EECE 376
Electromechanics

Spring 2015
Instructor: Dr. Juri Jatskevich
Class Webpage: http://courses.ece.ubc.ca/eece376

Module 3 (Read Chap. 3):
Part 1: Brushed DC Motors Fundamentals and Steady-State Analysis

Learning Objectives & Important Topics and Concepts

- Construction of DC machines, fundamentals
- Induced voltage and torque
- Equivalent circuit
- Basic types of dc machines, their characteristics
- Starting

DC Machines

Range of Sizes

- Miniature dc motors – mW range
- Fractional HP (1HP = 746W)
- Medium Size – 1 … 500 HP
- Large DC Machines – 500 HP and up

Applications

- Very popular and easy to use in various applications
- Tools, robotics, toys, medical, automotive & other industries

General Properties

- Very easy to control !
- Good torque-speed performance
- Brushes is a weak point of design
  - Limits the application
  - Limits the lifetime
  - Maintenance

Small Motors

Gear-head Motors

Motor (Machine) Speed

Transmitting power through the mechanical shaft

Source
(Prime Mover)

Load

Commonly used units of speed

$$\omega_m \text{ - } [\text{rad}/\text{sec}]$$

$$\text{m} = \frac{\omega}{2\pi} \ [\text{rev}/\text{sec}]$$

$$\text{n} = 60\frac{\omega}{2\pi} = \frac{30}{\pi}\omega \ [\text{rev}/\text{min}]$$

RPM = >Most commonly used

Mechanical Speed and Torque

Electric Motor

Mechanical Load

Electromagnetic torque T_e

Useful mechanical torque on the shaft T_m

Torque balance

$$T_e = J\frac{d\omega_r}{dt} + T_m + T_{mech\_loss}$$

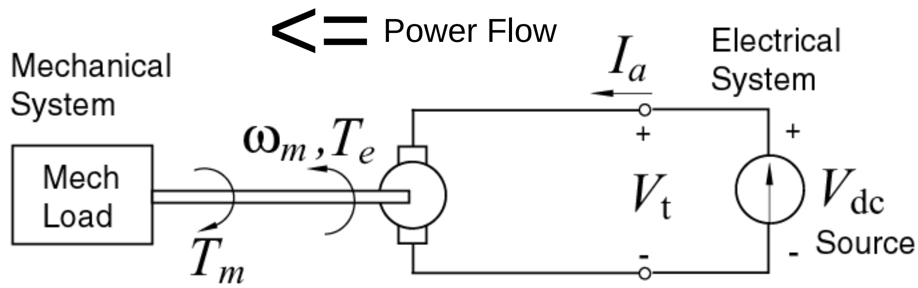
Total moment of inertia (motor and load combined)

$$J = J_{dc\_machine} + J_{mech\_load}, \quad [kg \cdot m^2]$$

Mechanical loss (friction)

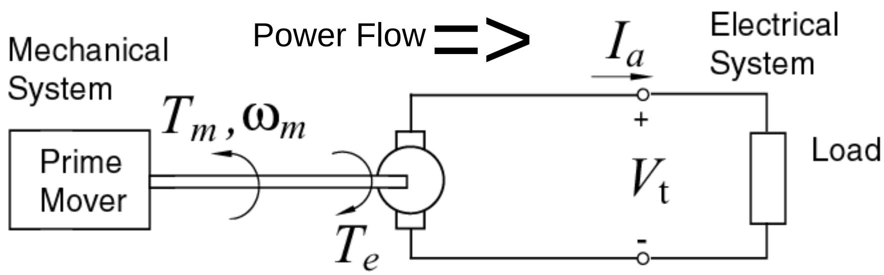
$$T_{mech\_loss} = T_{fric} = D_m\omega_r$$

DC Machine Motoring and Generating



Mechanical load torque is applied in the direction opposite to rotation

$$P_{out} = \omega_m T_m = P_m \qquad V_t I_a = P_{in}$$

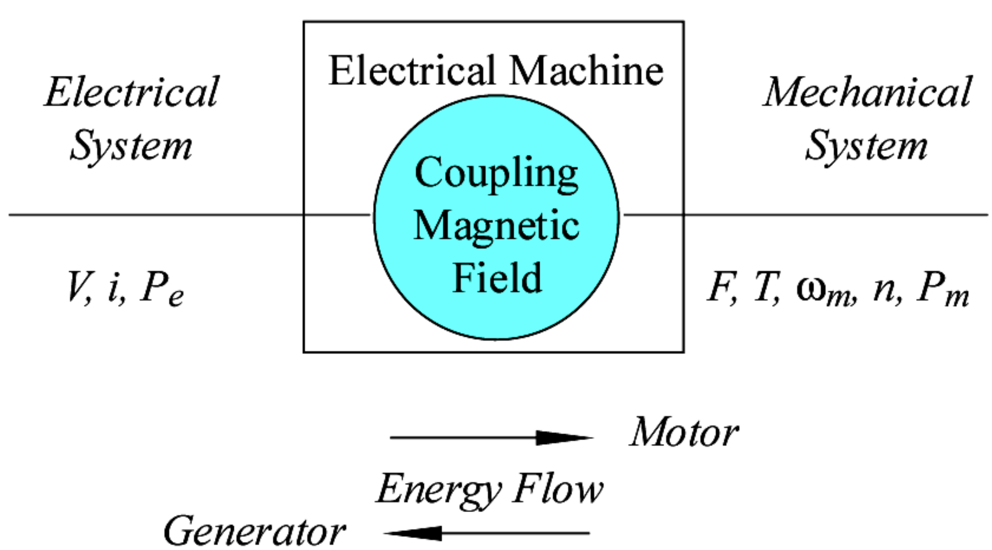


Mechanical torque is applied in the direction of rotation

$$P_{in} = \omega_m T_m = P_m \qquad V_t I_a = P_{out}$$

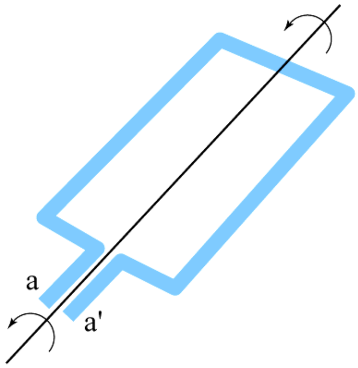
Electromechanical Interaction

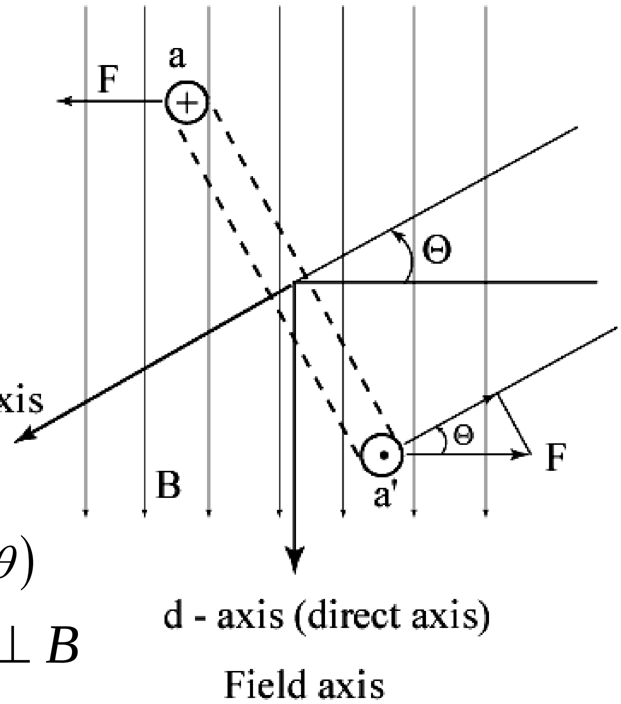
- Current-carrying conductor in magnetic field
  => mechanical force
- Conductor moves in magnetic field
  => voltage induced, emf



Force & Torque

Consider a conductor frame (single-coil)





Lorentz Force $F = lBi_a$

Torque $T_e = 2RlBi_a \cos(\theta)$

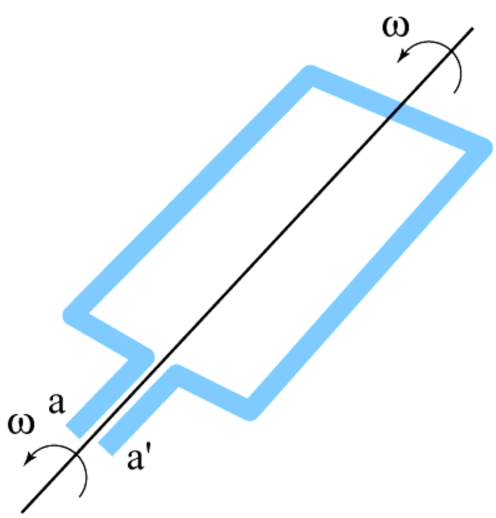
$$= ABi_a \cos(\theta) = \Phi_p i_a \cos(\theta)$$

NOTE: Maximum torque is when $a - axis \perp B$

and $T_e \sim \Phi_p i_a$

Induced Voltage

Consider a conductor frame (single-coil)



Faraday's Law $e = \dfrac{d\Phi}{dt}$

$$\Phi = BA\sin(\theta) = B2Rl\sin(\theta)$$

Let us rotate the frame with speed ω

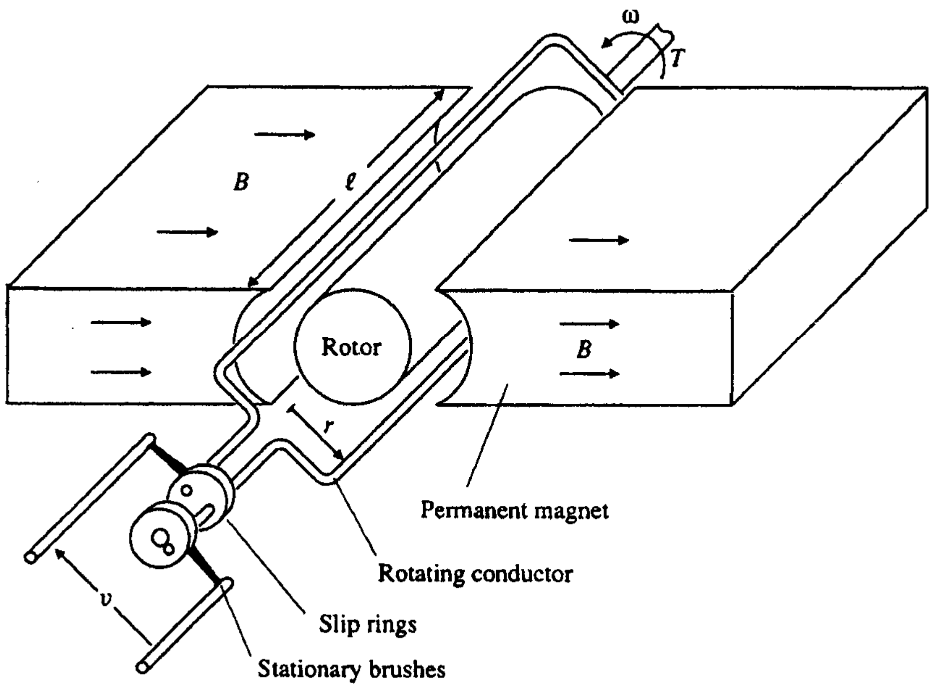
$$\theta = \omega t$$

$$\Phi = BA\sin(\omega t) = \Phi_p \sin(\omega t)$$

$$e = \frac{d\Phi}{dt} = \omega\Phi_p \cos(\omega t)$$ => $e \sim \omega\Phi_p$

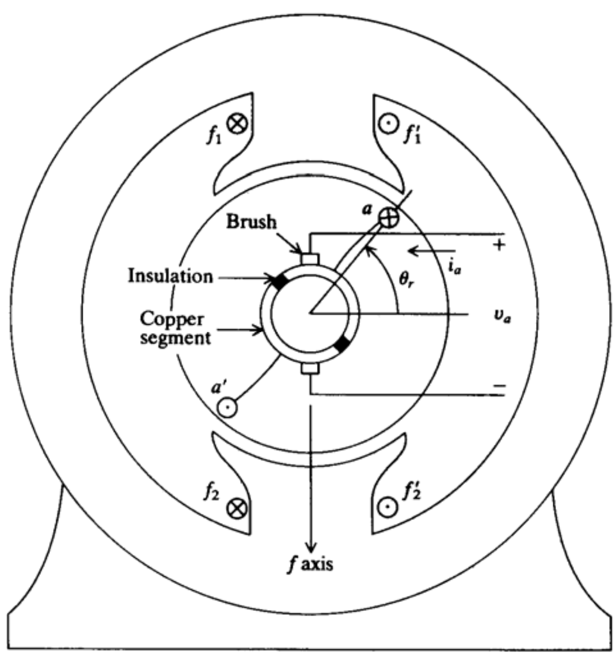
Conductor Frame (1-turn coil)

Consider a conductor frame placed on a rotor between magnetic poles



Elementary DC Machine

Consider a two-pole case



Voltage Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

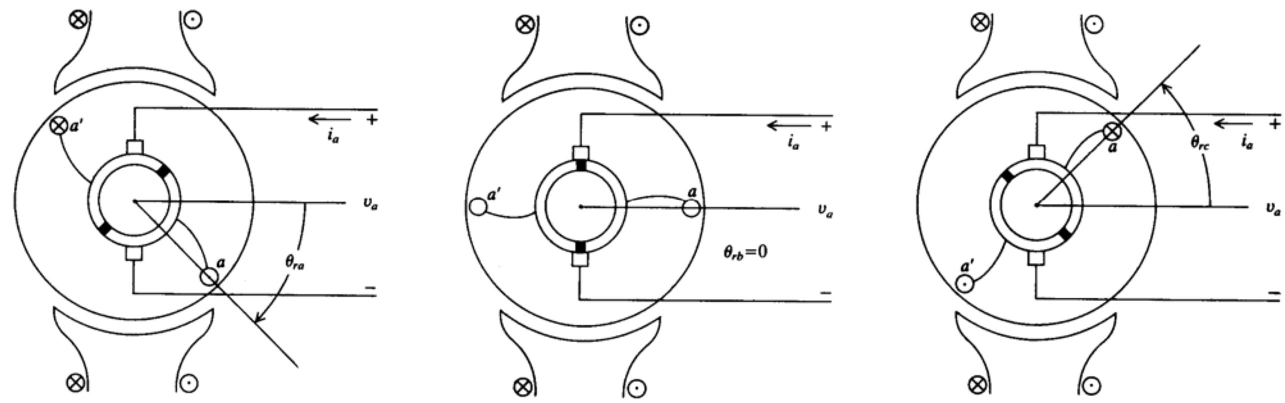
Flux linkages

$$\lambda_f = L_{ff} i_f + L_{fa} i_a$$

$$\lambda_a = L_a i_a + L_{af} i_f$$

Approximate the mutual inductance $L_{af} = L_{fa} = -L\cos(\theta_r)$

Commutation of Elementary DC Machine



Induced voltage

$$e_a = \omega_r L i_f \sin(\theta_r)$$

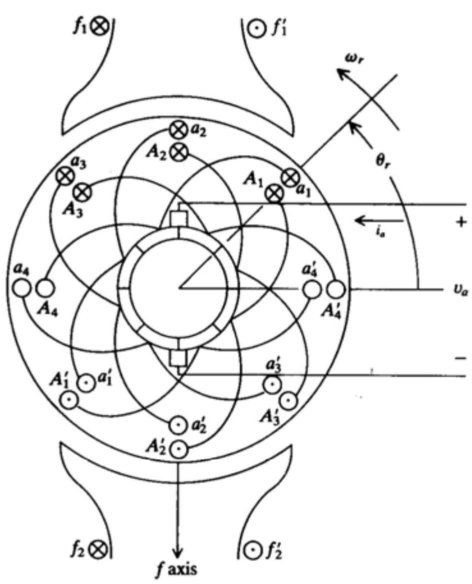
$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

Assume

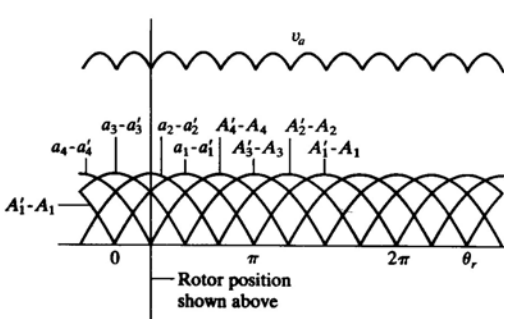
$i_a = 0$ and $\theta_r = \omega_r t$

$$\lambda_a = L_a i_a + L_{af} i_f$$

More Realistic DC Machine

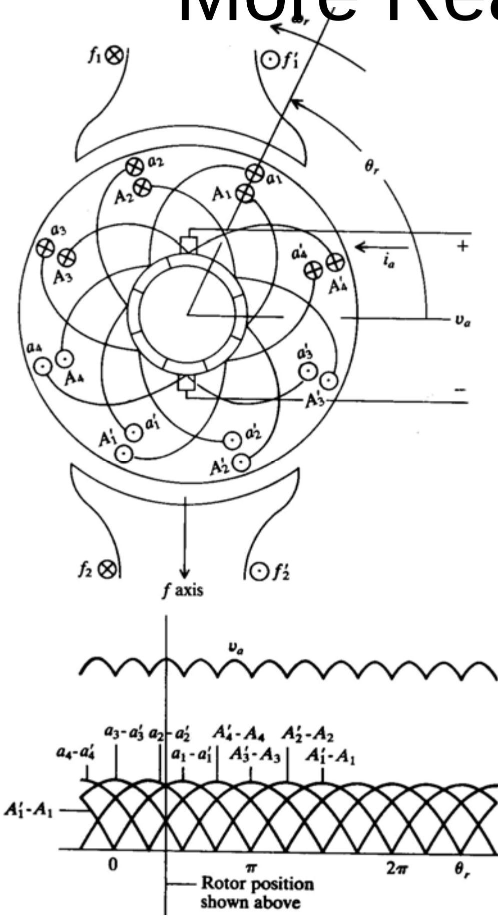


Resulted Winding Connection



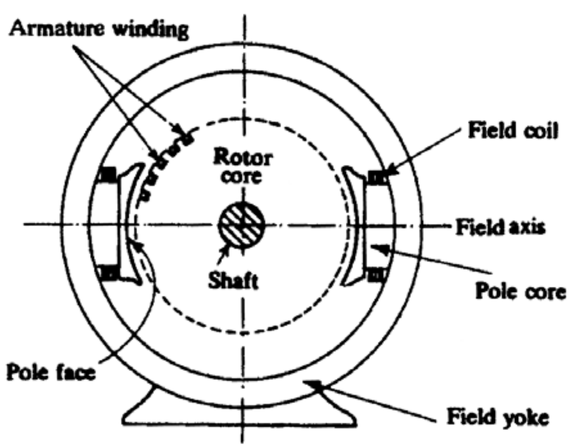
More Realistic DC Machine

Resulted Winding Connection

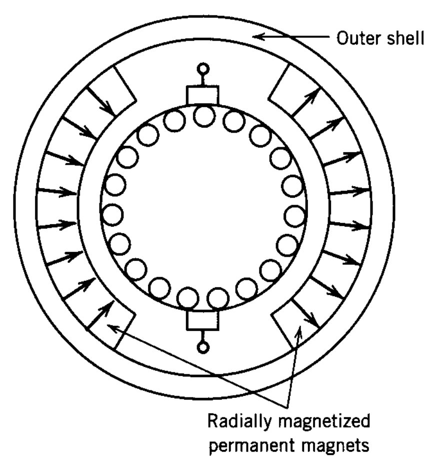


DC Machine Construction

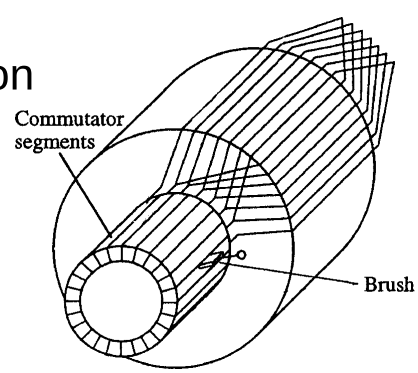
DC machine with field winding

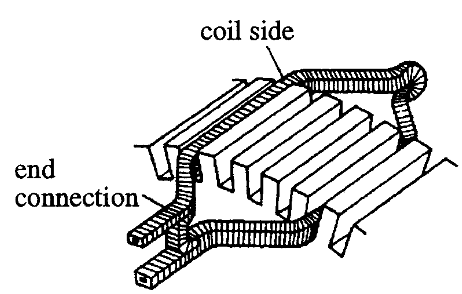


Permanent Magnet (PM) DC machine

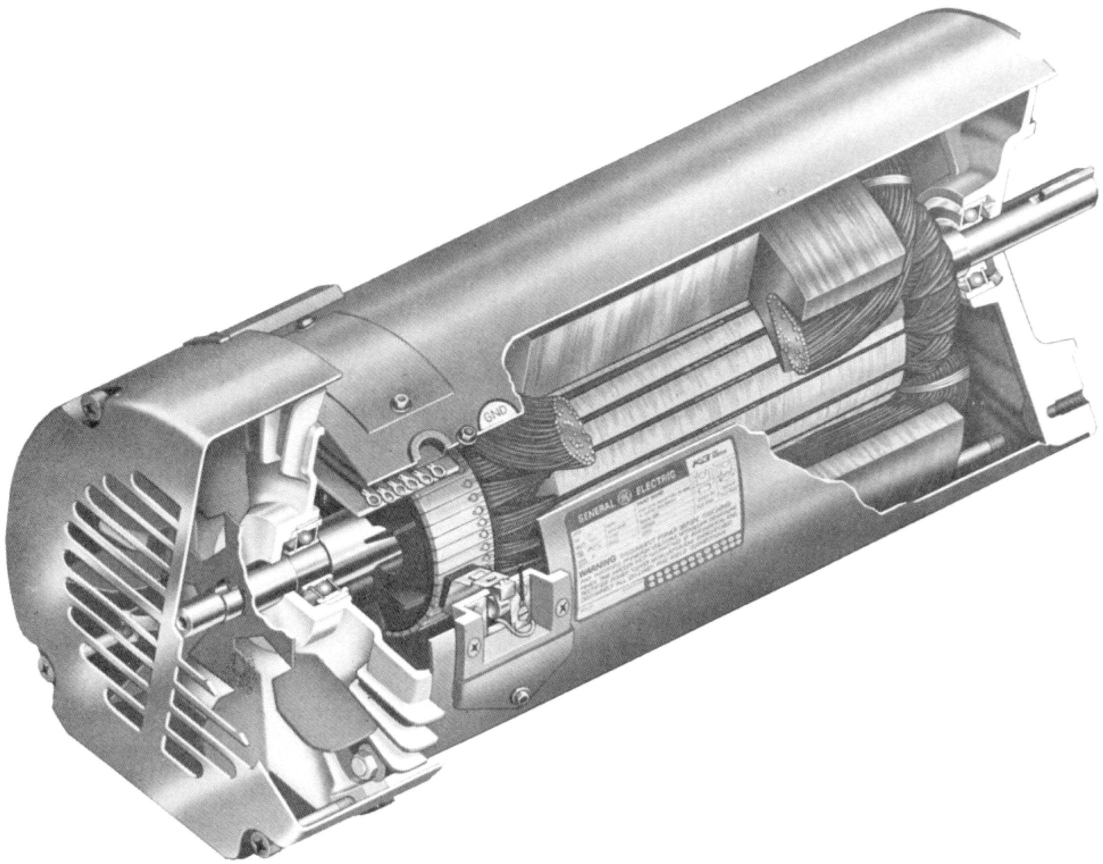


Rotor construction

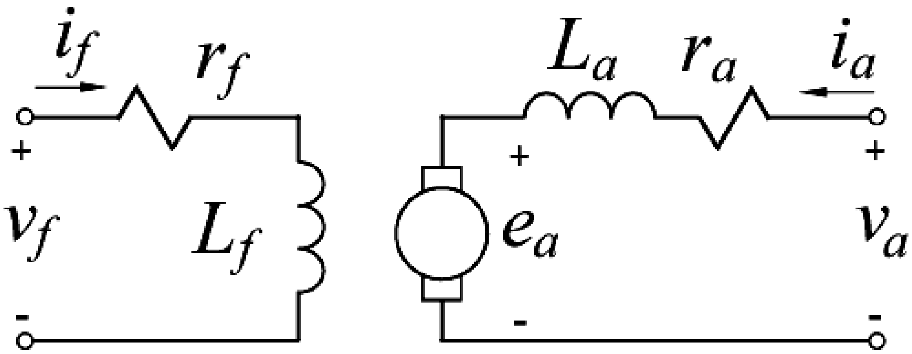




Cutaway View of a Two-Pole DC Machine



Equivalent Circuit



v_f - is the field voltage

i_f - is the field current

r_f - field winding resistance

L_f - field winding inductance

v_a - applied terminal voltage

i_a - is the armature current

L_a - armature winding inductance

r_a - armature winding + brush resistance

e_a - Induced back emf (voltage)

Dynamic Equivalent Circuit

Field Winding Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$\lambda_f = N_f \Phi_p = L_f i_f$$

Armature Equations

$$v_a = r_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$e_a = k_1 \omega_r \Phi_p$$

Electromagnetic Torque $T_e = k_2 \Phi_p i_a$

Power balance $T_e \omega_r = k_2 \Phi_p i_a \omega_r = e_a i_a = k_1 \omega_r \Phi_p i_a$

Equivalent Circuit

Recall flux $\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$

Induced voltage

$$e_a = k\Phi_p \omega_r = k\frac{L_f}{N_f} i_f \omega_r$$

Torque

$$T_e = k\Phi_p i_a = k\frac{L_f}{N_f} i_f i_a$$

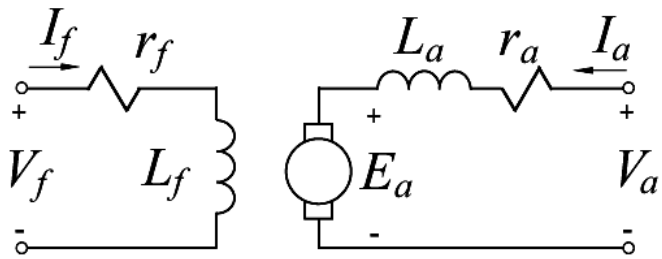
Define $L_{af} = k\frac{L_f}{N_f}$ - is the mutual inductance between field and rotating armature winding $L_{af} = \frac{N_a N_f}{\Re}$

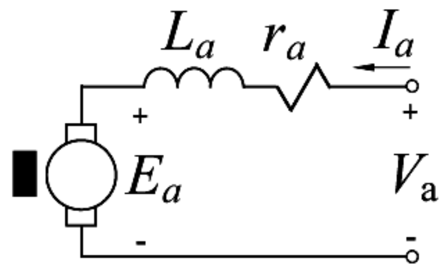
Re-define expression for back emf and torque

$$e_a = L_{af} i_f \omega_r = k_v \omega_r \qquad T_e = L_{af} i_f i_a = k_t i_a$$

Machine voltage/torque constant for PM machine $k_v = k_t = L_{af} i_f$

Steady-State





Field Winding Equations

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

Armature Equations

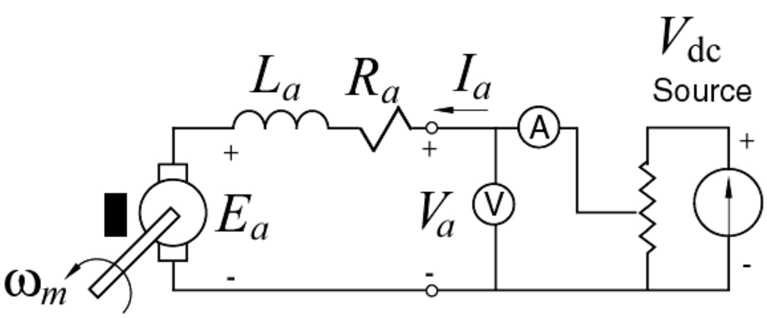
$$V_a = r_a I_a + E_a$$

$$E_a = \omega_r k_v = \omega_r L_{af} i_f$$

Electromagnetic torque $T_e = L_{af} I_f I_a = k_t I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$

Example 1: No Load Test PM Motor



Assume you know R_a

Measure I_a, V_a, ω_r

$$V_a = R_a I_a + E_a$$

$$P_e = I_a E_a = \omega_r T_e$$

$$T_e = T_{friction} = \omega_r K_{friction}$$

$$T_e = k_v I_a$$

$$E_a = k_v \omega_r$$

$k_t = k_v$ Torque / voltage machine constant

Example 2: PM DC Motor

Consider a small PM DC Motor with the following parameters:
$V_{t\_rated} = 6V$, $I_{no\_load} = 0.15A$, $R_a = 7\Omega$, and torque constant $K_t = 0.014$ [Vs/rad].
Find no-load speed n_{nl} [rpm], back emf E_a, and friction torque T_{fric} at $V_t = 6V$

$$V_t = R_a \cdot I_a + E_a = R_a I_a + K_t \cdot \omega$$

$$\omega_{nl} = \frac{V_t - R_a I_a}{K_t} = \frac{6 - 7 \cdot 0.15}{0.014} = 353.57 \text{ rad/sec}$$

$$n_{nl} = \frac{30}{\pi} \omega_{nl} = 3{,}376.4 \text{ rpm}$$

Back emf $E_a = K_t \cdot \omega = 0.014 \cdot 353.57 = 4.95 V$

Electromagnetic torque

$$T_e = K_t \cdot I_a = 0.014 \cdot 0.15 = 0.0021 \text{ N}\cdot\text{m}$$

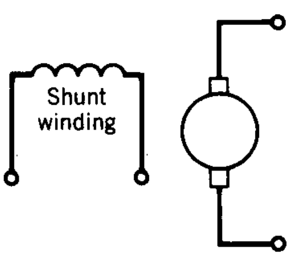
$$T_e = \frac{P_e}{\omega} = \frac{I_a \cdot E_a}{\omega} = \frac{0.15 \cdot 4.95}{353.57} = 0.0021 \text{ N}\cdot\text{m}$$

$$T_{fric} = T_e = 0.0021 \text{ N}\cdot\text{m}$$

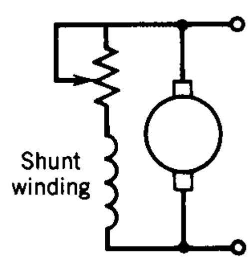
Basic Types of DC Machines

Basic DC Machines

- Shunt or Series field windings are possible

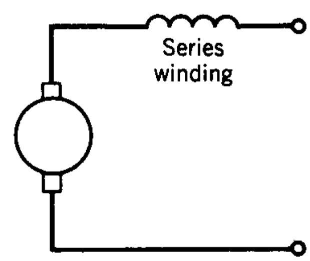


Separately
Excited
DC Machine



Shunt DC
Machine

- Field winding is designed for up to rated armature voltage
- Field winding has large number of turns
- Field current is small compared to armature current



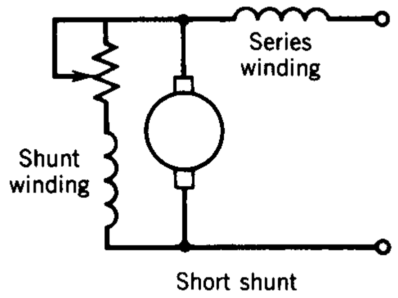
Series DC
Machine

- Field winding is designed for up to rated armature current
- Field winding has small number of turns
- Field current is the same as the armature current

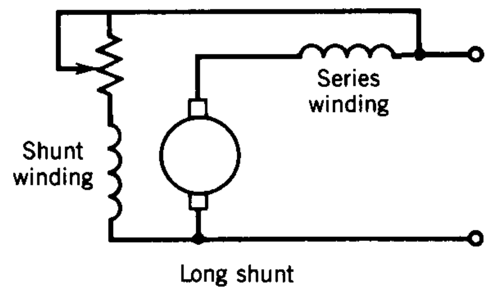
Basic Types of DC Machines

Compound DC Machines

- Both Shunt and Series field windings are present



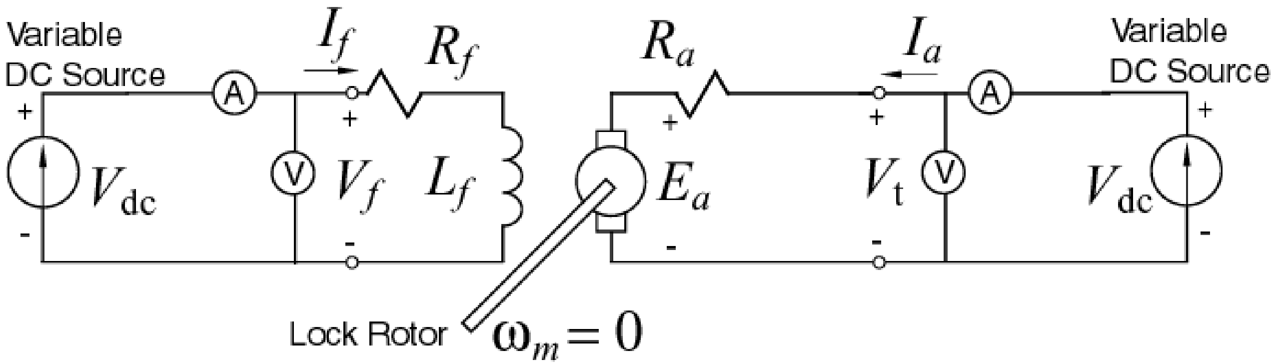
Short-Shunt
Compound
DC Machine



Long-Shunt
Compound
DC Machine

Parameters of DC Machine

- Locked-Rotor Test (DC Measurements)



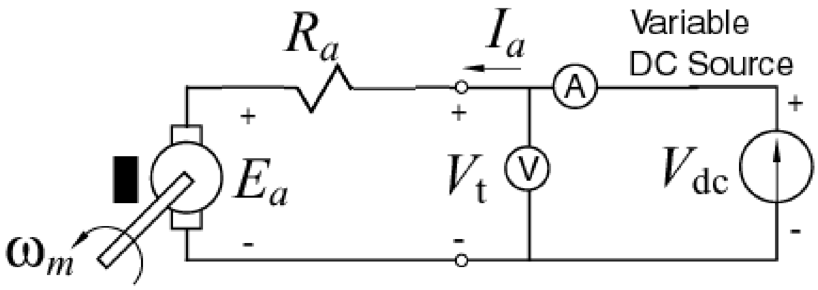
$$R_f = \frac{V_f}{I_f}$$

$$R_a = \frac{V_t}{I_a}$$

(armature + brush combined resistance at no speed!)

Parameters of DC Machine

- No Load Test for PM Motors (Friction vs. Speed)



Assume you know R_a

Measure I_a, V_t, ω_m

$$V_t = R_a I_a + E_a$$

$$T_e = K_a \Phi_p I_a = K_t I_a$$

$$P_e = I_a E_a = \omega_m T_e$$

$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

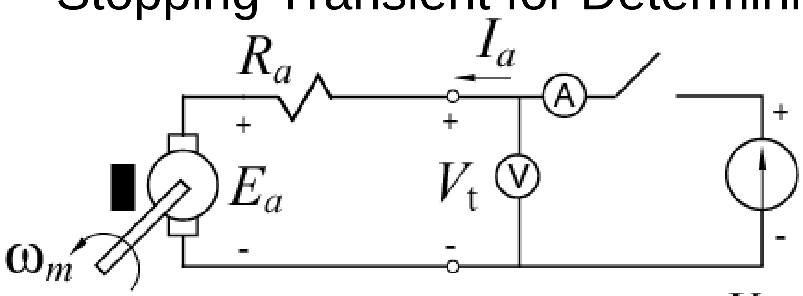
$$T_e = T_{fric}(\omega_m) = \omega_m K_{fric}$$

$K_t = K_v$ Torque / voltage machine constant

Measure Friction Torque-Speed Characteristic

Parameters of DC Machine

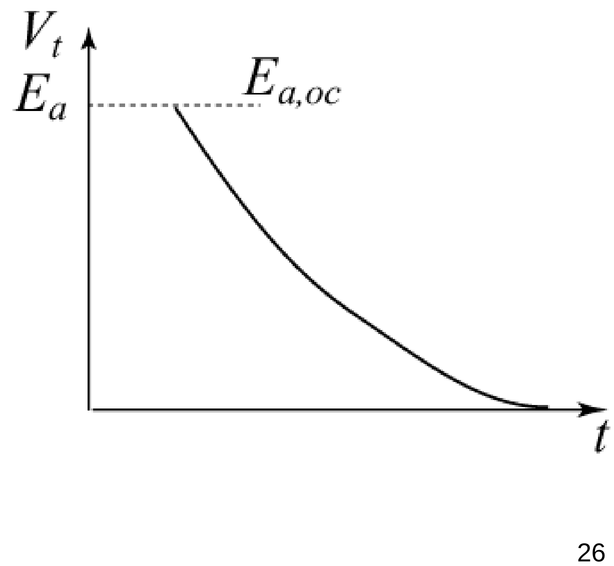
- Stopping Transient for Determining Inertia



Assume you know $T_{fric}(\omega_m)$

Measure & Record I_a, V_t, ω_m

$$T_e = T_m + T_{fric}(\omega_m) + J\frac{d\omega_m}{dt}$$

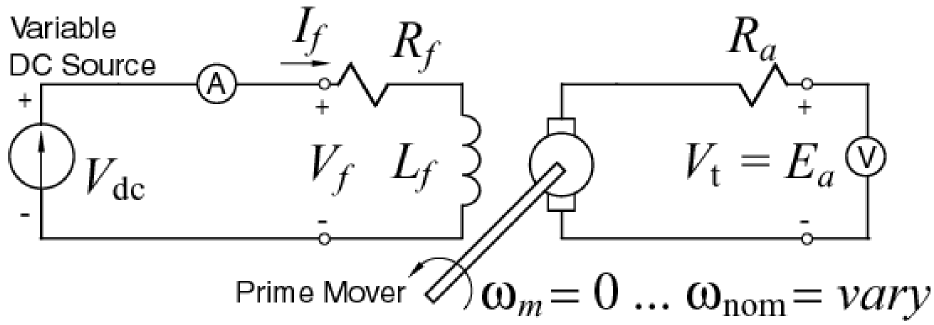


$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

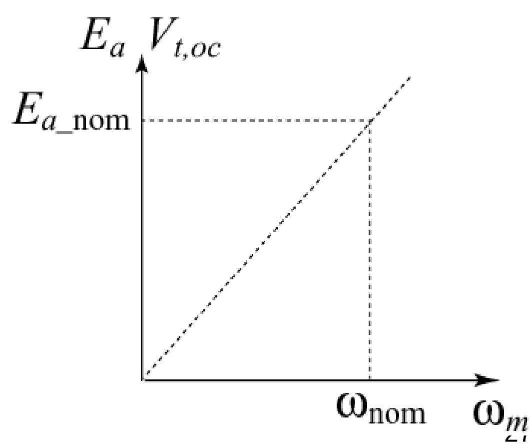
$$T_{fric}(\omega_m) = -J\frac{\Delta\omega_m}{\Delta t} = -J\frac{\Delta E_a}{K_v \Delta t}$$

Parameters of DC Machine

- Open-Circuit Test (Generated Voltage vs. Speed)



$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$
$$= L_{af} I_f \omega_m$$
$$= K_v \omega_m$$



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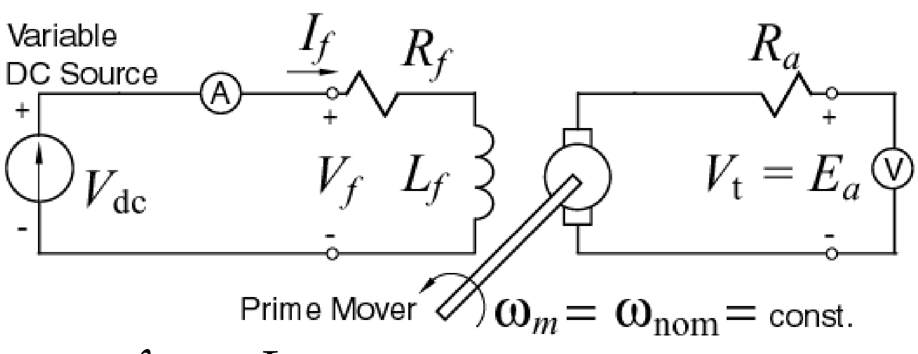
Parameters of DC Machine

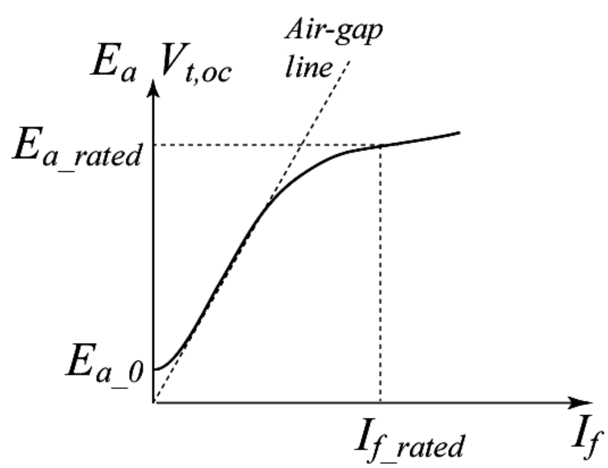
- Open-Circuit Test (Generated Voltage vs. Field Currents)



$$L_{af} = \frac{E_a}{I_f \omega_m}$$

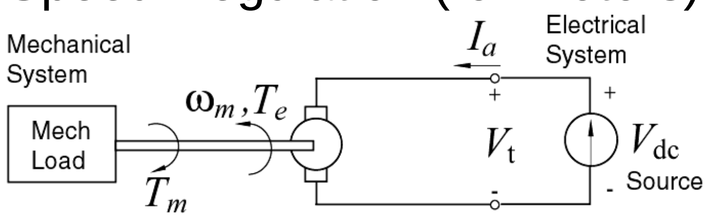
$$\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$$

$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$
$$= L_{af} I_f \omega_m$$



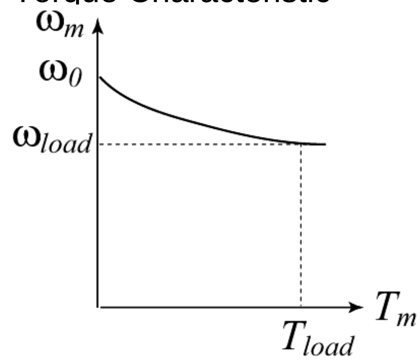
DC Machines Characteristics:

Speed Regulation (for Motors)

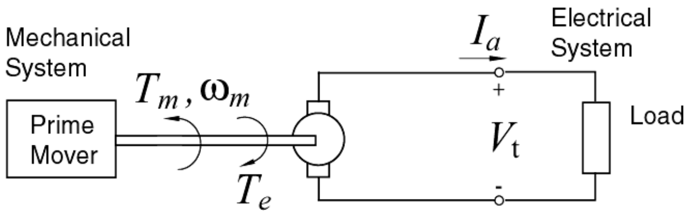


$$SR = \frac{\omega_{m,no\_load} - \omega_{m,load}}{\omega_{m,load}} 100\% = \frac{n_{no\_load} - n_{load}}{n_{load}} 100\%$$

Speed-Torque Characteristic

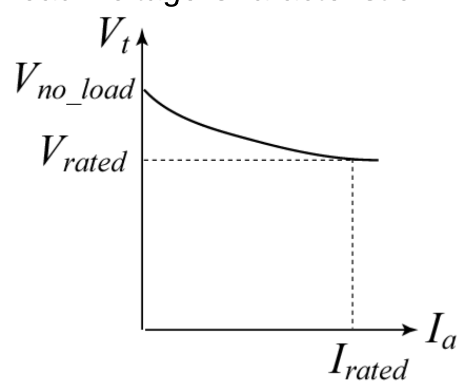


Voltage Regulation (for Generators)



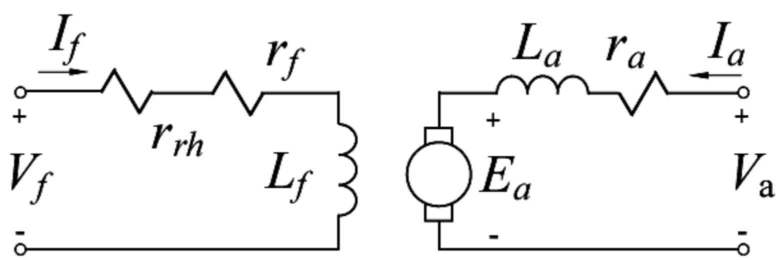
$$VR = \frac{V_{t,oc} - V_{t,load}}{V_{t,load}} 100\% = \frac{V_{no\_load} - V_{nom}}{V_{nom}} 100\%$$

Load-Voltage Characteristic



Separately Excited

Field Winding Equations



Armature Equations

$$R_f = r_f + r_{rh}$$

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

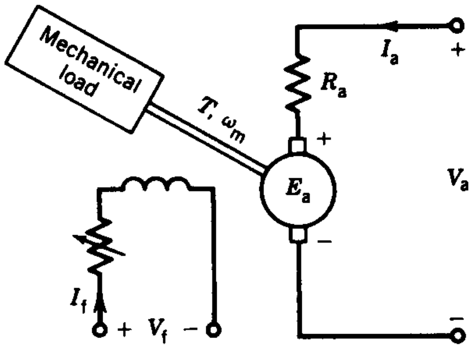
$$V_a = r_a I_a + E_a$$

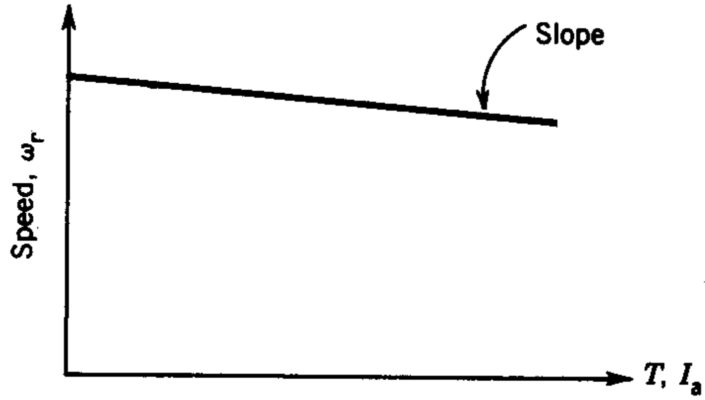
$$E_a = \omega_r k_v = \omega_r L_{af} i_f$$

Electromagnetic torque $T_e = L_{af} I_f I_a = k_t I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$

Separately-Excited DC Motor





Speed-Torque Characteristic

$$V_a = E_a + R_a I_a$$

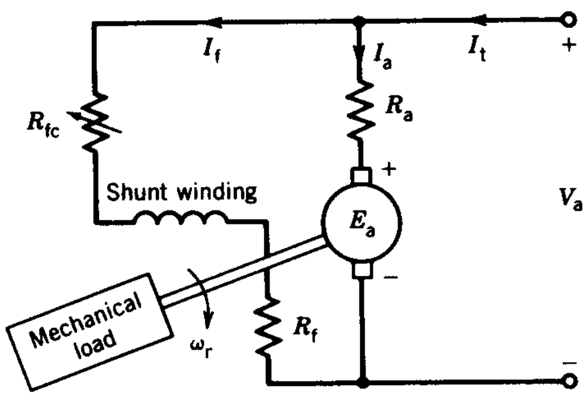
$$T = L_{af} I_f I_a = k_t I_a$$

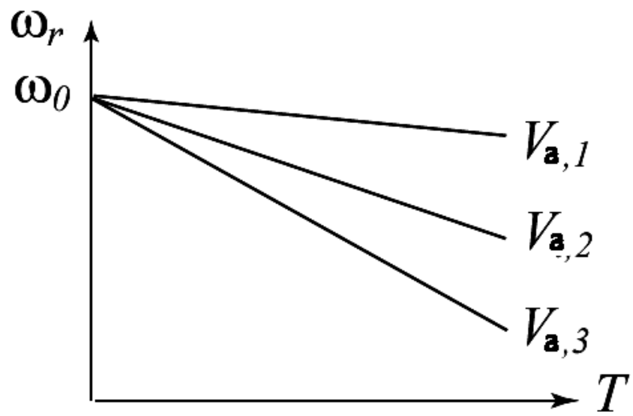
$$E_a = L_{af} I_f \omega_r = k_v \omega_r$$

$$\omega_r = \frac{V_a - I_a R_a}{k_v}$$

$$\omega_r = \frac{V_a}{k_v} - \frac{R_a}{k_v^2} T$$

Shunt DC Motor





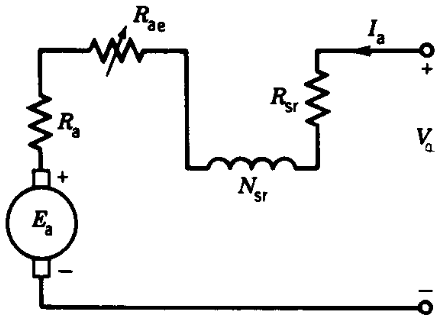
Speed-Torque Characteristic

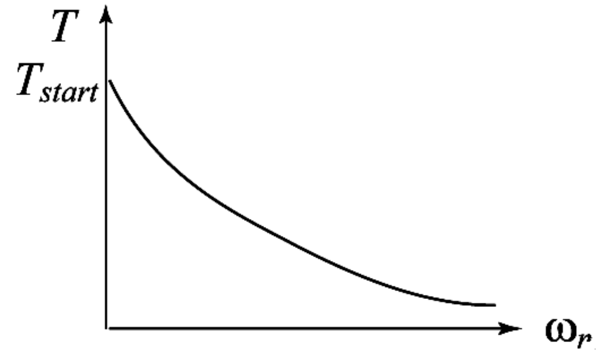
$$\omega_r = \frac{V_a}{k_v} - \frac{R_a}{k_v^2} T$$

$$k_v = L_{af} I_f = \frac{L_{af} V_a}{R_f + R_{fc}}$$

$$\omega_r = \frac{R_{f,total}}{L_{af}} - \frac{R_a R_{f,total}^2}{\left(L_{af} V_a\right)^2} T$$

Series DC Motor





Torque-Speed Characteristic

$$V_t = I_a\left(R_a + R_{sr} + R_{ae}\right) + L_{af} I_a \omega_m$$

$$I_a = \frac{V_a}{R_a + R_{sr} + R_{ae} + L_{af}\omega_m}$$

$$T = \frac{L_{af} V_a^2}{\left(R_a + R_{sr} + R_{ae} + L_{af}\omega_m\right)^2}$$

$$T_e = L_{af} I_a^2$$

Starting Considerations

Consider a 1.2kW, 120V, Shunt DC Motor with Ra = 2Ω, and Ia_max = 20A

Rated current $I_{a,rated} = \frac{1200}{120} = 10\text{A}$

Starting current $I_{a,start} = \frac{V_a - E_a}{R_a} = \frac{120 - 0}{2} = 60\text{A}$

Too high ! => Burn the motor !

1. Use additional starting resistor in series with armature
2. Use reduced voltage
3. Use Power Electronic Drive & Controller

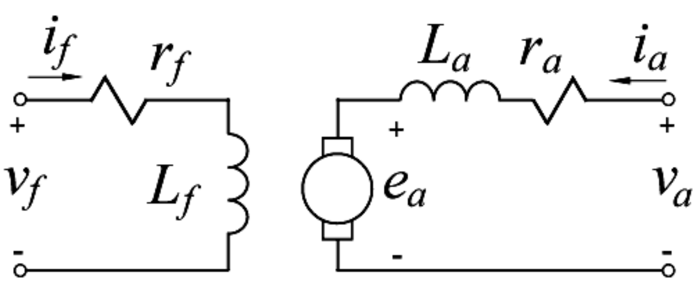
Part 2: Brushed DC Motor Drives and Dynamics

Important Topics & Concepts

- Basic types of dc motor drives
- DC to DC choppers (Chap. 3.8)
- AC to DC controlled rectifiers
- Dynamic (State) equations of the separately-excited motor
- Possible model & implementation/solution
- Dynamic (State) equations of the PM motor
- Possible model & implementation/solution

Speed/Torque Control of DC Motors

Recall the Separately Excited DC Motor



$$k_v = L_{af} i_f$$

$$v_a = r_a i_a + k_v \omega_r$$

$$T_e = k_v i_a$$

$$\omega_r = \frac{v_a - i_a r_a}{k_v}$$

$$\omega_r = \frac{v_a}{k_v} - \frac{r_a}{k_v^2} T_e$$

Control methods:

1. Varying the armature resistance
2. Varying the field
3. Varying the terminal voltage
4. Varying the armature current

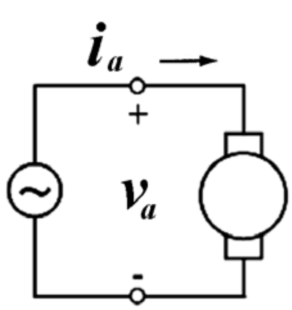
Solid-State Converters for DC Motors

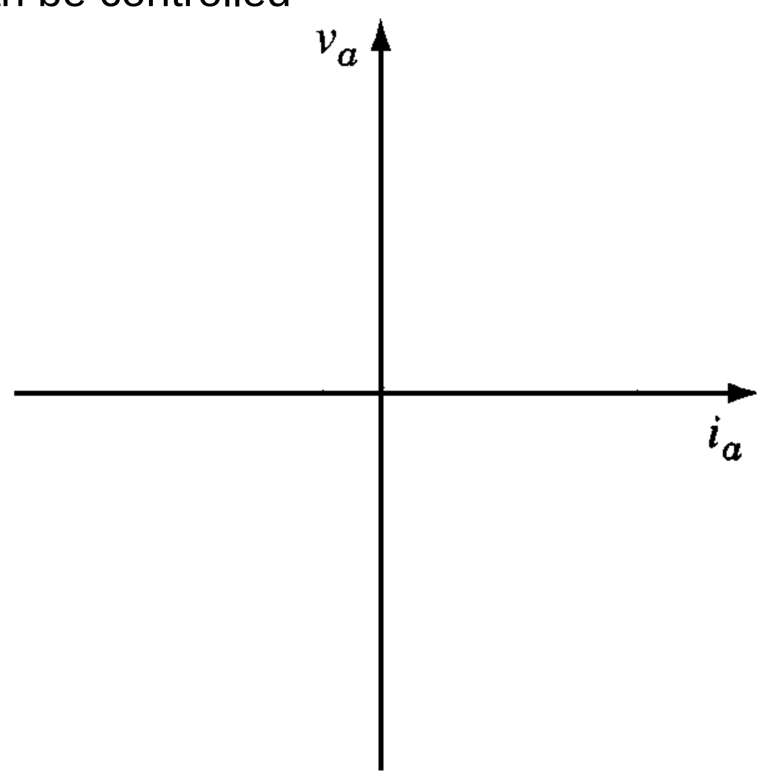
DC-DC Converters (Choppers) – used with small & large motors
- Voltage Source (VS) (Pulse Width Modulation – PWM)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant
- Current Source (CS) (Hysteresis & Delta Modulation)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant

AC-DC Controlled Rectifiers – used with large motors
- Single-Phase
  - Half-wave
  - Full bridge (full wave)
- Three-Phase
  - Half-wave
  - Full bridge (full wave)

DC-DC Converters (Choppers)

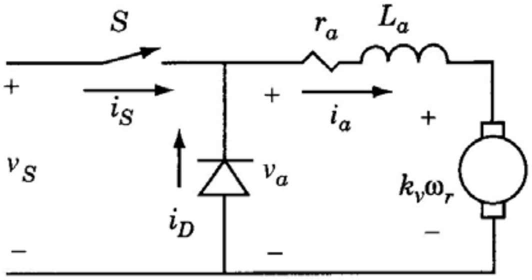
Assume a dc voltage source wherein the averaged output voltage can be controlled

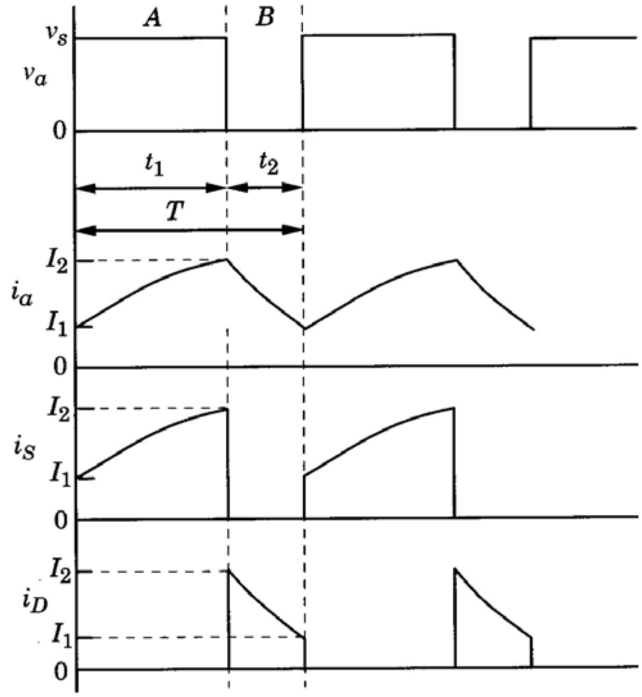




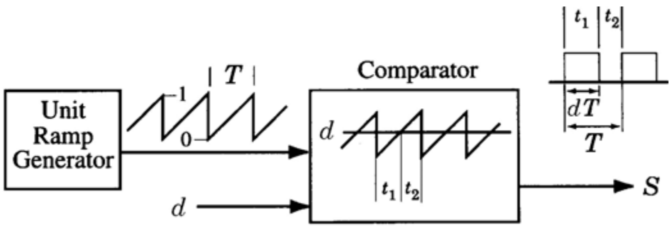
$$\overline{V}_a = \frac{1}{T}\int_0^T V_a(t)\,dt$$

One-Quadrant VS DC-DC Converter





Pulse Width Modulation (**PWM**)

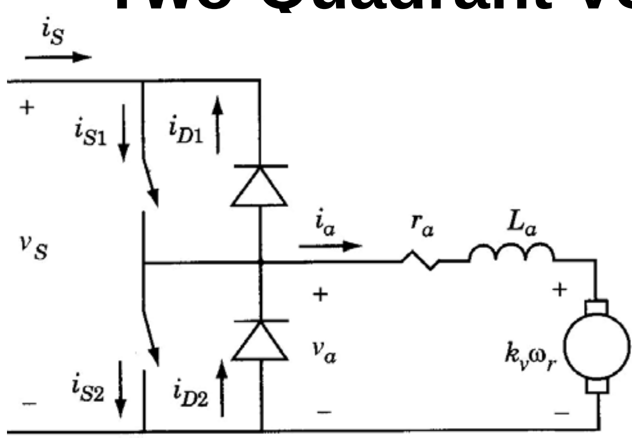


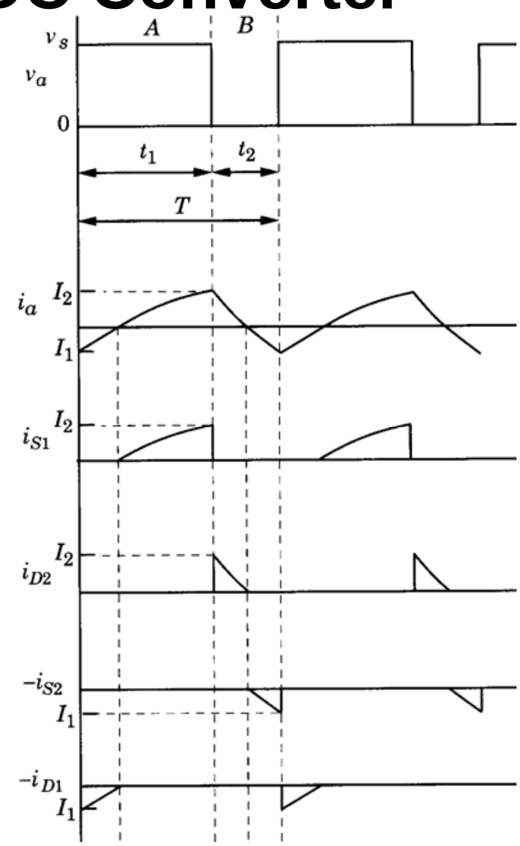
Switch can be realized using:

- Bipolar Junction Transistor (**BJT**)
- Insulated Gate Bipolar Transistor (**IGBT**)
- Metal Oxide Semiconductor Field Effect Transistor (**MOSFET**)

$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = \frac{t_1}{t_1+t_2}V_s$$

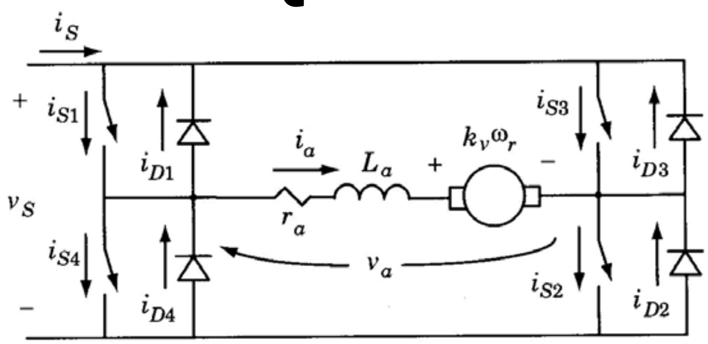
Two-Quadrant VS DC-DC Converter

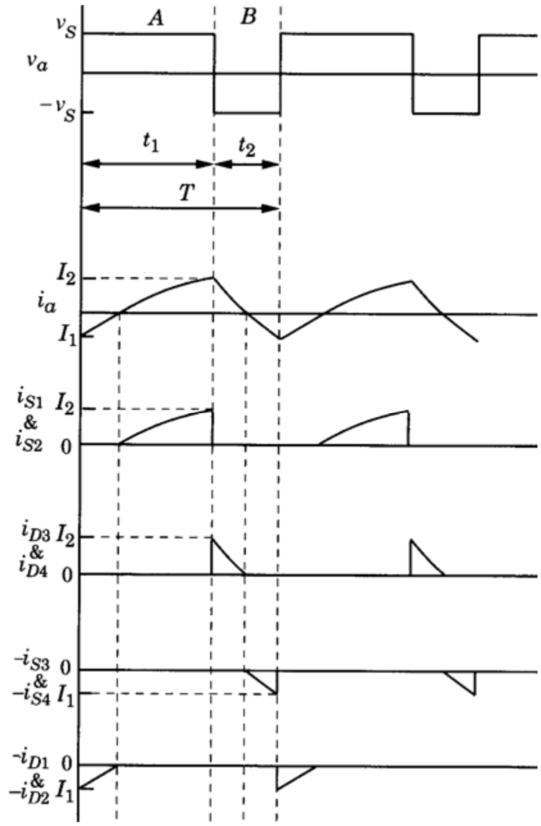




Pulse Width Modulation (**PWM**)

Four-Quadrant VS DC-DC Converter



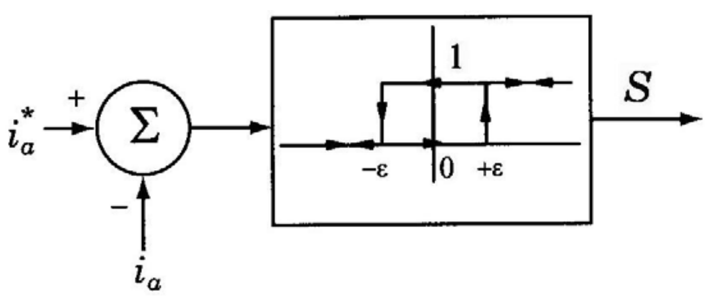


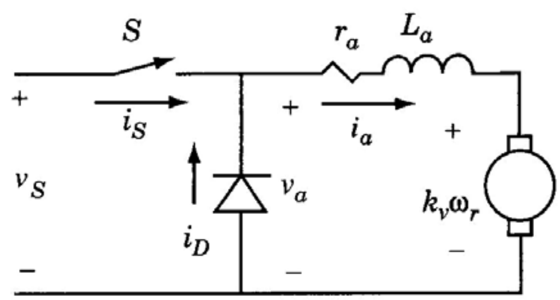
Pulse Width Modulation (**PWM**)

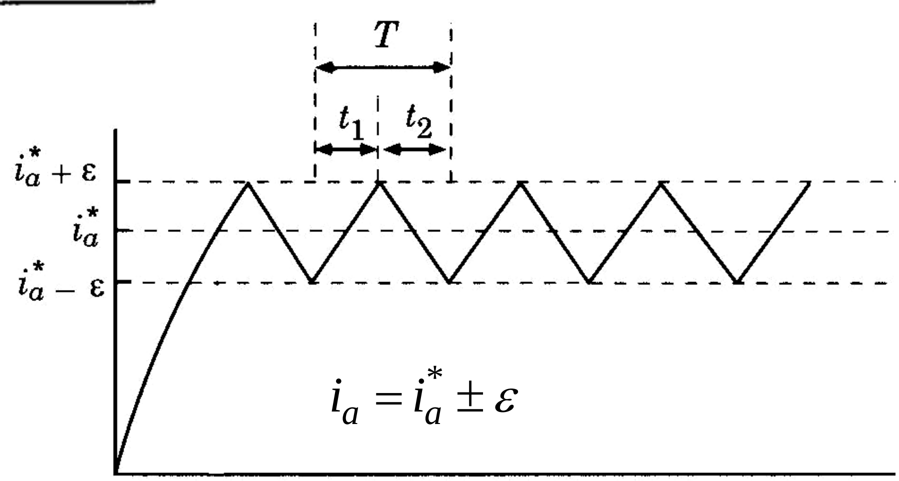
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = dV_s$$

Current Source DC-DC Converter

Hysteresis Modulation (**HM**)







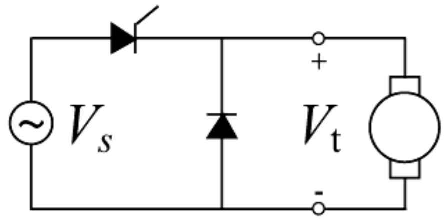
The same modulation can be applied to two- and four-quadrant choppers

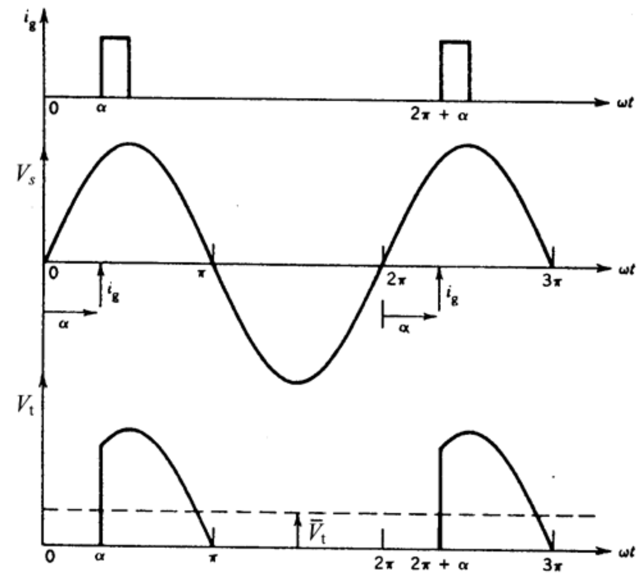
$$i_a = i_a^* \pm \varepsilon$$

Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

a) Single-phase, half-wave





$$V_s = \sqrt{2} V_{rms} \sin(\omega t)$$

$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)dt = \frac{V_{rms}}{\sqrt{2}\pi}(1+\cos(\alpha))$$

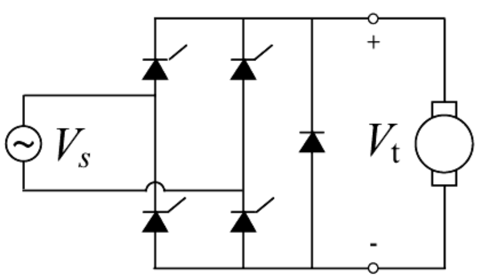
$$0 \le \alpha \le \pi$$

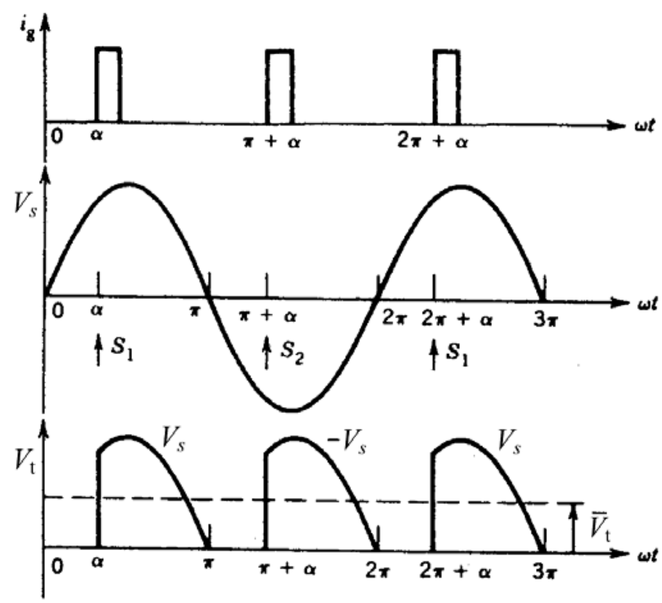
$$0 \le \overline{V_t} \le \frac{\sqrt{2}}{\pi} V_{rms}$$

Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

b) Single-phase, full-wave





$$V_s = \sqrt{2} V_{rms} \sin(\omega t)$$

$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)dt = \frac{\sqrt{2}V_{rms}}{\pi}(1+\cos(\alpha))$$

$$0 \le \alpha \le \pi$$

$$0 \le \overline{V_t} \le \frac{2\sqrt{2}}{\pi} V_{rms}$$

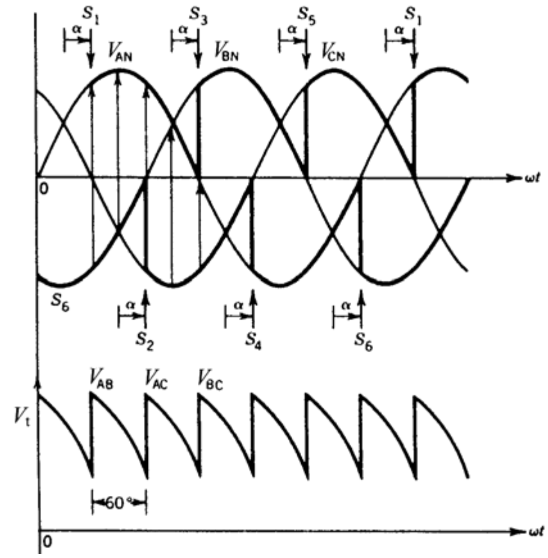
Thyristor Controlled Rectifiers

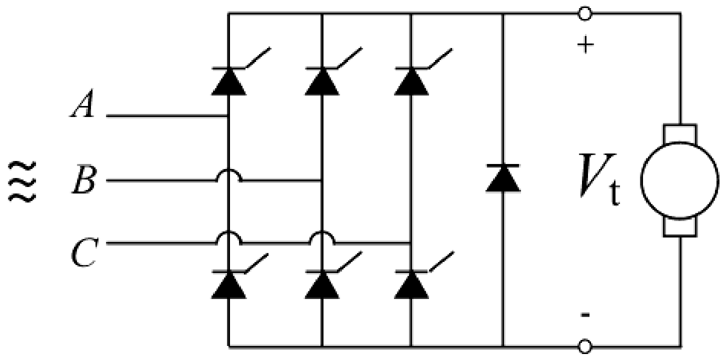
Control the Averaged Output Voltage
d) Three-phase, full-wave

$$V_A = \sqrt{2}V_{rms}\sin(\omega t)$$

$$V_B = \sqrt{2}V_{rms}\sin(\omega t - 120^\circ)$$

$$V_C = \sqrt{2}V_{rms}\sin(\omega t + 120^\circ)$$

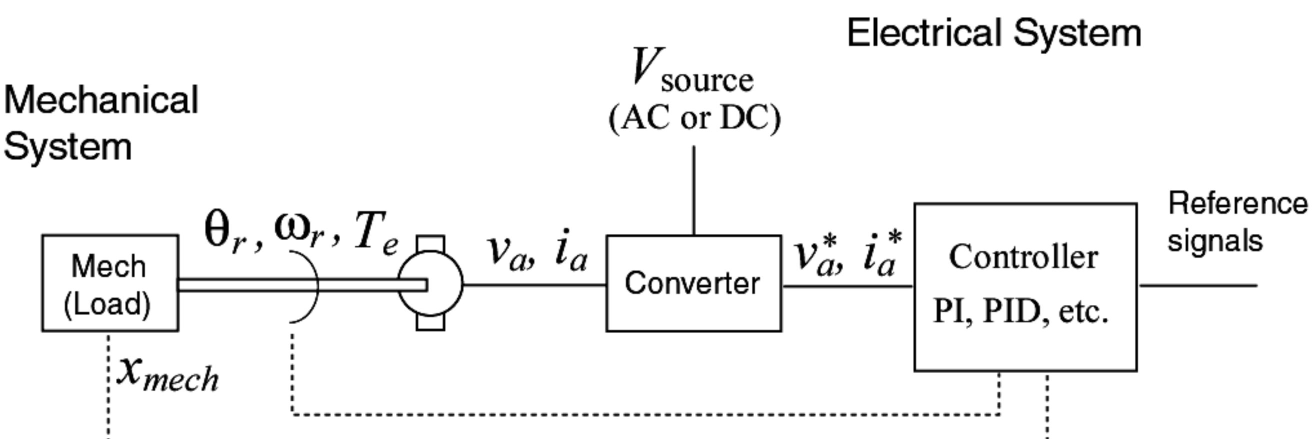




$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)\,dt \qquad 0 \le \alpha \le \pi$$

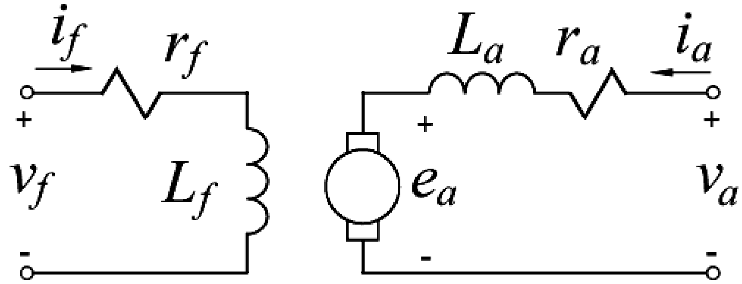
$$0 \le \overline{V_t} \le \frac{3\sqrt{6}}{\pi}V_{rms}$$

Closed-Loop Electric Drive System



Dynamic Modeling

Circuit is valid for transient analysis



Voltage Equations

$$v_a = r_a i_a + \frac{d\lambda_a}{dt} + e_a = r_a i_a + e_a + L_a \frac{di_a}{dt}$$

$$v_f = r_f i_f + \frac{d\lambda_f}{dt} = r_f i_f + L_f \frac{di_f}{dt}$$

Speed Equation

$$T_e = J \frac{d\omega_r}{dt} + T_m + D_m \omega_r$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$

Re-Write the Equations to Express Derivatives

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J} \omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$

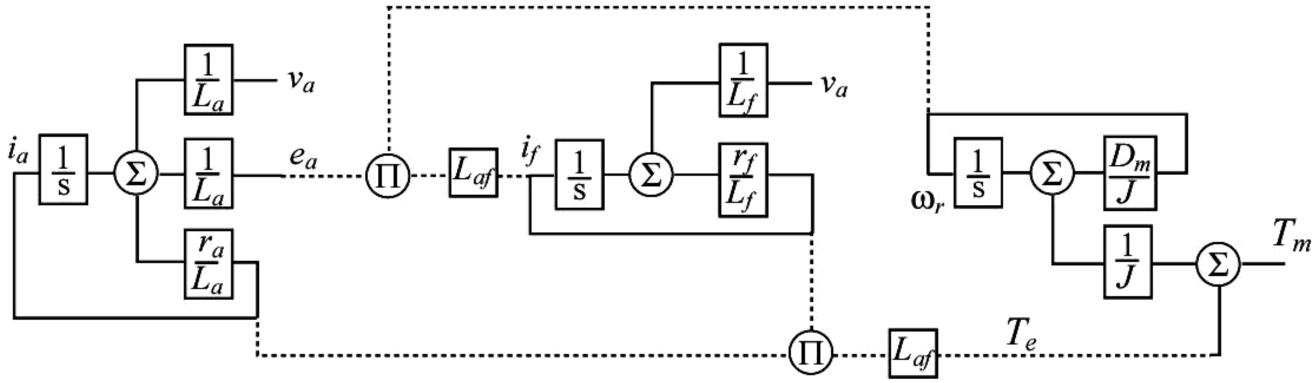
Are these equations coupled or decoupled ?

Are these equations linear or non-linear ?

How do we solve these equations ?

Implementation of State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a \qquad \frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$

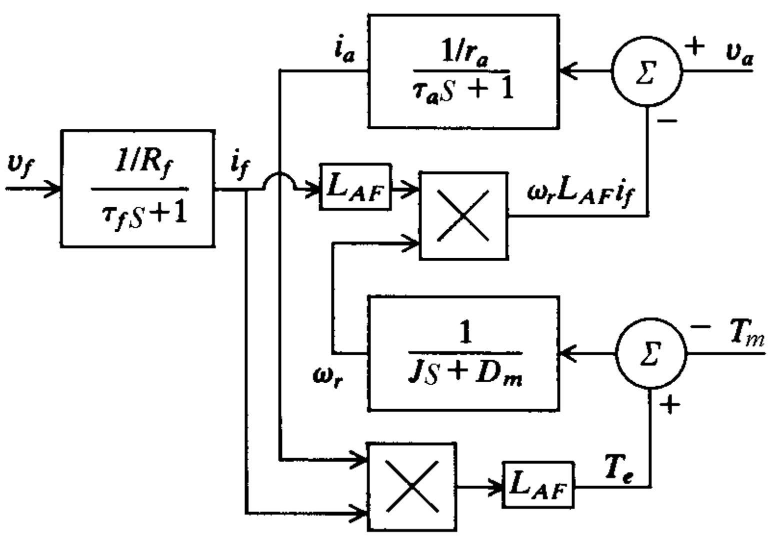


$$e_a = \omega_r L_{af} i_f \qquad \frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f \qquad T_e = L_{af} i_f i_a$$

Try to implement it in Simulink ?

Is this a unique implementation ?

Implementation of State Equations



Re-write the equations

$$v_f = r_f(1 + \tau_f s) i_f$$

$$v_a = r_a(1 + \tau_a s) i_a + e_a$$

$$e_a = \omega_r L_{af} i_f$$

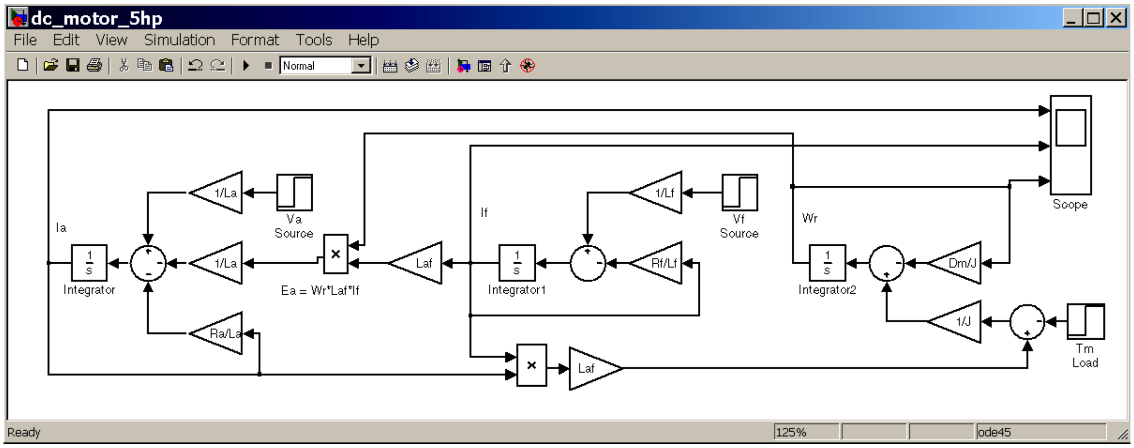
$$T_e - T_m = (D_m + Js)\omega_r$$

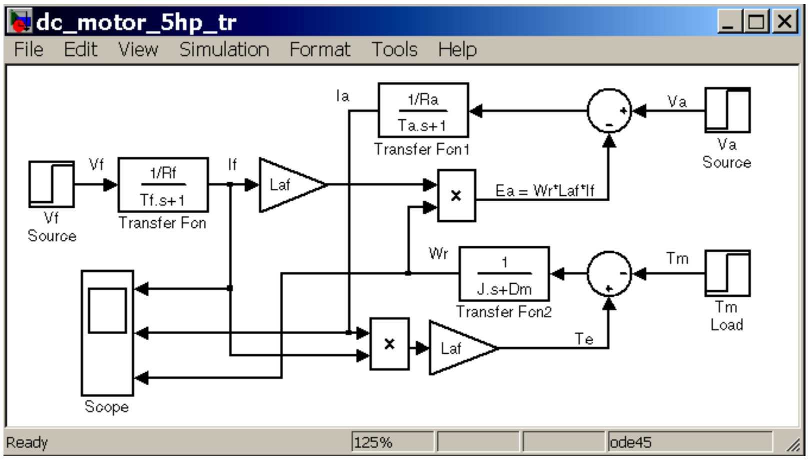
$$T_e = L_{af} i_f i_a$$

Define time constants

$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

5-HP Shunt DC Motor Simulink Model





Try to do it yourself !

Dynamic Response of a 5-HP DC Motor

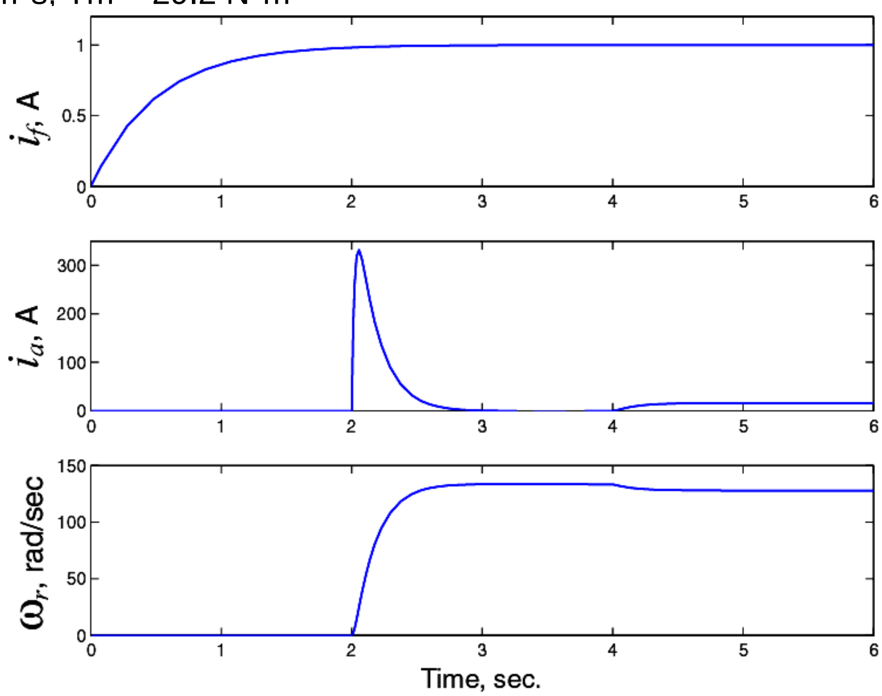
Motor parameters:

Va = Vf = 240V, Rf = 240Ω, Lf = 120H, Ra = 0.6Ω, La = 0.012H, Laf = 1.8H,
J = 1kg*m^2, Dm = 1e-4 N*m*s, Tm = 29.2 N*m

In this computer study:

Zero initial conditions assumed

Step-1: field winding is energized
Step-2: armature is energized
Step-3: load torque is applied



PM DC Motor Dynamics

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ?

Are these equations linear or non-linear ?

How do we solve these equations ?

Standard State-Space Form

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}$$

PM DC Motor Implementations

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

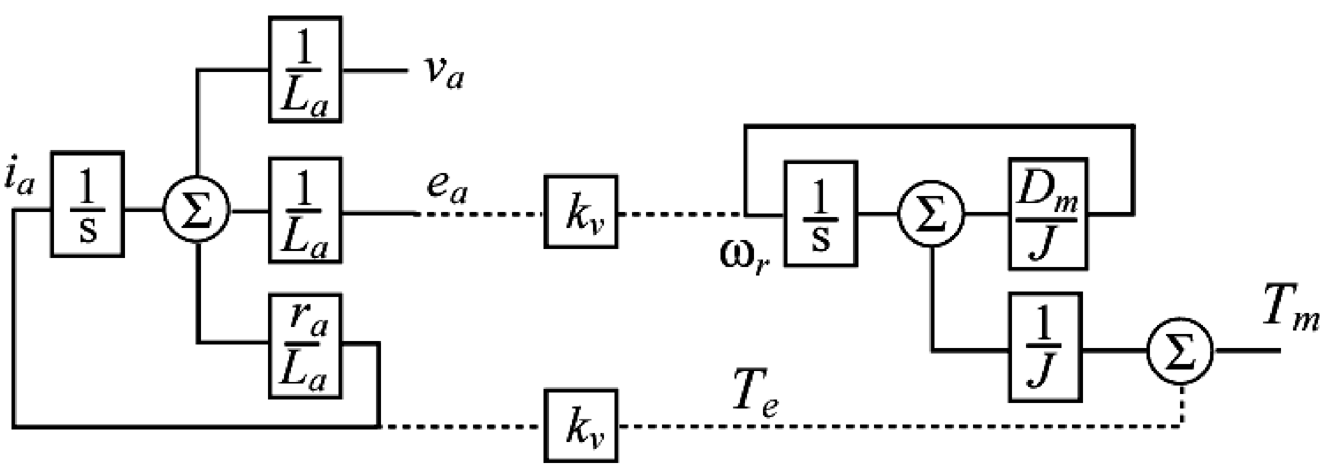
Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

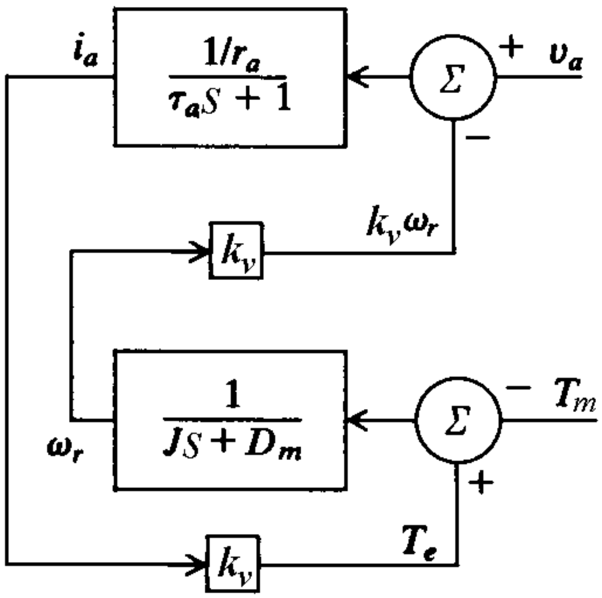
Are these equations coupled or decoupled ?

Are these equations linear or non-linear ?

How do we solve these equations ?



Implementation of State Equations



Re-write the equations

$$v_a = r_a(1+\tau_a s)i_a + e_a$$

$$e_a = k_v\omega_r$$

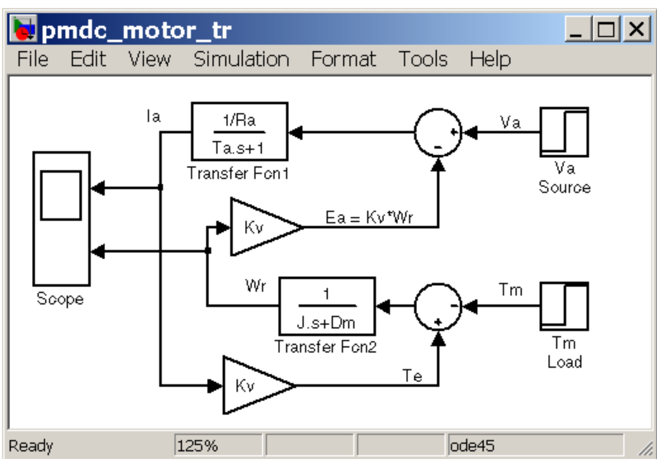
$$T_e - T_m = (D_m + Js)\omega_r$$

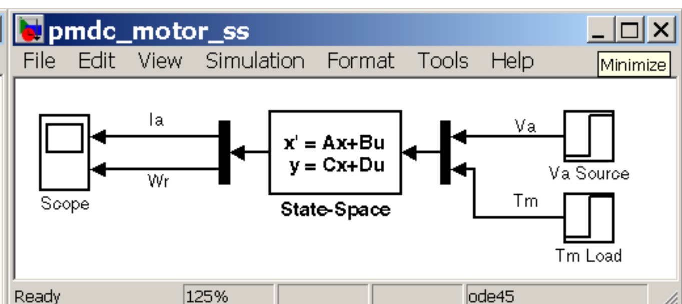
$$T_e = k_v i_a$$

Define time constants

$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

Simulink Model of a PM DC Motor





$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$i_a = \frac{1}{r_a(1+\tau_a s)}(v_a - e_a)$$

$$e_a = k_v\omega_r$$

$$\omega_r = \frac{1}{D_m + Js}(T_e - T_m)$$

$$T_e = k_v i_a$$

$$\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

Dynamic Response of a 6V PM DC Motor

Motor parameters:
Va = 6V, Ra = 7Ω, La = 120mH, Kv = 0.0141 N*A/m, J = 1.06e-6 kg*m^2,
Dm = 6.01e-6 N*m*s, Tm = 3.53e-3 N*m

In this computer study:

Zero initial conditions assumed

Step-1: voltage is applied

Step-2: load torque is applied