k where $\frac{dy}{dt} = ky$ with $y(0) = 4$ and $y(5) = 3$.

A: $\frac{3}{4}$	B: $\ln\left(\frac{3}{4}\right)$	C: $\frac{1}{5}\ln\left(\frac{3}{4}\right)$	D: 4	E: $\frac{1}{5}\ln\left(\frac{4}{3}\right)$

375. If $\frac{dy}{dt} = 5y$ and when $t = 0$, $y = 10$, then find y when $t = \frac{1}{5}\ln 3$

A: 30	B: 45	C: 50	D: $10e^5$	E: $5e^6$

376. Suppose $\frac{dy}{dt} = 2y$ and $y(0) = 4$. Find $y(2)$.

A: $2e^2$	B: $4e^4$	C: $2e^4$	D: e^4	E: $4e^2$

377. Suppose $\frac{dy}{dx} = kx$, $y(0) = 2$ and $y(1) = 5$. Find k.

A: $\ln 5$	B: $\ln 2$	C: $\ln\left(\frac{5}{2}\right)$	D: $\ln 5 + \ln 2$	E: 5

378. A certain substance decays according to the differential equation $\frac{dy}{dt} = -0.03y$, where y is the amount in grams after t years. After how many years will 25% of the initial amount remain?

A: 0.75	B: $-\frac{\ln 0.03}{0.25}$	C: $\frac{\ln 0.03}{0.25}$	D: $-\frac{\ln 0.25}{0.03}$	E: $\frac{\ln 0.25}{0.03}$

379. A certain substance decays exponentially according to the differential equation $\frac{dy}{dt} = ky$. Initially, 36 grams of the substance are present. After 1485 years, 9 grams will remain. Determine the decay constant k.

A: $-\frac{\frac{1}{4}}{1485}$	B: $-\frac{\ln\left(\frac{1}{4}\right)}{1485}$	C: $\frac{\ln\left(\frac{1}{4}\right)}{1485}$	D: $-\frac{1}{4}e^{1485}$	E: $-\frac{1}{4}\ln(1485)$

380. At noon, there are 600 bacteria in a dish. At 5:00pm, there are 1800. Assuming exponential growth, how many will there be at midnight?

A: $600e^{\left(\frac{12\ln 3}{5}\right)}$	B: $1800e^{\left(\frac{5\ln 12}{3}\right)}$	C: $\frac{12}{5}\cdot 600$	D: $600e^{\left(\frac{5\ln 12}{3}\right)}$	E: $600e^{\left(\frac{12\ln 5}{3}\right)}$

381. A bacteria culture starts with 1,000 bacteria and grows at a rate proportional to its size. After 2 hours there are 3,000 bacteria. After how many hours will there be 10,000 bacteria?

A: $\frac{2\ln 2}{\ln 3}$	B: $\frac{2\ln 10}{\ln 3}$	C: $\frac{\ln 3}{2\ln 10}$	D: $\frac{\ln 2}{2\ln 10}$	E: $\frac{2\ln 10}{\ln 2}$

382. The population $y(t)$ of a certain city increases exponentially over time (t) according to the differential equation $\frac{dy}{dt} = ky$. In January 2000, the population was 3 million and in January 2005, the population was 4 million. What will the population of this city be in January 2010?

A: $4\frac{1}{3}$ million	B: 5 million	C: $5\frac{1}{3}$ million
D: 6 million	E: Cannot be determined	

383. The population $y(t)$ of a certain city increases exponentially according to the differential equation $\frac{dy}{dt} = ky$. In 1980, the population was 2 million. In the year 2000, the population was 3 million. Determine the growth constant k.

A: $\frac{1}{20}\ln\left(\frac{2}{3}\right)$	B: $\frac{1}{20}\ln(1.5)$	C: $\frac{1}{2000}\ln\left(\frac{3}{2}\right)$	D: $\frac{3}{2}$	E: $\frac{2}{3}e^{20}$

384. The population of a certain country is 20 million on January 1, 2000 and is 21 million on January 1, 2003. Assume that the exponential growth model is appropriate. Let $y(t)$ be the population of the country in millions, t years after January 1, 2000. Construct the mathematical model for this situation.

A: $\frac{dy}{dt} = ke^y$ $y(0) = 20$ $y(3) = 21$	B: $\frac{dy}{dt} = ky$ $y(0) = 20$ $y(3) = 21$	C: $\frac{dy}{dt} = k\ln y$ $y(0) = 20$ $y(3) = 21$
D: $\frac{dy}{dt} = y + k$ $y(0) = 20$ $y(3) = 21$	E: None of A, B, C or D	

385. Use the following information for parts a and b.

A bacteria culture starts with 1,000 bacteria and grows exponentially according to the differential equation $\frac{dy}{dt} = ky$. After 2 hours, there are 3,000 bacteria.

a) The growth constant k has the value $\frac{\ln 3}{2}$

A: True	B: False	C: Cannot be determined

b) After how many hours will there be 10,000 bacteria?

A: $\frac{(\ln 3)(\ln 10)}{2}$	B: $\frac{\ln 3}{2\ln 10}$	C: $\frac{2\ln 10}{\ln 3}$	D: $\ln 10$	E: $\frac{2}{\ln 3}$

386. A radioactive element decays with a half-life of 100 years. If we have 10 grams of this element today, how many grams will we have in 50 years?

A: $7\frac{1}{2}$	B: 10	C: $5\sqrt{\frac{1}{2}}$	D: $10\sqrt{\frac{1}{2}}$	E: $10\sqrt{2}$

Applications of Differential Equations

387. The slope of a curve at the point (x, y) is equal to $\dfrac{2x}{3y^2+1}$ and the curve passes through the point $(1, 2)$. Find an equation for the curve.

A: $y^3 + y = x^2 + 9$	B: $y^3 = x^2 + 7$	C: $\ln(3y^2+1) = x^2 - 1 + \ln(13)$
D: $\ln(y^3+3) = x^2 - 1 + \ln(10)$	E: None of A, B, C or D	

388. The slope of a certain curve at the point (x, y) is given by $\dfrac{3x^2 - 2x}{y}$. This curve passes through the point $(1, 2)$. Find an equation for the curve.

A: $\ln\lvert y\rvert = x^3 - x^2 + 2$	B: $y^2 = x^3 - x^2 + 2$	C: $y^2 = x^3 - x^2$
D: $y^2 = 2x^3 - 2x^2 + 2$	E: $y^2 = 2x^3 - 2x^2 + 4$	

389. A certain curve passes through the point $(0, 0)$. The slope of the tangent line to the curve at the point (x, y) is equal to $\dfrac{\sin x}{\cos x}$. Find the equation for this curve.

A: $\cos x + \sin y = 1$	B: $\cos x = -\sin y$	C: $\tan\left(\dfrac{x}{y}\right) = 0$
D: $\tan x - \tan y = 0$	E: $\tan x - \tan y = 1$	

390. A large tank initially contains 1000 litres of water and 10 kg. of salt solution. A salt solution containing 0.2 kg of salt per litre is allowed to slow into the tank at the rate of 25 litres per minute. The well-stirred solution flows out of the tank at 25 litres per minute. Find the mathematical model needed to find $y(t)$, the total amount of salt in the tank at time t.

A: $\dfrac{dy}{dt} = 0.2 - 25y$, $y(0) = 10$	B: $\dfrac{dy}{dt} = 0.2 - \dfrac{y}{40}$, $y(0) = 10$	C: $\dfrac{dy}{dt} = 5 - \dfrac{y}{4}$, $y(0) = 10$
D: $\dfrac{dy}{dt} = 5 - \dfrac{y}{200}$, $y(0) = 10$	E: $\dfrac{dy}{dt} = 5 - \dfrac{y}{40}$, $y(0) = 10$	

391. A tank with a capacity of 1000 litres is initially filled with a solution that contains 15 kilograms of salt. A salt solution with a concentration of 0.5 kilograms per litre is run into the tank at 20 litres per minute. The solution in the tank is continuously stirred and flows out of the tank at the same rate. What is the mathematical model for the amount of salt $y(t)$ present in the tank after t minutes?

A: $\dfrac{dy}{dt} = 10 - \dfrac{y}{50}$ $y(0) = 15$	B: $\dfrac{dy}{dt} = 15 - \dfrac{y}{50}$ $y(0) = 0$	C: $\dfrac{dy}{dt} = 20 - 20y$ $y(0) = 15$
D: $\dfrac{dy}{dt} = 15 - yt$ $y(0) = 0$	E: $\dfrac{dy}{dt} = 10 - \dfrac{y}{1000}$ $y(0) = 15$	

392. A tank contains 420 litres of a solution, with 20 kilograms of salt in this solution. A salt solution with a concentration $\frac{5}{7}$ kilograms per litre enters the tank at a rate of 7 litres per minute. The well-stirred solution flows out of the tank at the same rate. Determine the mathematical model for $y(t)$, the number of kilograms of salt in the tank after t minutes.

A: $\dfrac{dy}{dt} = \dfrac{5}{7}y$ $y(0) = 20$	B: $\dfrac{dy}{dt} = 7 - \left(\dfrac{20}{420}\right) \cdot (7t)$ $y(0) = 20$	C: $\dfrac{dy}{dt} = 20 - \dfrac{y}{420}$ $y(0) = 20$
D: $\dfrac{dy}{dt} = 5 - \dfrac{y}{60}$ $y(0) = 20$	E: $\dfrac{dy}{dt} = 20 - yt$ $y(0) = 20$	

393. A tank contains 100 litres of a solution with 50 grams of a certain chemical in the solution. At a certain time, John opens a valve allowing a solution containing the same chemical in a concentration of 2 grams per litre to enter the tank at a rate of 5 litres per minute. While continuously stirring the solution in the tank, he drains off this solution at 5 litres per minute. What is the mathematical model for the amount $y(t)$?

A: $\dfrac{dy}{dt} = 2y$ $y(0) = 50$	B: $\dfrac{dy}{dt} = 5 - \dfrac{y}{100}$ $y(0) = 50$	C: $\dfrac{dy}{dt} = 10 - \dfrac{y}{100}$ $y(0) = 50$
D: $\dfrac{dy}{dt} = 5 - \dfrac{y}{20}$ $y(0) = 50$	E: $\dfrac{dy}{dt} = 10 - \dfrac{y}{20}$ $y(0) = 50$	

394. Initially, a tank contains 10 litres of pure water. A valve is opened, allowing a brine solution containing 1 gram of salt per litre to enter the tank at a rate of 2 litres per minute. The solution in the tank is stirred constantly, keeping its concentration uniform. The brine drains out of the tank at a constant rate of 2 litres per minute. Let $y(t)$ equal the number of grams of salt in the tank t minutes after the valve is opened. Construct the mathematical model for this situation.

A: $\dfrac{dy}{dt} = 2 + \dfrac{1}{5}y$ $y(0) = 0$	B: $\dfrac{dy}{dt} = 2 + 5y$ $y(0) = 0$	C: $\dfrac{dy}{dt} = 2 - \dfrac{1}{5}y$ $y(0) = 0$
D: $\dfrac{dy}{dt} = \dfrac{2}{5} + y$ $y(0) = 0$	E: $\dfrac{dy}{dt} = \dfrac{2}{5} - 5y$ $y(0) = 0$	

395. A tank initially contains 300 litres of water, with 70 grams of a chemical in solution. A valve is opened, allowing a solution containing the same chemical in a concentration of 5 grams/litre to enter the tank at a rate of 8 litres per minute. The well-stirred solution is drained off at the same rate. Construct the mathematical model for this situation, with $y = y(t)$ denoting the amount in grams of chemical in the tank after t minutes.

A: $\frac{dy}{dt} = 40 - \frac{y}{60}$ $y(0) = 70$	B: $\frac{dy}{dt} = 70 - \frac{2y}{15}$ $y(0) = 70$	C: $\frac{dy}{dt} = 40 - \frac{2y}{75}$ $y(0) = 70$
D: $\frac{dy}{dt} = 300 - \frac{5}{8}y$ $y(0) = 70$	E: $\frac{dy}{dt} = \frac{5}{8} - 300y$ $y(0) = 70$	

396. Blood enters and leaves Jack's liver at the rate of 3.5 cubic centimetres per second. The capacity of Jack's liver is 350 cubic centimetres of blood. Determine a mathematical model for the amount $y(t)$ of a drug (in grams) in Jack's previously drug-free liver, t seconds after blood carrying the drug, in a concentration of 0.1 grams per cubic centimetre, first enters the liver.

A: $\frac{dy}{dt} = 0.35 - \frac{y}{100}$ $y(0) = 0$	B: $\frac{dy}{dt} = 3.5 - \frac{y}{350}$ $y(0) = 0$	C: $\frac{dy}{dt} = 35 - \frac{y}{35}$ $y(0) = 0$
D: $\frac{dy}{dt} = 0.35 - 3.5y$ $y(0) = 0$	E: $\frac{dy}{dt} = 350e^{0.35t}$ $y(0) = 0$	

397. If the time rate of change in a quantity Q is jointly proportional to t and the sum of R and S, then we have

A: $\frac{dQ}{dt} = kRS$	B: $\frac{dQ}{dt} = ktRS$	C: $\frac{dQ}{dt} = k + (R + S)$
D: $\frac{dQ}{dt} = kt + (R + S)$	E: $\frac{dQ}{dt} = kt\,(R + S)$	

398. The rate at which an epidemic spreads in a city is jointly proportional to the number of people who are infect and the number of people who are not infected. Let $P(t)$ be the number of people infected, at time t days after the first reported infection, for a city of 100,000 people. Construct the mathematical model for this situation.

A: $\frac{dP}{dt} = kP(1 - P)$ $P(0) = 1$	B: $\frac{dP}{dt} = kP(1 - P)$ $P(0) = 0$
C: $\frac{dP}{dt} = kP(100,000 - P)$ $P(0) = 1$	D: $\frac{dP}{dt} = kP(100,000 - P)$ $P(0) = 0$
E: None of A, B, C or D	

399. On a planet where the acceleration due to gravity is 24 meters per second per second, a steel ball is dropped from a height of 1500 m. Find the height of the ball in meters after 10 seconds.

A: 300	B: 500	C: 1000	D: 240	E: 150

400. A dense object is dropped from a helicopter flying at an altitude of 10,000 feet. Assume that air resistance can be ignored and that the acceleration of gravity is 32 feet per second per second. What is the altitude of the object 10 seconds after it is dropped?

A: 7200 feet	B: 4800 feet	C: 3400 feet	D: 5600 feet	E: 8400 feet

401. A dense object is dropped from the top of a cliff which is 5000 feet above the ground. Let $s(t)$ be the height above the ground (in feet) of the object t seconds after it has been dropped. Assume that the acceleration due to gravity is 32 feet per second per second. Construct the mathematical model for this situation.

A: $s''(t) = -32$, $s(0) = 5000$	B: $s''(t) = 32$, $s(0) = 5000$	C: $s''(t) = -32$, $s'(0) = 0$, $s(0) = 5000$
D: $s''(t) = 32$, $s'(0) = 0$, $s(0) = 5000$	E: Cannot be determined	

402. A solid object is thrown upward from sea level at an initial velocity of 22 m/sec. Assume the acceleration due to gravity is 9.8 m/sec^2. Construct a mathematical model for this situation. $s(t)$ is the distance of the object from sea level at time t.

A: $s''(t) = -9.8$, $s'(0) = 22$	B: $s''(t) = -9.8$, $s'(0) = 0$	C: $s''(t) = -9.8$, $s'(0) = 22$, $s(0) = 0$
D: $s''(t) = -9.8$, $s'(0) = -22$, $s(0) = 0$	E: $s''(t) = -9.8$	

Solutions

1. B
2. C
3. B
4. B
5. B
6. B
7. D
8. B
9. C
10. B
11. C
12. E
13. E
14. B
15. C
16. D
17. D
18. C
19. B
20. C
21. D
22. B
23. C
24. C
25. A
26. A
27. D
28. B
29. E
30. A
31. A
32. D
33. A
34. D
35. A
36. D
37. E
38. C
39. E
40. D
41. E
42. B
43. D
44. D
45. E
46. D
47. A
48. B
49. B
50. A
51. D
52. B
53. D
54. B
55. E
56. A
57. D
58. D
59. E
60. B
61. D
62. E
63. E
64. B
65. E
66. A
67. D
68. C
69. D
70. A
71. C
72. B
73. A
74. B
75. D
76. B
77. D
78. B
79. D
80. D
81. E
82. D
83. E
84. A
85. D
86. C
87. E
88. E
89. B
90. E
91. D
92. C
93. A
94. D
95. E
96. D
97. A
98. B
99. C
100. C
101. E
102. B
103. A
104. E
105. D
106. A
107. E
108. A
109. B
110. D
111. B
112. E
113. A
114. B
115. B
116. a) C
     b) D
117. A
118. D
119. B
120. E
121. A
122. B
123. C
124. D
125. B
126. C
127. A
128. C
129. D
130. B
131. C
132. B
133. C
134. D
135. B
136. D
137. C
138. C
139. A
140. B
141. D
142. D
143. D
144. B
145. C
146. A
147. E
148. D
149. A
150. B
151. E
152. D
153. A
154. C

155. A
156. B
157. E
158. D
159. A
160. A
161. C
162. D
163. E
164. C
165. B
166. A
167. E
168. A
169. D
170. E
171. A
172. E
173. E
174. C
175. C
176. E
177. E
178. D
179. A
180. B
181. B
182. C
183. D
184. C
185. A
186. C
187. D
188. C
189. D
190. C
191. D
192. E
193. D
194. E
195. A
196. A
197. E
198. B
199. a) B
     b) A
     c) A
200. a) D
     b) E
     c) A
201. a) D
     b) E
202. a) C
     b) A
     c) E
203. a) A
     b) B
     c) D
204. a) D
     b) E
     c) A
205. a) D
     b) D
206. a) C
     b) A
207. a) B
     b) D
208. a) D
     b) C
209. A
210. E
211. C
212. E
213. B
214. E
215. C
216. C
217. B
218. E
219. C
220. D
221. A
222. D
223. D
224. D
225. D
226. B
227. C
228. E
229. C
230. A
231. E
232. D
233. D
234. E
235. B
236. A
237. A
238. E
239. C
240. A
241. B
242. D
243. B
244. D
245. C
246. C
247. C
248. B
249. a) D
     b) D
     c) C
250. a) D
     b) A
     c) B
251. a) D
     b) A
     c) C
252. a) A
     b) D
     c) C
253. a) B
     b) A
     c) D
254. A
255. C
256. a) D
     b) C
257. D
258. a) D
     b) B
     c) A
259. a) C
     b) D
     c) B
260. a) D
     b) C
261. a) D
     b) B
     c) A
262. a) C
     b) C
263. A
264. A
265. E
266. D
267. A
268. C
269. C
270. D
271. C
272. E
273. E
274. D
275. a) B
     b) D
276. a) C
     b) B
     c) D
277. D
278. C
279. A
280. E
281. E
282. B
283. D
284. D
285. E
286. C
287. E
288. E
289. C
290. D

291. E
292. D
293. C
294. A
295. D
296. B
297. E
298. E
299. B
300. A
301. A
302. B
303. B
304. B
305. D
306. E
307. B
308. A
309. A
310. A
311. A
312. D
313. A
314. B
315. B
316. C
317. D
318. B
319. B
320. E
321. E
322. B
323. C
324. C
325. E
326. A
327. A
328. B
329. A
330. A
331. E
332. C
333. D
334. E
335. E
336. B
337. B
338. C
339. E
340. E
341. C
342. C
343. B
344. B
345. C
346. B
347. B
348. B
349. C
350. E
351. B
352. E
353. C
354. D
355. D
356. D
357. E
358. A
359. A
360. C
361. D
362. A
363. E
364. A
365. C
366. C
367. C
368. D
369. A
370. B
371. A
372. B
373. C
374. C
375. A
376. B
377. C
378. D
379. C
380. A
381. B
382. C
383. B
384. B
385. a) A
     b) C
386. D
387. A
388. E
389. A
390. E
391. A
392. D
393. E
394. C
395. A
396. A
397. E
398. C
399. A
400. E
401. A
402. C