EECE 376

Electromechanics

Module 7: Brushless DC Motors (Chap. 8)

Spring 2015

Instructor: Dr. Juri Jatskevich

Class Webpage:

http://courses.ece.ubc.ca/376/

Important Topics & Concepts

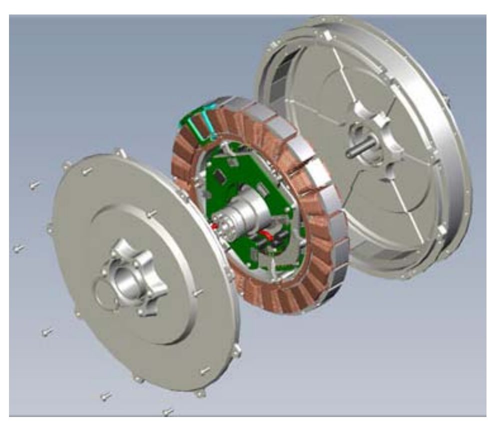
- Types and construction of commonly Brushless DC and Servo Motors
- Principle of operation & torque production
- 2-phase BLDC Motor model
- Model in qd-Rotor Reference Frame (RRF)
- Steady-state analysis characteristics
- Similarity with conventional brushed DC Motors
- 3-phase BLDC Motor model
- 6-step 180/120 deg inverter operation
- Principle of PWM drive of Brushless DC Motors
- Simulink Model implementation

Typical Brushless DC Motors

Computers

Propulsion & Transportation

In-Wheel motors



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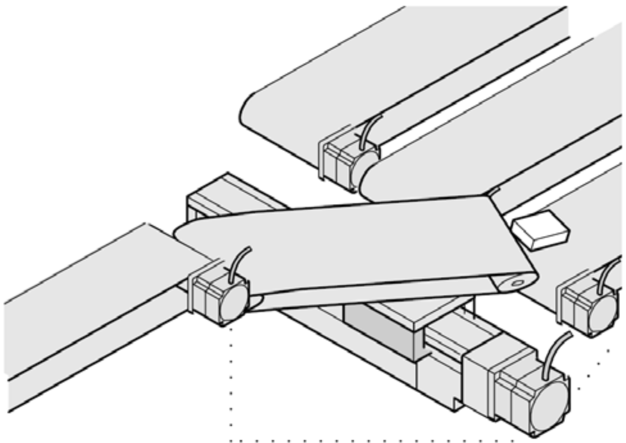
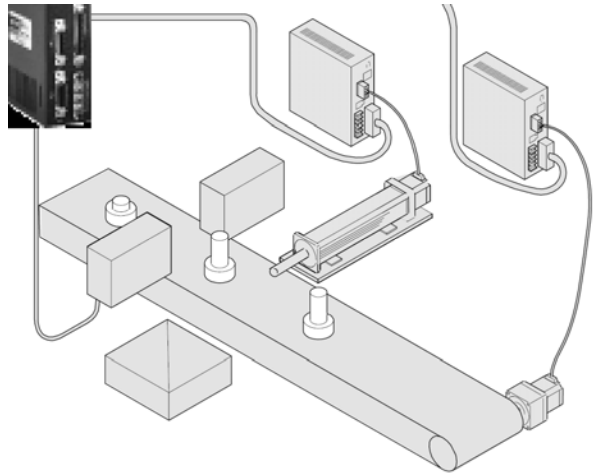
Typical Brushless DC Motors

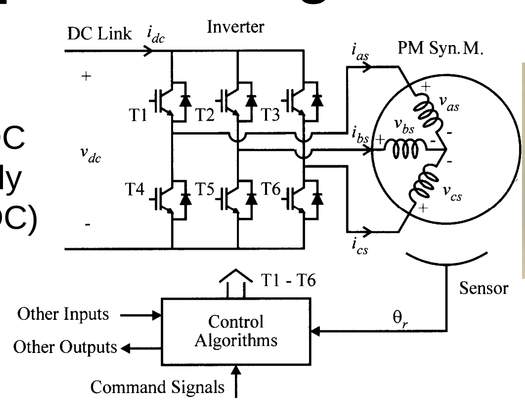
Servo motors for positioning and industrial automation



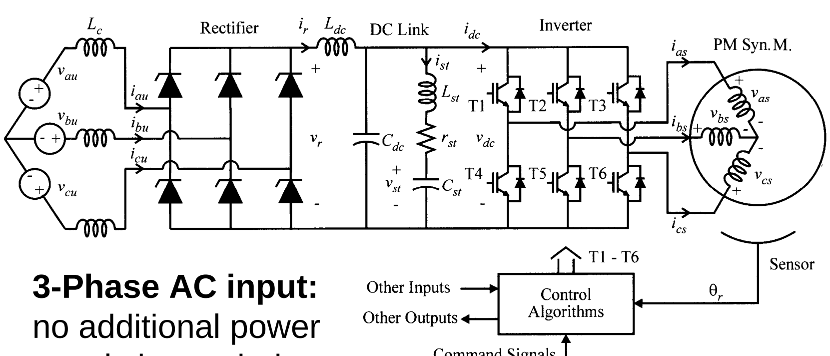
Typical Configurations of BLDC Motors

DC input: requires additional DC power supply (24 – 48V DC)



BLDC Motor Drivers


Brushless AC or DC Servo Motors

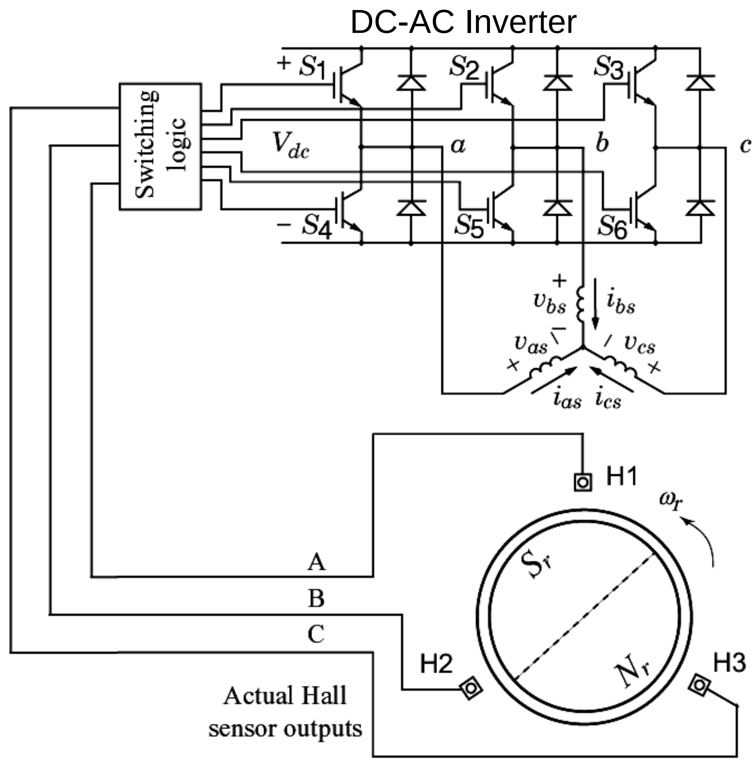


3-Phase AC input: no additional power supply is needed (208/120 V AC)

Hall-Sensor-Driven BLDC Machine

Features

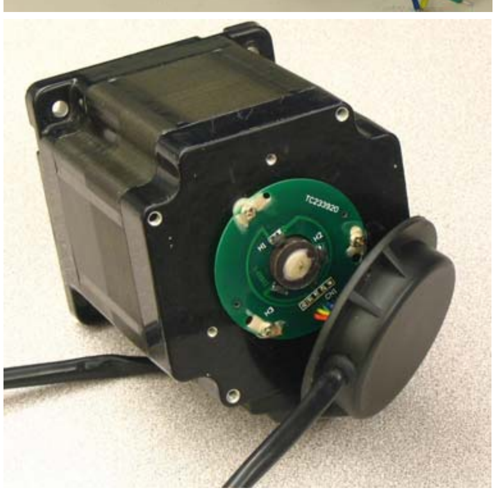
- Simple & low cost
- Work with various loads
- Drivers work with different motors
- Motors work with different drivers
- Good speed/torque characteristics
- High dynamic response
- High efficiency
- Wide speed ranges

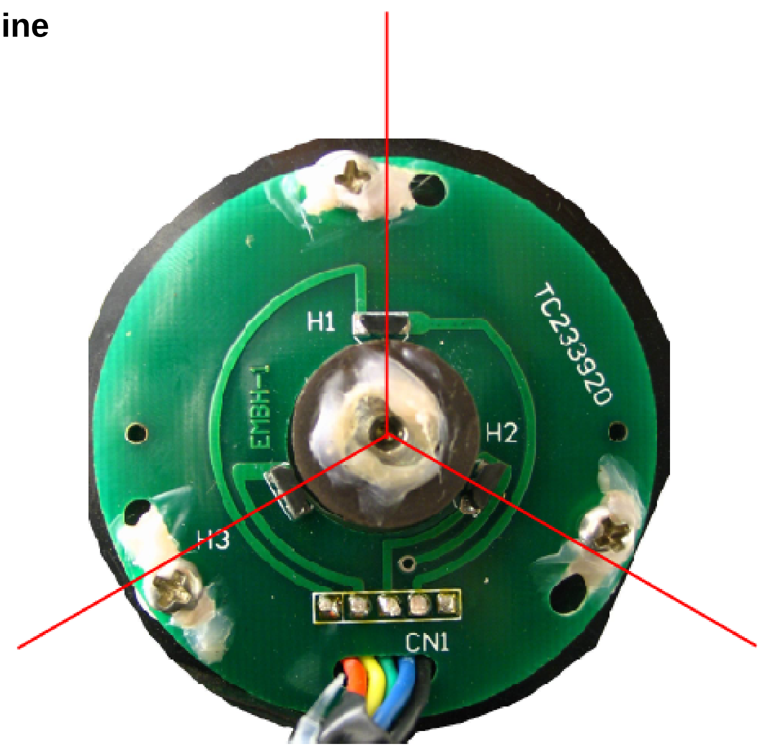


Permanent Magnet Synchronous Machine (PMSM)

Hall Sensors in BLDC Motors

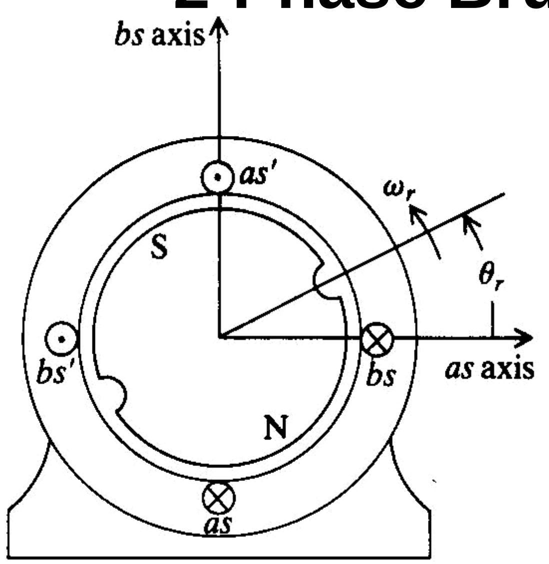
Typical Industrial BLDC Machine

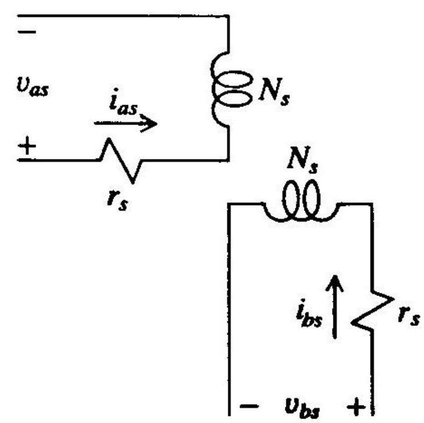




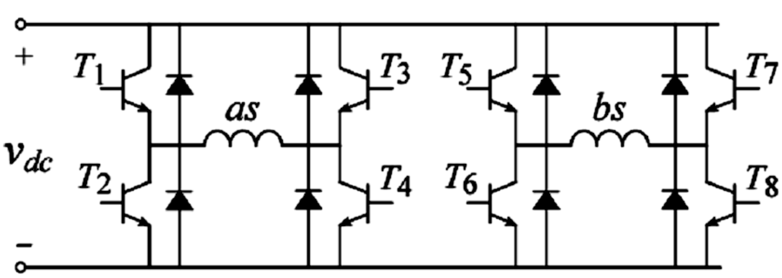
Brushless DC Motor

2-Phase Brushless DC Motor





Assume inverter circuit

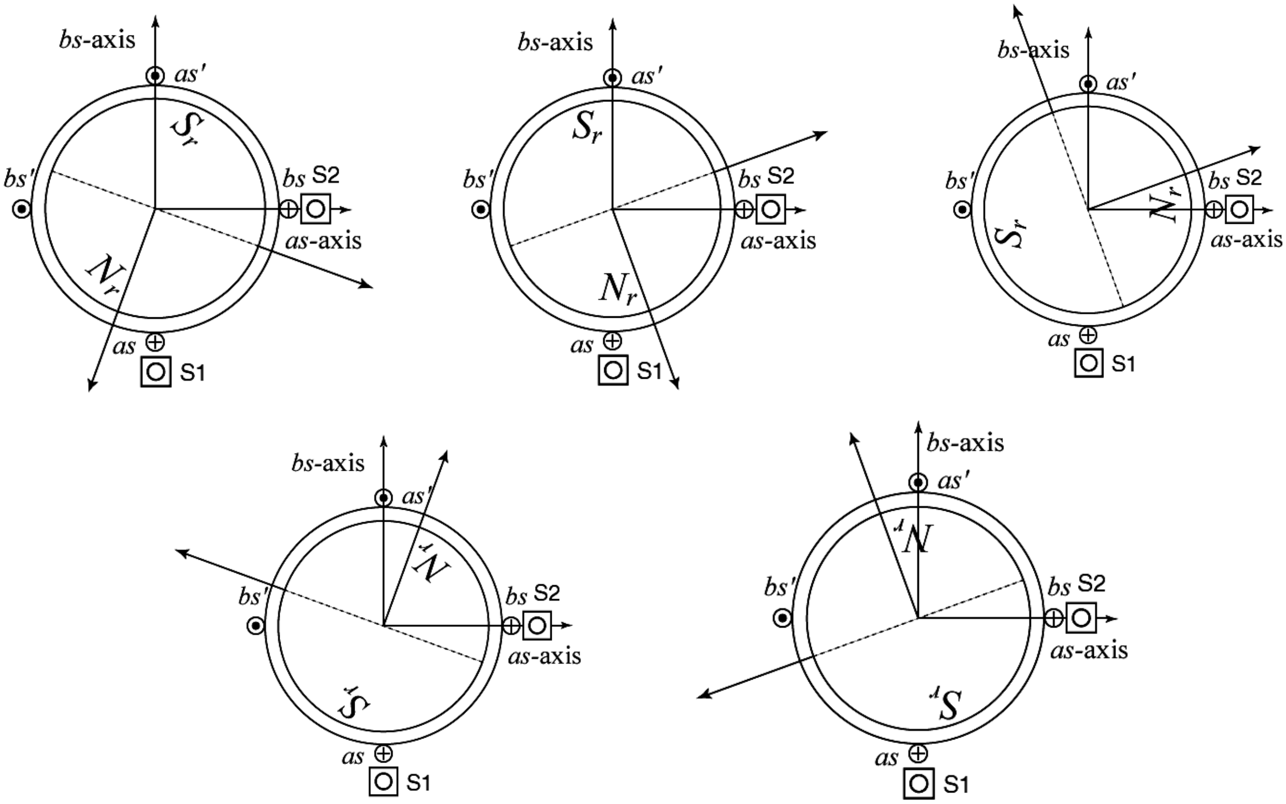


Hall-effect sensors control the inverter

T1 & T4 => as positive
T2 & T3 => as negative

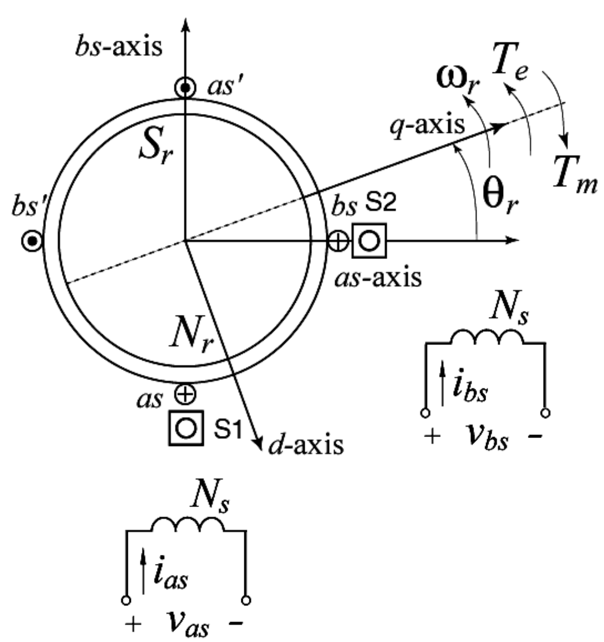
T5 & T8 => bs positive
T6 & T7 => bs negative

Brushless DC Motor Principle



Hall-effect sensors: Under N – on (positive), under S – off (negative)

2-Phase Brushless DC Motor Model



Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 \\ 0 & L_{bsbs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix}$$

$$\boldsymbol{\lambda}_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \boldsymbol{\lambda}_m$$

Assume stator self inductances

$$L_{asas} = L_{ls} + L_{ms}$$

$$L_{bsbs} = L_{ls} + L_{ms}$$

Flux linkage due to Permanent Magnet

$$\boldsymbol{\lambda}_m = \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix} = \lambda_m \begin{bmatrix} \sin(\theta_r) \\ -\cos(\theta_r) \end{bmatrix}$$

2-Phase Brushless DC Motor Model

Electromagnetic Torque $T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$

Assume magnetically linear system $W_c = W_f$

$$W_f = \frac{1}{2}\mathbf{i}_{abs}^T(\mathbf{L}_s - L_{ls}\mathbf{I})\mathbf{i}_{abs} + \mathbf{i}_{abs}^T\boldsymbol{\lambda}_m$$

$$= \frac{1}{2}L_{ms}i_{as}^2 + \frac{1}{2}L_{ms}i_{bs}^2 + i_{as}\lambda_{asm} + i_{bs}\lambda_{bsm}$$

$$= \frac{1}{2}L_{ms}\left(i_{as}^2 + i_{bs}^2\right) + \lambda_m\left[i_{as}\sin(\theta_r) - i_{bs}\cos(\theta_r)\right]$$

$$T_e = \frac{P}{2}\lambda_m\left[i_{as}\cos(\theta_r) + i_{bs}\sin(\theta_r)\right]$$

Mechanical System

$$J_{total}\frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

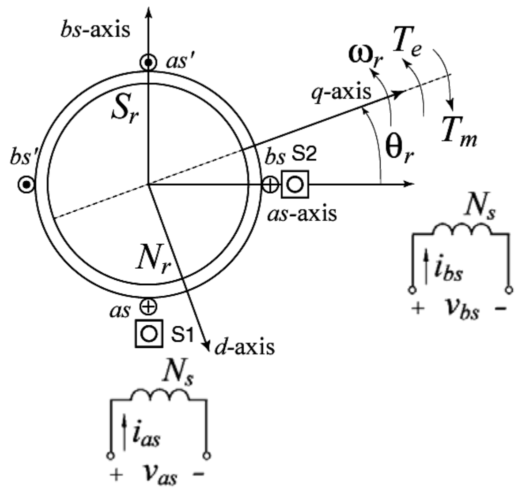
Rotor position $\theta_r = \theta_r(0) + \int \omega_r dt$

Recall for P-pole motors we can define:

$$\omega_r = \frac{P}{2}\omega_{rm}$$

$$\theta_r = \frac{P}{2}\theta_{rm}$$

2-Phase Brushless DC Motor Model



Voltage equations = State equations

$$\frac{d\boldsymbol{\lambda}_{abs}}{dt} = -\mathbf{r}_s\mathbf{i}_{abs} + \mathbf{v}_{abs}$$

$$= -\mathbf{r}_s\mathbf{L}_s^{-1}(\boldsymbol{\lambda}_{abs} - \boldsymbol{\lambda}_m) + \mathbf{v}_{abs}$$

Flux linkage equations

$$\boldsymbol{\lambda}_{abs} - \boldsymbol{\lambda}_m = \mathbf{L}_s\mathbf{i}_{abs}$$

$$\mathbf{i}_{abs} = \mathbf{L}_s^{-1}(\boldsymbol{\lambda}_{abs} - \boldsymbol{\lambda}_m)$$

Electromagnetic Torque

$$T_e = \frac{P}{2}\lambda_m\left[i_{as}\cos(\theta_r) + i_{bs}\sin(\theta_r)\right]$$

Mechanical System

$$J_{total}\frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

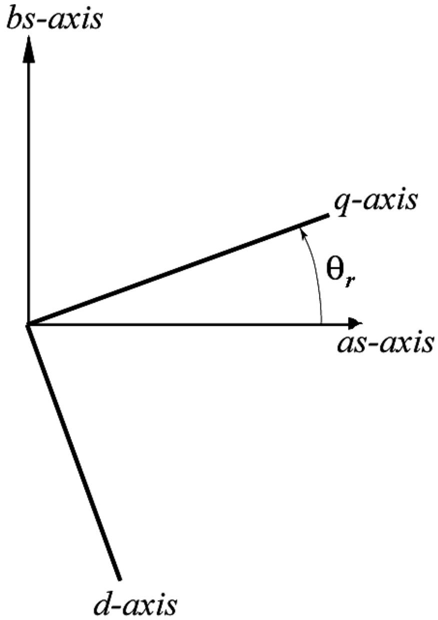
$$\theta_r = \theta_r(0) + \int \omega_r dt$$

$$\omega_r = \frac{P}{2}\omega_{rm} \qquad \theta_r = \frac{P}{2}\theta_{rm}$$

How similar is this to a conventional brushed DC motor ?

Apply Rotor Frame Reference (RRF)

qd-frame is fixed on the rotor



Phases have variables $f = i, v, \lambda, \ldots$

Define a vector $\mathbf{f}_{abs} = \begin{bmatrix} f_{as} & f_{bs} \end{bmatrix}^T$

Define change of variables to *qd*-coordinate

$$\mathbf{f}_{qds} = \begin{bmatrix} f_{qs} & f_{ds} \end{bmatrix}^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$\mathbf{f}_{qds} = \mathbf{K}_s^r \mathbf{f}_{abs}$$

$$\mathbf{f}_{abs} = \left(\mathbf{K}_s^r\right)^{-1} \mathbf{f}_{qds}$$

$$\left(\mathbf{K}_s^r\right)^{-1} = \mathbf{K}_s^r = \left(\mathbf{K}_s^r\right)^T$$

Rotor position $\theta_r = \theta_r(0) + \int \omega_r dt$

Express all equations in transformed *qd* variables !

Voltage Equations in RRF

Original voltage equation

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1} \mathbf{v}_{qds} = \mathbf{r}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{v}_{qds} = \mathbf{K}_s^r \mathbf{r}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{K}_s^r \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right] = \mathbf{K}_s^r \left[\frac{d}{dt}\left(\mathbf{K}_s^r\right)^{-1}\right] \boldsymbol{\lambda}_{qds} + \mathbf{K}_s^r \left(\mathbf{K}_s^r\right)^{-1} \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

$$\begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \omega_r \begin{bmatrix} -\sin(\theta_r) & \cos(\theta_r) \\ \cos(\theta_r) & \sin(\theta_r) \end{bmatrix} = \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{\lambda}_{qds} = \omega_r \begin{bmatrix} \boldsymbol{\lambda}_{ds} \\ -\boldsymbol{\lambda}_{qs} \end{bmatrix} = \omega_r \boldsymbol{\lambda}_{dqs}$$

Resulted voltage equation

$$\mathbf{v}_{qds} = \mathbf{r}_s \mathbf{i}_{qds} + \omega_r \boldsymbol{\lambda}_{dqs} + \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

Flux Linkage Equations in RRF

Original equation

$$\begin{bmatrix}\lambda_{as}\\ \lambda_{bs}\end{bmatrix}=\begin{bmatrix}L_{ls}+L_{ms} & 0\\ 0 & L_{ls}+L_{ms}\end{bmatrix}\cdot\begin{bmatrix}i_{as}\\ i_{as}\end{bmatrix}+\lambda_m\begin{bmatrix}\sin(\theta_r)\\ -\cos(\theta_r)\end{bmatrix}$$

$$\boldsymbol{\lambda}_{abs}=\mathbf{L}_s\mathbf{i}_{abs}+\boldsymbol{\lambda}_m$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1}\boldsymbol{\lambda}_{qds}=\mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds}+\boldsymbol{\lambda}_m$$

$$\boldsymbol{\lambda}_{qds}=\mathbf{K}_s^r\mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds}+\mathbf{K}_s^r\boldsymbol{\lambda}_m$$

where

$$\mathbf{K}_s^r\mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1}=\begin{bmatrix}L_{ls}+L_{ms} & 0\\ 0 & L_{ls}+L_{ms}\end{bmatrix}$$

$$\begin{bmatrix}\cos(\theta_r) & \sin(\theta_r)\\ \sin(\theta_r) & -\cos(\theta_r)\end{bmatrix}\cdot\lambda_m\begin{bmatrix}\sin(\theta_r)\\ -\cos(\theta_r)\end{bmatrix}=\lambda_m\begin{bmatrix}0\\ 1\end{bmatrix}$$

Resulted flux linkage equation

$$\begin{bmatrix}\lambda_{qs}\\ \lambda_{ds}\end{bmatrix}=\begin{bmatrix}L_{ls}+L_{ms} & 0\\ 0 & L_{ls}+L_{ms}\end{bmatrix}\cdot\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix}+\lambda_m\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\lambda_{qs}=\left(L_{ls}+L_{ms}\right)i_{qs}=L_{ss}i_{qs}$$

$$\lambda_{ds}=\left(L_{ls}+L_{ms}\right)i_{ds}+\lambda_m=L_{ss}i_{ds}+\lambda_m$$

2-Phase Brushless DC Motor Model in qd RRF

Expanded Voltage Equations

$$v_{qs}=r_si_{qs}+\omega_r\lambda_{ds}+\frac{d\lambda_{qs}}{dt}=r_si_{qs}+\omega_rL_{ss}i_{ds}+\omega_r\lambda_m+L_{ss}\frac{di_{qs}}{dt}$$

$$v_{ds}=r_si_{ds}-\omega_r\lambda_{qs}+\frac{d\lambda_{ds}}{dt}=r_si_{ds}-\omega_rL_{ss}i_{qs}+L_{ss}\frac{di_{ds}}{dt}$$

$$\frac{d}{dt}\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix}=\begin{bmatrix}L_{ss} & 0\\ 0 & L_{ss}\end{bmatrix}^{-1}\left\{\begin{bmatrix}-r_s & -\omega_rL_{ss}\\ \omega_rL_{ss} & -r_s\end{bmatrix}\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix}+\begin{bmatrix}-\omega_r\lambda_m\\ 0\end{bmatrix}+\begin{bmatrix}v_{qs}\\ v_{ds}\end{bmatrix}\right\}$$

Electromagnetic Torque

$$\begin{bmatrix}i_{as}\\ i_{bs}\end{bmatrix}=\begin{bmatrix}\cos(\theta_r) & \sin(\theta_r)\\ \sin(\theta_r) & -\cos(\theta_r)\end{bmatrix}\begin{bmatrix}i_{qs}\\ i_{ds}\end{bmatrix}=\begin{bmatrix}i_{qs}\cos(\theta_r)+i_{ds}\sin(\theta_r)\\ i_{qs}\sin(\theta_r)-i_{ds}\cos(\theta_r)\end{bmatrix}$$

$$T_e=\frac{P}{2}\lambda_m\left[i_{as}\cos(\theta_r)+i_{bs}\sin(\theta_r)\right]$$

$$=\frac{P}{2}\lambda_m\left[i_{qs}\cos^2(\theta_r)+i_{ds}\sin(\theta_r)\cos(\theta_r)+i_{qs}\sin^2(\theta_r)-i_{ds}\sin(\theta_r)\cos(\theta_r)\right]$$

$$=\frac{P}{2}\lambda_mi_{qs}$$

Steady-State Operation

Voltage Equations

$$V_{qs} = r_s I_{qs} + \omega_r L_{ss} I_{ds} + \omega_r \lambda_m$$

$$V_{ds} = r_s I_{ds} - \omega_r L_{ss} I_{qs}$$

$$v_{qs} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss} \frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss} \frac{di_{ds}}{dt}$$

Electromagnetic Torque $T_e = \frac{P}{2} \lambda_m I_{qs}$

$$T_e = \frac{P}{2} \frac{\lambda_m r_s}{r_s^2 + \omega_r^2 L_{ss}^2} \left(V_{qs} - \frac{\omega_r L_{ss}}{r_s} V_{ds} - \omega_r \lambda_m \right)$$

Assume applied voltages

$$V_{as} = \sqrt{2} V_s \cos(\omega_e t + \phi_v)$$

$$V_{bs} = \sqrt{2} V_s \sin(\omega_e t + \phi_v)$$

$$\phi_v = \theta_e - \theta_r$$

$$= \theta_e(0) - \theta_r(0) + \int (\omega_e - \omega_r) dt$$

Voltages in RRF $V_{qs} = \sqrt{2} V_s \cos(\phi_v)$

$$V_{ds} = -\sqrt{2} V_s \sin(\phi_v)$$

Common **O**peration **M**ode: Set $\phi_v = 0$

$$I_{ds} = \frac{\omega_r L_{ss}}{r_s} I_{qs}$$

$$V_{qs} = r_s I_{qs} + \omega_r L_{ss} \frac{\omega_r L_{ss}}{r_s} I_{qs} + \omega_r \lambda_m$$

Steady-State Operation

Brushless DC Motor in COM

- Voltage equation

$$V_{qs} = \frac{r_s^2 + \omega_r^2 L_{ss}^2}{r_s} I_{qs} + \omega_r \lambda_m$$

- Back emf

$$E_{a,peak} = \omega_r \lambda_m$$

- Electromagnetic torque

$$T_e = \frac{P}{2} \lambda_m I_{qs}$$

- Torque-speed characteristic

$$T_e = \frac{P}{2} \frac{\lambda_m r_s}{r_s^2 + \omega_r^2 L_{ss}^2} \left(V_{qs} - \omega_r \lambda_m \right)$$

PM Brushed DC Motor

- Voltage equation

$$V_a = r_a I_a + \omega_r k_v$$

- Back emf

$$E_a = \omega_r k_v$$

- Electromagnetic torque

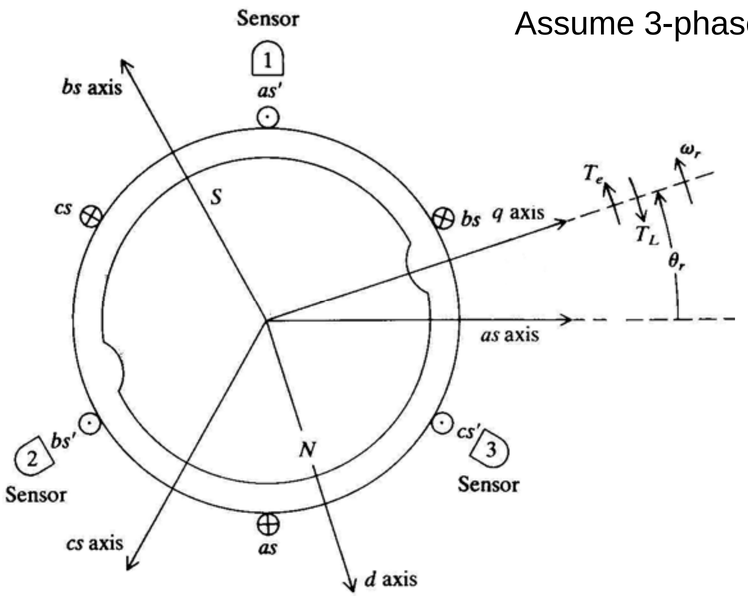
$$T_e = k_v I_a$$

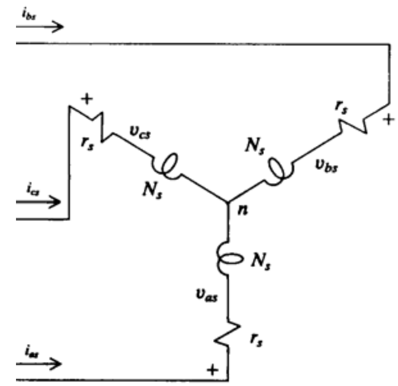
- Torque-speed characteristic

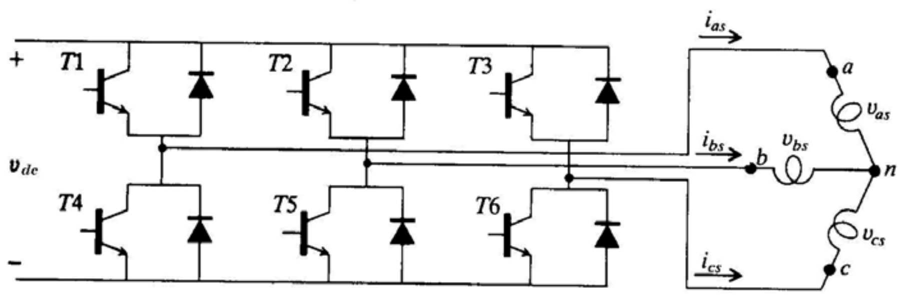
$$T_e = \frac{k_v}{r_s} \left(V_a - \omega_r \lambda_m \right)$$

3-Phase Brushless DC Motor

Assume 3-phases are symmetric and 120 deg apart

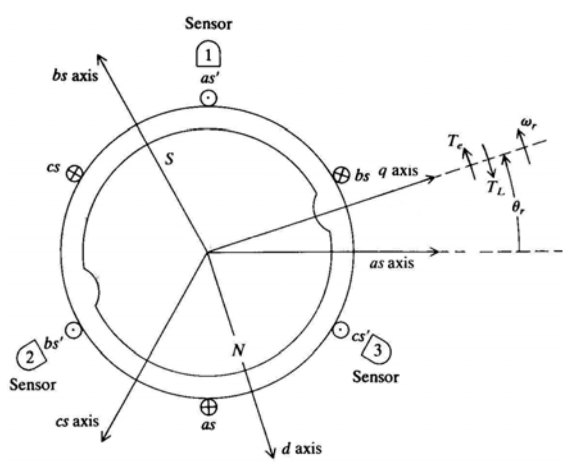






Assume 3-phase (leg) inverter circuit

3-Phase Brushless DC Motor Model



Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_{ls} + L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_m \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 120) \\ \sin(\theta_r + 120) \end{bmatrix}$$

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \boldsymbol{\lambda}_m$$

3-Phase Brushless DC Motor Model

Electromagnetic Torque $T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$

Assume magnetically linear system $W_c = W_f$

$$W_f = \frac{1}{2} \mathbf{i}_{abcs}^T (\mathbf{L}_s - L_{ls}\mathbf{I}) \mathbf{i}_{abcs} + \mathbf{i}_{abcs}^T \boldsymbol{\lambda}_m$$

$$T_e = \frac{P}{2} \lambda_m \left[\left(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs} \right) \cos(\theta_r) + \frac{\sqrt{3}}{2} (i_{bs} - i_{cs}) \sin(\theta_r) \right]$$

Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

Rotor position

$$\theta_r = \theta_r(0) + \int \omega_r dt$$

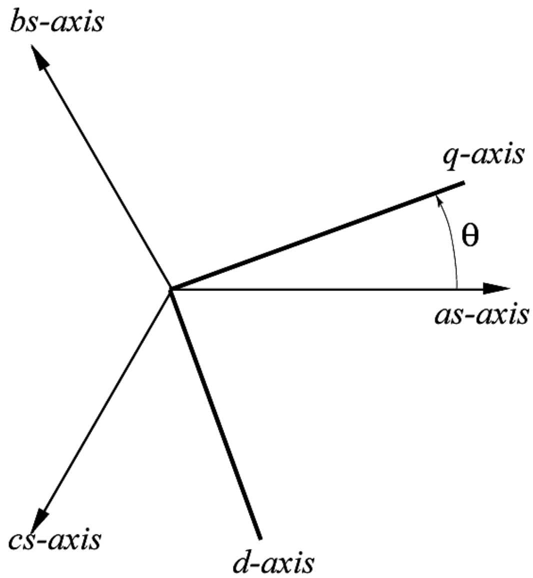
Recall for P-pole motors:

$$\omega_r = \frac{P}{2} \omega_{rm}$$

$$\theta_r = \frac{P}{2} \theta_{rm}$$

Apply Rotating Frame of Reference (RRF)

qd-frame is fixed on the rotor



Consider that each phase has variables

$$f = i, v, \lambda, ...$$

Define a vector $\mathbf{f}_{abcs} = [f_{as} \quad f_{bs} \quad f_{cs}]^T$

Define change of variables to *qd*-coordinate

$$\mathbf{f}_{qds} = [f_{qs} \quad f_{ds}]^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ \sin(\theta) & \sin(\theta - 120^\circ) & \sin(\theta + 120^\circ) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

$$\mathbf{f}_{qds} = \mathbf{K}_s \mathbf{f}_{abcs}$$

$$\mathbf{f}_{abcs} = (\mathbf{K}_s^r)^{-1} \mathbf{f}_{qds}$$

$$(\mathbf{K}_s^r)^{-1} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \cos(\theta_r - 120) & \sin(\theta_r - 120) \\ \cos(\theta_r + 120) & \sin(\theta_r + 120) \end{bmatrix}$$

Pseudo inverse !

Voltage Equations in RRF

Original voltage equation

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1}\mathbf{v}_{qds} = \mathbf{r}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds} + \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1}\boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{v}_{qds} = \mathbf{K}_s^r\mathbf{r}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds} + \mathbf{K}_s^r\frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1}\boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{K}_s^r\frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1}\boldsymbol{\lambda}_{qds}\right] = \mathbf{K}_s^r\left[\frac{d}{dt}\left(\mathbf{K}_s^r\right)^{-1}\right]\boldsymbol{\lambda}_{qds} + \mathbf{K}_s^r\left(\mathbf{K}_s^r\right)^{-1}\frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

Following the same procedure as for 2-phase case, the resulted voltage equation is

$$\mathbf{v}_{qds} = \mathbf{r}_s\mathbf{i}_{qds} + \omega_r\boldsymbol{\lambda}_{dqs} + \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

where

$$\boldsymbol{\lambda}_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\boldsymbol{\lambda}_{qds}$$

Flux Linkage Equations in RRF

Original equation

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_{ls}+L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_{ls}+L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_{asm}\begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 120) \\ \sin(\theta_r + 120) \end{bmatrix}$$

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s\mathbf{i}_{abcs} + \boldsymbol{\lambda}_m$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1}\boldsymbol{\lambda}_{qds} = \mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds} + \boldsymbol{\lambda}_m$$

$$\boldsymbol{\lambda}_{qds} = \mathbf{K}_s^r\mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1}\mathbf{i}_{qds} + \mathbf{K}_s^r\boldsymbol{\lambda}_m$$

where

$$\mathbf{K}_s^r\mathbf{L}_s\left(\mathbf{K}_s^r\right)^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ms} \end{bmatrix}$$

$$\mathbf{K}_s^r\boldsymbol{\lambda}_m = \lambda_m\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Resulted flux linkage equation

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_m\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda_{qs} = L_{ss}i_{qs}$$

$$\lambda_{ds} = L_{ss}i_{ds} + \lambda_m$$

Define

$$L_{ss} = L_{ls} + \frac{3}{2}L_{ms}$$

3-Phase Brushless DC Motor Model in qd RRF

Expanded Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + \frac{d\lambda_{qs}}{dt} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss}\frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + \frac{d\lambda_{ds}}{dt} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss}\frac{di_{ds}}{dt}$$

$$\frac{d}{dt}\begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -r_s & -\omega_r L_{ss} \\ \omega_r L_{ss} & -r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} -\omega_r \lambda_m \\ 0 \end{bmatrix} + \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} \right\}$$

Electromagnetic Torque

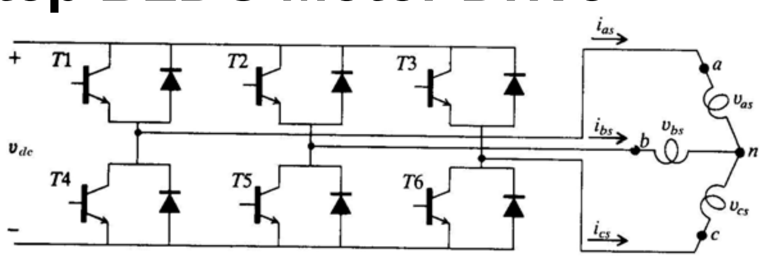
$$T_e = \frac{P}{2}\lambda_m \left[\left(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs} \right) \cos(\theta_r) + \frac{\sqrt{3}}{2}(i_{bs} - i_{cs}) \sin(\theta_r) \right]$$

$$= \frac{3}{2}\frac{P}{2}\lambda_m i_{qs}$$

Steady-State Operation = Same as 2-phase motor !

180-Deg., 6-Step BLDC Motor Drive

Step	Rotor Position	Active Transistors
1	-30 to 30	T1, T5, T6
2	30 to 90	T1, T2, T6
3	90 to 150	T2, T4, T6
4	150 to 210	T2, T3, T4
5	210 to 270	T3, T4, T5
6	270 to 330	T1, T3, T5

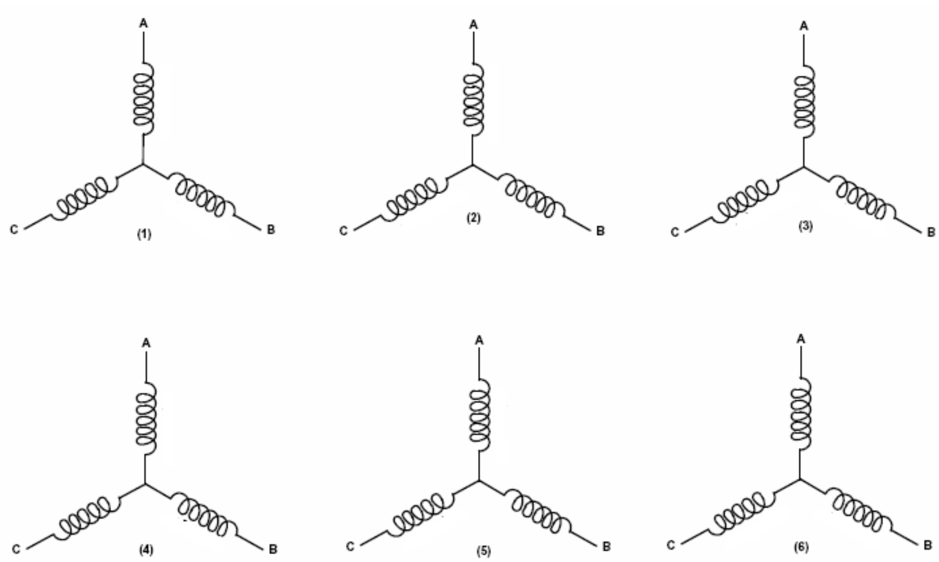


Hall-effect sensors control the inverter

T1 => as positive
T4 => as negative

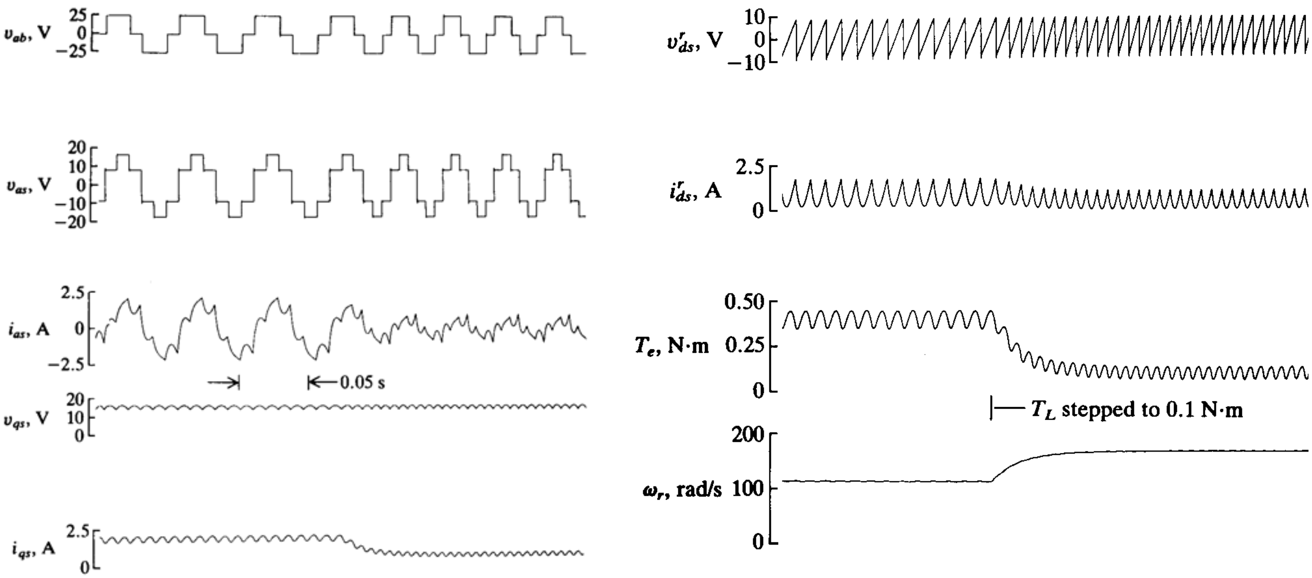
T2 => bs positive
T5 => bs negative

T3 => cs positive
T6 => cs negative



3-Phase Brushless DC Motor Drive

6-Step, 180 deg. Operation:
Typical waveforms



Note: current harmonics cause the torque ripple !

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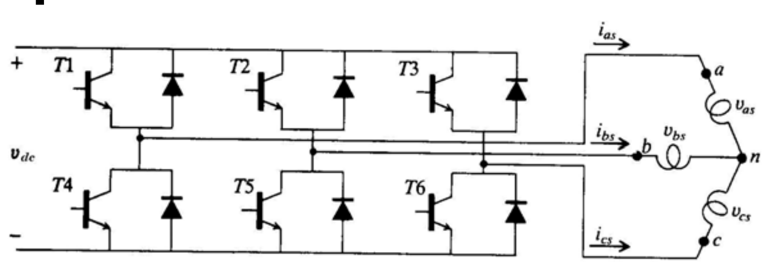
120-Deg., 6-Step BLDC Motor Drive

Step	Rotor Position	Active Transistors
1	-30 to 30	T1, T5
2	30 to 90	T1, T6
3	90 to 150	T2, T6
4	150 to 210	T2, T4
5	210 to 270	T3, T4
6	270 to 330	T3, T5

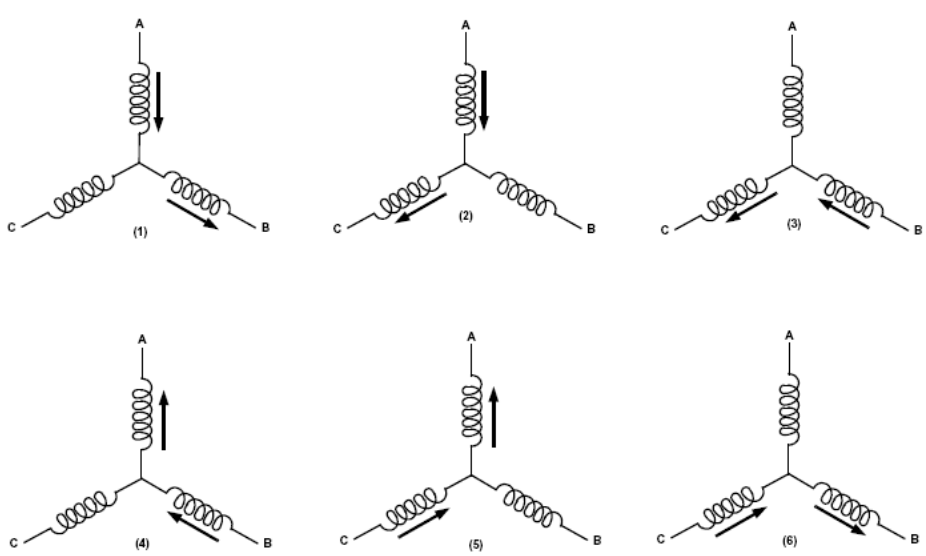


Hall-effect sensors control the inverter

T1 => as positive
T4 => as negative

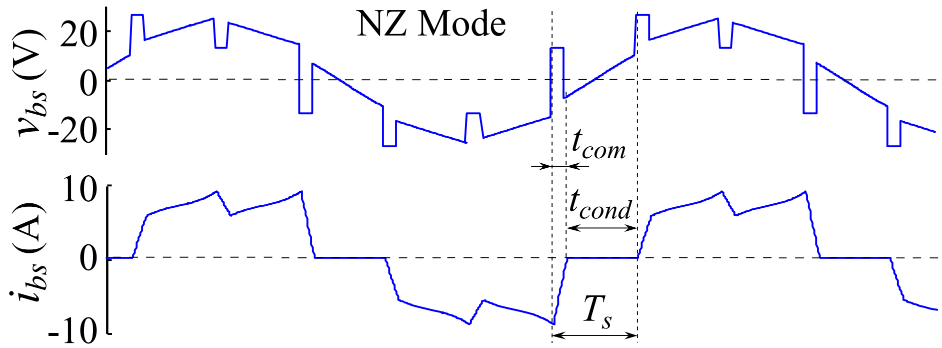
T2 => bs positive
T5 => bs negative

T3 => cs positive
T6 => cs negative

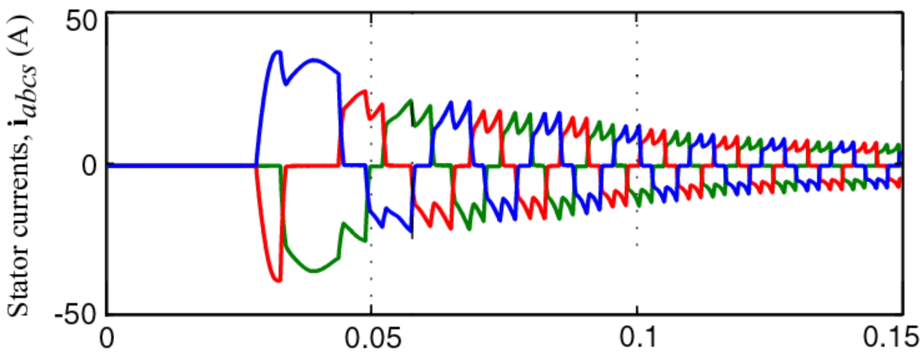


3-Phase Brushless DC Motor Drive

6-Step, 120 deg. Operation: Typical steady state waveforms



Startup transient

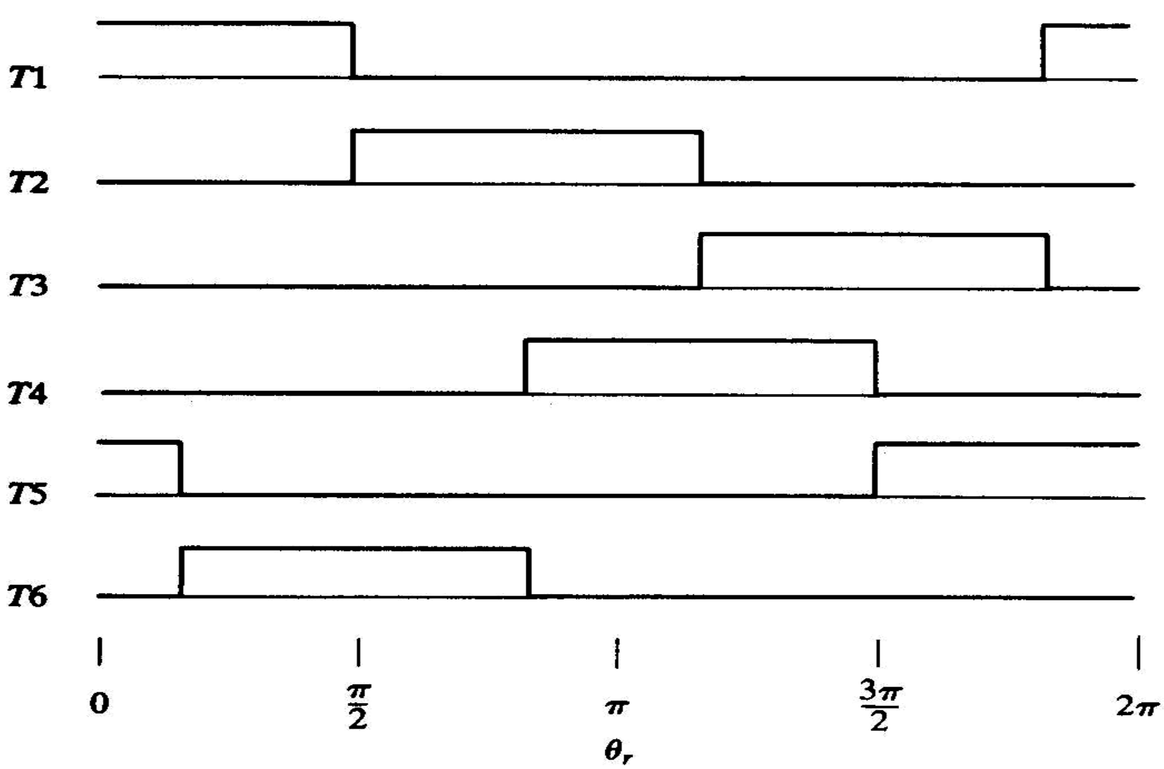


Note: current harmonics will cause the torque ripple !

3-Phase Brushless DC Motor Drive

6-Step, 120 deg. Operation with PWM

Apply PWM to control the average voltage !



Possible Implementation

Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss}\frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss}\frac{di_{ds}}{dt}$$

$$v_{qs} = r_s(1+\tau_s s) i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m$$

$$v_{ds} = r_s(1+\tau_s s) i_{ds} - \omega_r L_{ss} i_{qs}$$

$$i_{qs} = \frac{1/r_s}{\tau_s s + 1}\left(v_{qs} - \omega_r L_{ss} i_{ds} - \omega_r \lambda_m\right)$$

$$i_{ds} = \frac{1/r_s}{\tau_s s + 1}\left(v_{ds} + \omega_r L_{ss} i_{qs}\right)$$

Electromagnetic Torque

$$T_e = \frac{P}{2}\lambda_m I_{qs}$$

Stator time constant $\tau_s = \dfrac{L_{ss}}{r_s}$

Mechanical System

Rotor Position

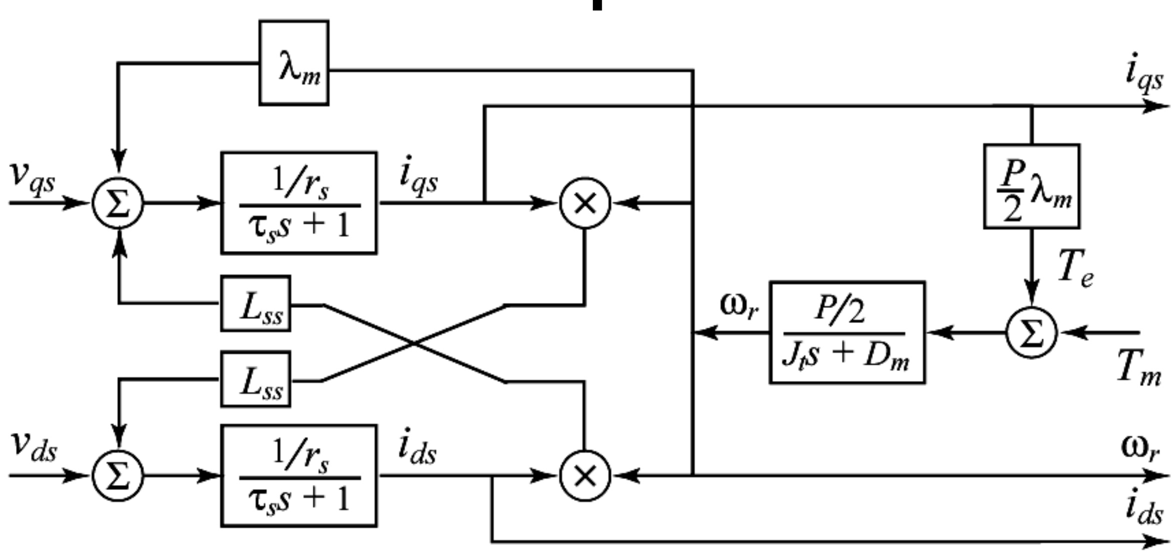
$$\theta_r = \theta_r(0) + \int \omega_r dt$$

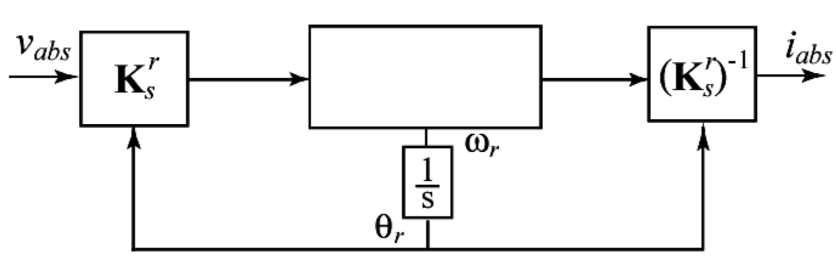
$$\theta_r = \theta_r(0) + \frac{1}{s}\omega_r$$

$$J_t \frac{2}{P}\frac{d\omega_r}{dt} = T_e - D_m \frac{2}{P}\omega_r - T_m$$

$$\omega_r = \frac{P/2}{J_t s + D_m}\left(T_e - T_m\right)$$

Possible Implementation





Possible Implementation