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Mathématiques et statistique
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2010
Midterm II
Professor: Victor G. LeBlanc
Time limit: 80 minutes. Closed books. No calculators.

Name:

ID Numb

Instructio

- You have 80 minutes to complete the test.
- You are not allowed to use any books, course notes, calculators, cell phones, pagers or other electronic devices.
- Read each question carefully before answering.
- There are 5 questions, and each requires a full, clearly-written and detailed solution. Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test.
- Good luck!

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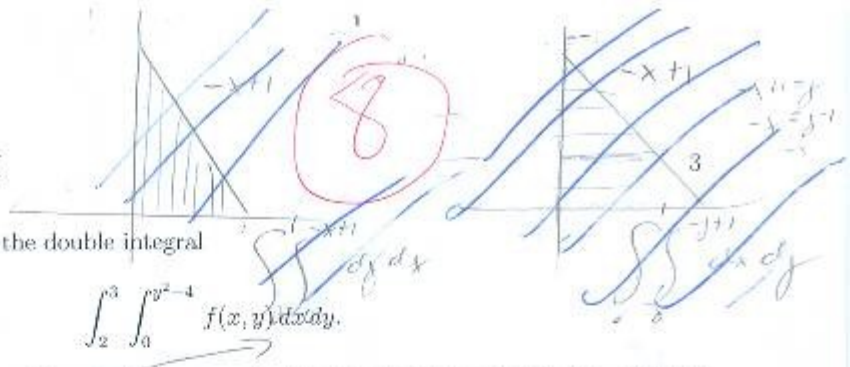
1. **(10 marks)** A triangular metal plate has its vertices at the points $(0,0)$, $(1,0)$ and $(0,3)$ in the x-y plane. The mass-density of the plate is $\delta(x,y) = xy$, in appropriate units. Using a double integral, find the total mass of the plate.



2. (10 marks) Consider the double integral

$$\int_2^3 \int_0^{y^2-4} f(x,y)\,dx\,dy.$$

Sketch the region of integration, and then rewrite the double integral with the order of integration reversed. **Do not evaluate the integral.**

3. (10 marks) Using a change of coordinates, evaluate the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{1-x^2-y^2}(x^2+y^2)^4\,dz\,dy\,dx.$$

4. **(10 marks)** Consider the parametrized curve

$$\vec{r}(t) = (t\sin(3t))\,\vec{i} + t^2\,\vec{j} + (t\cos(3t))\,\vec{k}, \quad t \in [0, 2\pi].$$

(a) Compute the tangent vector $\dfrac{d\vec{r}}{dt}(t)$, and compute its length $\left\|\dfrac{d\vec{r}}{dt}(t)\right\|$. For the length, simplify your answer as much as possible using appropriate trig identities.

(b) Using your answer in part (a) above, write an integral (**but do not evaluate this integral**) which would give the total length of the curve $\vec{r}(t)$ for $t \in [0, 2\pi]$.

5. (10 marks) Consider the portion of the sphere $x^2 + y^2 + z^2 = 1$ which lies between the planes $z = -\frac{\sqrt{2}}{2}$ and $z = \frac{\sqrt{2}}{2}$. (See figures on next page).

(a) Give a parametrization of this surface.

(b) Compute the total area of this surface.

Two different views of the surface described in problem 5.