ECON 4021 Winter 2016
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Assignment 1

Due date: Monday, February 8, in class
Total Marks: 100

1. (10 marks) In your own words, (briefly) describe the "Lucas critique", and explain how the "modern micro-founded" method of macroeconomic modeling attempts to address the issue that the Lucas critique raised about the "traditional" approach to macroeconomics that was dominant up to the 1970s.

2. (10 marks) Imagine that we have a time series of real GDP per capita, Y_t, where Y_t is the level of the observation of the series for period t. Assume that through some transformation we obtain a measure of a "trend" or growth component of Y_t, which we call Y_t^g. Defining $d_t = \frac{Y_t - Y_t^g}{Y_t^g}$ as the fractional deviation of the level of real GDP per capital from its trend component, show that for small deviations, we can approximate this deviation by computing the difference between a series constructed with the log of Y_t, and the log of Y_t^g.

3. (10 marks) Robert Lucas said that "...business cycles are all alike", yet from Chapter 3, we see that the times series of deviations from trend in real GDP is "choppy", and there is no regularity in either the amplitude or frequency of fluctuations in real GDP about trend. What did Lucas mean by this statement?

4. (40 marks) Consider the consumer's optimization problem from the Appendix to chapter 4 in Williamson,

$$\begin{aligned} & max_{c,l} U(c,l) \\ & \text{subject to} \qquad c = w(h-l) + \pi - T, \end{aligned} \tag{1}$$

where $U(\cdot)$ is increasing in both arguments, strictly quasiconcave, and twice differentiable.

   (a) (3 marks) Write down the three first-order conditions for the problem, using the Langranian method, and defining λ as the Lagrange multiplier.

   (b) (10 marks) Now totally differentiate these three first-order conditions, except that instead of substituting out the Lagrange multipler λ as Williamson does, keep it in the problem, so that you obtain a system in the form

$$\mathbf{A}\begin{pmatrix} dc \\ dl \\ d\lambda \end{pmatrix} = \mathbf{D},$$

where **A** and **D** are comformable matrices, and the matrix **D** contains the differentials of the exogenous variables as in Williamson.

(c) (2 marks) Do you notice anything special about the form of the matrix **A**?

(d) (5 marks) Calculate the determinant of **A**, $|\mathbf{A}|$.

(e) (3 marks) Calculate the determinant of the Bordered Hessian.

(f) (2 marks) What does the assumption that $u(\cdot)$ is strictly quasiconcave imply about the sign of the Bordered Hessian?

(g) (10 marks) Determine expressions for the derivatives $\frac{\partial C}{\partial \pi}$, $\frac{\partial l}{\partial \pi}$

(h) (5 marks) What does the assumption that c and l are normal goods imply about the sign of $\frac{\partial C}{\partial \pi}$ and $\frac{\partial l}{\partial \pi}$?

5. (30 marks) Over the past century in the U.S., measures of labour input per person have been remarkably constant, displaying little or no trend, which is potentially surprising, since the real wage has displayed a distinct positive trend over this period. In order to address this and other balanced growth facts, King, Plosser and Rebelo (1988) developed a class of preferences of the general form

$$U(c,l) = \frac{1}{1-\sigma}\{[cv(l)]^{1-\sigma} - 1\}, \tag{2}$$

where $v(l)$ is some twice differentiable function of leisure with specific regularity conditions that for the purpose of this problem we can assume that implies that $U(c,l)$ is strictly quasiconcave.

(a) (5 marks) Show that for these preferences,

$$\lim_{\sigma \to 1} u(c,l) = \ln c + \ln(v(l)). \tag{3}$$

Hint: Use L'hopital's rule

(b) (15 marks) Derive the expression for the comparative statics derivative $\frac{\partial l}{\partial w}$ for the consumer's optimization problem

$$max_{c,l} U(c,l) \qquad \text{subject to} \qquad c = w(h-l) + \pi - T, \tag{4}$$

where

$$U(c,l) = \frac{1}{1-\sigma}\{[cv(l)]^{1-\sigma} - 1\}. \tag{5}$$

Note that you can work directly from the comparative statics expressions obtained in the consumer's optimization problem in the Appendix to Chapter

4 in Williamson, since all we are doing are is placing some restrictions on the functional form of the utility function $u(c,l)$ that Williamson uses (ie there is no need to re-do the total differentiations etc).

(c) (5 marks) Now assume that all variables in the expression above for $\frac{\partial l}{\partial w}$ are equilibrium quantities and/or prices. Impose the following equilibrium condition on your expression for $\frac{\partial l}{\partial w}$,

$$C = w(h - l), \tag{6}$$

(which corresponds to $\pi - T = 0$, implying that in aggregate there is wage income only) , and simplify.

(d) (3 marks) Can you interpret the meaning your result? *Hint: think about the the income and substitution effects.*

(e) (2 marks) Although this is a one-period model and so technically speaking there is no "growth", can you speculate (briefly) about how in a model of multiple periods (ie where you can loosely assume that your comparative static expression holds in each period), this analytical result could help a model be consistent with the empirical observation that as the wage grows over time, hours-worked seems relatively constant? Note: no calculations are necessary for this answer.