Solutions

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2013

TEST 4

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name : _____

Student Number: _____

Tutorial Section (A1, A4, ...): _____

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[3 marks]** Evaluate $\int\int_{\mathcal{R}} 2x\ dA.$ where $\mathcal{R} = \{(x,y) : 0 \le x \le 2,\ 0 \le y \le 1\}$
   (a) 8, (b) 4, (c) 0, (d) -1, (e) none of these.

2. **[3 marks]** Evaluate $\int\int_{\mathcal{R}} (x^3 + 2y)\ dA.$ where $\mathcal{R} = \{(x,y) : 0 \le x \le 2,\ x^2 \le y \le 2x\}$
   (a) 32/5, (b) 12, (c) 7/5, (d) -16, (e) none of these.

3. **[3 marks]** Which of the following integrals is equal to the given integral: $\int\int_{\mathcal{R}} e^{-x^2}\ dx\,dy$ where $\mathcal{R} = \{(x,y) : 2y \le x \le 2,\ 0 \le y \le 1\}$
   (a) $\int_0^2 \int_0^1 e^{-x^2}\ dy\,dx$, (b) $\int_1^2 \int_0^x e^{-x^2}\ dy\,dx$, (c) $\int_0^2 \int_0^{x/2} e^{-x^2}\ dy\,dx$, (d) $\int_0^2 \int_0^{2x} e^{-x^2}\ dy\,dx$, (e) none of these.

4. **[3 marks]** Evaluate the double integral $\int\int_{\mathcal{R}} 3y\ dA.$ where $\mathcal{R}$ is the disk of radius 2 centered at the origin.
   (a) -1, (b) 1, (c) 0, (d) 2, (e) none of these

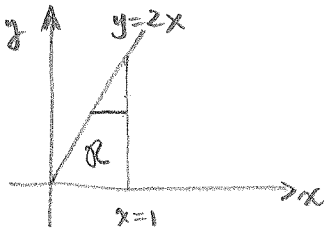
5. **[3 marks]** Let T denote the solid sphere centered at the origin having radius 9. Then $\int\int\int_T dV$ is equal to the volume of T.
   (a) True, (b) False, (c) none of these

PART II: Show all work here and give details. No additional pages will be accepted

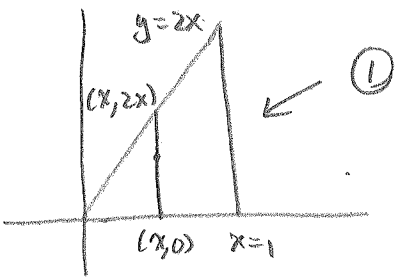
6. **[5+5 marks]** a) Use Fubini's theorem to evaluate the double integral $\int_0^2 \int_{y/2}^1 e^{y/x}\ dx\,dy.$ $= I$

   b) Evaluate the iterated integral $\int_0^1 \int_0^z \int_0^y 2xz\ dx\,dy\,dz.$ $= J$

a) $\mathcal{R} = \{(x,y) : \frac{y}{2} \le x \le 1,\ 0 \le y \le 2\}$ ∴ uses horizontal slices



Now use vertical slices.



(1) ← (1)

(1) (1)

∴ $\mathcal{R} = \{(x,y) : 0 \le y \le 2x,\ \underbrace{0 \le x \le 1}_{(1)}\}$.

$$\therefore I = \int_0^1 \int_0^{2x} e^{y/x}\,dy\,dx = \int_0^1 x e^{y/x}\Big|_{y=0}^{y=2x} = \int_0^1 x(e^2-1)\,dx = \boxed{\frac{e^2-1}{2}}$$

(1) ←