

ENGR-233: Applied Advanced Calculus Winter 2016

Midterm sample 1

Problem 1. Find the parametric equation of the line of intersection of the two planes:

$$P_1: x+y-8z=4 \text{ and } P_2: 3x-y+4z=0 .$$

Problem 2. Position vector of a moving particle is given by

$$\mathbf{r}(t) = (3t^2+1, 2t^2-7t+3, (t-1)^2) .$$

(a) At what time(s) does the particle pass the xz -plane?

(b) What are the particle (i) coordinates, (ii) velocity, (iii) speed, (iv) acceleration at $t=2$?

Problem 3. Find the directional derivative of $F(x, y, z) = 15x^2e^{-z} + 3y^2$ in the direction $\mathbf{u} = (4, -4, 2)$ at the point $(1, 2, 0)$.

Problem 4. Let $\mathbf{F} = (x(x^2+y^2+z^2)^m, y(x^2+y^2+z^2)^m, z(x^2+y^2+z^2)^m)$.

(a) Find $\nabla \cdot \mathbf{F}$; (b) Find m such that $\nabla \cdot \mathbf{F} = 0$ for $x^2+y^2+z^2 > 0$.

Problem 5. Let

$$\mathbf{F}(x, y, z) = (a \cos y + b \sin z, c \cos z + d \sin x, e \cos x + f \sin y) .$$

(a) Find $\nabla \times \mathbf{F}$; (b) Find the values of a, b, c, d, e, f such that $\nabla \times \mathbf{F} \equiv \mathbf{F}$.

Problem 6. Find the work done by the force $\mathbf{F}(x, y, z) = (x-y, x^2, -z)$ moving a particle along a **line segment** from a point $P(1, 2, 3)$ to a point $Q(2, 1, 2)$.

Hint: Find the parametric equation of the line connecting P and Q , then evaluate the integral.

Problem 7. Let $\mathbf{F}(x, y, z) = (y e^{xy}, x e^{xy} - \sin(y+z), 3z^2 - \sin(y+z))$.

(a) Show that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of the path;

(b) Compute the integral for any path C from the point $A(2, -1, 1)$ to the point $B(3, 2, -2)$.