ADM 2304 -- ASSIGNMENT 3 - Solutions
(88 Marks)

Due Date: Wednesday, March 25, at 11:59 pm.

Question 1

Q1a) Chi-square test for independence (1 mark)

Q1b) (1 mark)

H0: there is no relationship between defect rate and years of experience (1 mark)
H1: there is a relationship between defect rate and years of experience

Q1c) (3 marks)

Cell11 (E=37*38/160=8.7875, Chi-square=(11-8.7875)^2/8.7875= 0.5571 (1.5 marks)
Cell23 (E=67*79/160=33.0813, Chi-square=(35-33.0813)^2/33.0813=0.1113 (1.5 marks)

Minitab Output

Chi-Square Test for Association: Worksheet rows, Worksheet columns

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Pearson Chi-Square = 2.807, DF = 4, P-Value = 0.591
Likelihood Ratio Chi-Square = 2.843, DF = 4, P-Value = 0.585
```

Q1d) (3 marks)

Since Chi-Square=2.807 < Chi-Square(.05,4)=9.488, we fail to reject Ho.

We have insufficient evidence to reject the claim that there is no relationship between defect rate and years of experience.

(3 marks: 1 mark the correct value of Chi-Square, 1 mark for the right decision, 1 mark for the right managerial statement)

Q1e) (4 marks)

P-Value = P_r(Chi-Square$_4$ > Chi-Square=2.807)= 0.591 (1 mark)
(The P-value is the area in the upper tail of the Chi-square model above the computed Chi-square value.)

Since P-value=0.591 is greater than alpha=0.05, we fail to reject Ho. (1 mark)

We have insufficient evidence to reject the claim that there is no relationship between defect rate and years of experience. (1 mark)

This conclusion is the same as the one reached in part 'd' above. (1 mark)

Q1f) (7marks)

Hypothesis Testing:

$H_0 : p_1 - p_2 = 0$

$H_a : p_1 - p_2 \neq 0$ (1 mark)

$\hat{p}_1 = \frac{X_1}{n_1} = 11/36 = 0.2973$

$\hat{p}_2 = \frac{X_2}{n_2} = 14/67 = 0.2090$ (1 mark)

Pool estimate of proportion

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} \Rightarrow z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

(1 mark for using the right formula)

$$\hat{p} = \frac{11+14}{37+67} = 0.2404 \Rightarrow z = \frac{0.0883}{\sqrt{.2404 * .7596\left(\frac{1}{37} + \frac{1}{67}\right)}} = 1.0093$$ (1 mark)

Since $|z_{test}| = 1.0093 < z_{crit(\alpha/2)} = 1.96$, we fail to reject H_o. (1 mark)

and

Since $\text{P-value} = 2 * \Pr(z > |z_{stat}|) = 2 * 0.1587 = 0.3174 > 0.05$, we fail to reject H_o. (1 mark)

The critical value approach and the p-value approach suggest that there is insufficient evidence to claim that the proportion of workers with a high defect rate differs between the group with "< 1 Year" of experience and the group with "5 - 9 Years" of experience. (1 mark)

Question 2

Q2a) (6 marks)

t-test assuming equal variance

$H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$
$H_a: \mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$

Minitab Output

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Two-sample T for A vs B

   N   Mean  StDev  SE Mean
A  5   78.8   10.5      4.7
B  5  70.60   9.13      4.1


Difference = μ (A) - μ (B)
Estimate for difference:  8.20
95% CI for difference:  (-6.18, 22.58)
T-Test of difference = 0 (vs ≠): T-Value = 1.31  P-Value = 0.225  DF = 8
Both use Pooled StDev = 9.8615
```

Since t = 1.31 < $t_{0.025}(8)$ = 2.306, we fail to reject H_o.
Or
Since p-value =0.225 > 0.05, we fail to reject H_o.

There is insufficient evidence to reject the claim that the two samples (A & B) come from populations with the same means.

(6 marks: 1.5 marks for Hypotheses, 1 mark for the Minitab output, 2 marks for the right decision based on the critical value or p-value approach (1 mark off for every mistake), 1.5 marks for the right managerial statement)

Q2b) (6 marks)

One-way ANOVA F-statistic

$H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$
$H_a: \mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$

Minitab Output

```
Method (Anova)

Factor  Levels  Values
Factor       2  A, B

Analysis of Variance

Source  DF  Adj SS  Adj MS  F-Value  P-Value
Factor   1   168.1  168.10     1.73    0.225
Error    8   778.0   97.25
```

```
Total     9   946.1

Model Summary

      S     R-sq  R-sq(adj)  R-sq(pred)
9.86154   17.77%      7.49%       0.00%
```

Since $F = 1.73 < F_{0.05}(1, 8) = 5.32$, we fail to reject H_o.
Or
Since p-value = 0.225 > 0.05, we fail to reject H_o.

There is insufficient evidence to reject the claim that the two samples (A & B) come from populations with the same means.

(6 marks: 1.5 marks for Hypotheses, 1 mark for the Minitab output, 2 marks for the right decision based on the critical value or p-value approach (1 mark off for every mistake), 1.5 marks for the right managerial statement)

Q2c) (3 marks)

t^2= 1.315^2= 1.73 = F (2 marks)

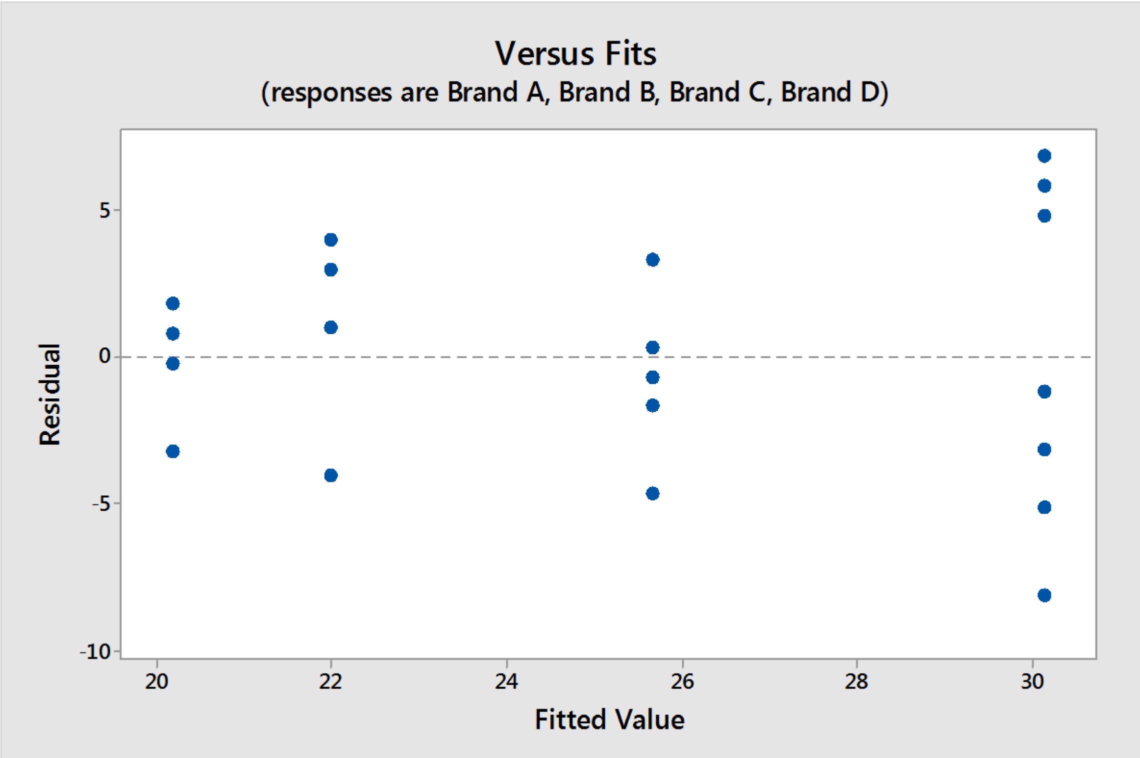
Alternatively

P-value: $t_{0.025}(8)$^2 = 2.306^2= 5.32 = $F_{0.05}(1, 8)$

F and t^2 (*or p-values*) are the same (1 mark)

Question 3

Q3a) (3 marks)



Equal variance assumption
The residuals plot shows no pattern and no systematic change in spread. However, someone could argue that the equal variance assumption is not satisfied because the residuals for the largest fitted value (Brand D) are more widely scattered than others.

Since we are not expected to perform the Levene test of homogeneity of variance, we will conclude that the residual plot does not show a systematic change in variance and, therefore, the pooled estimate of 4.16 (refer to the Minitab output in Q1b) for the error standard deviations seems reasonable for all four brands.

Alternatively (according to a conservative approach), we may conclude that there is a distinct difference in the spread of the residuals depending on the fitted value. Consequently, we should perform the analysis using a statistical method that does not assume homogeneity of variance.

Additional Comments:
-When faced with a small to moderate difference in spread, many researchers would perform Anova with a caveat that the reported probability levels (the Anova results - Q1b) may not be exactly correct. In our example, since the significance value is 0.003, even if we discount the value by an order of magnitude, the result would still be significant at the .05 level.

Normality assumption
Since we have a small number of observations for each brand, it is difficult to assess whether the distribution of five or seven numbers is Normal. However, the residual plot shows a reasonably equal number of points above as below the zero line. Moreover, there are no residuals more than 2 SD from zero (2*SD=4.16) -- there are no outliers. Thus, the normality assumption seems valid.

(3 marks – 1 mark for the plot, 1 mark for the discussion of the normality assumption, 1 mark for the discussion of the equal variance assumption).

Q3b) (6 marks)

H_0 : *all means are the same* (μ1 = μ2 = μ3 = μ4)
H_a: *not all means are the same* (at least one mean is different)

Minitab Output

Factor Information

Factor	Levels	Values
Factor	4	Brand A, Brand B, Brand C, Brand D

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	3	348.0	115.99	6.70	0.003
Error	19	329.0	17.32		
Total	22	677.0			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
4.16116	51.40%	43.73%	31.13%

Means

Factor	N	Mean	StDev	95% CI
Brand A	5	20.200	1.924	(16.305, 24.095)
Brand B	5	22.00	3.81	(18.11, 25.89)
Brand C	6	25.67	3.08	(22.11, 29.22)
Brand D	7	30.14	5.90	(26.85, 33.43)

Pooled StDev = 4.16116

Since $F = 6.70 > F_{0.025(3, 19)} = 3.90$, we reject H_o.
OR
Since p-value = 0.003 < alpha = 0.025, we reject H_o.

There is sufficient evidence to warrant rejection of the claim that the four brands have the same mean.

(6 marks: 1.5 marks for Hypotheses, 1 mark for the Minitab output, 2 marks for the right decision based on the critical value or p-value approach (1 mark off for every mistake), 1.5 marks for the right managerial statement)

Q3c) (6 marks)

Since the F test indicates that population differences exist, we will pursue the pairwise group comparisons.

Minitab Output

Means

Factor	N	Mean	StDev	95% CI
Brand A	5	20.200	1.924	(16.305, 24.095)
Brand B	5	22.00	3.81	(18.11, 25.89)
Brand C	6	25.67	3.08	(22.11, 29.22)
Brand D	7	30.14	5.90	(26.85, 33.43)

Pooled StDev = 4.16116

Thus with 4 means (4 brands) and therefore 6 pairwise comparisons, each Bonfeerroni test will be performed at the .0083 level.

$t_{0.0042}(19) = 2.94$ (one-tailed)
Pooled StDev = 4.1612

Confidence intervals are:

$$\text{D vs A}: \left(9.94 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{7}+\frac{1}{5}}\right) = (2.78, 17.11)$$

$$\text{D vs B}: \left(8.14 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{7}+\frac{1}{5}}\right) = (0.98, 15.31)$$

$$\text{D vs C}: \left(4.48 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{7}+\frac{1}{6}}\right) = (-2.33, 11.28)$$

$$\text{C vs A}: \left(5.47 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{6}+\frac{1}{5}}\right) = (-1.94, 12.87)$$

$$\text{C vs B}: \left(3.67 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{6}+\frac{1}{5}}\right) = (-3.74, 11.07)$$

$$\text{B vs A}: \left(1.80 \pm 2.94 \times 4.16 \times \sqrt{\frac{1}{5}+\frac{1}{5}}\right) = (-5.94, 9.54)$$

Since zero is not included in the first two intervals, we conclude that we are 95% confident that the means of Brands D and A and Brands D and B differ from each other.

(6 marks –1 mark for the right t-value, 1 mark for the right Pooled StDev, 3 marks for the correct CI's (.5 each*6=3), 1 mark the correct conclusion)

Q3d) (5 marks)

H_0 : *all medians are the same*
H_a: *not all medians are the same*

Minitab Output

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Kruskal-Wallis Test on Brand A

C1          N  Median  Ave Rank      Z
Brand A     5   21.00       5.1  -2.57
Brand B     5   23.00       8.7  -1.23
Brand C     6   25.50      13.9   0.81
Brand D     7   29.00      17.6   2.64
Overall    23              12.0

H = 11.68  DF = 3  P = 0.009
H = 11.77  DF = 3  P = 0.008  (adjusted for ties)
```

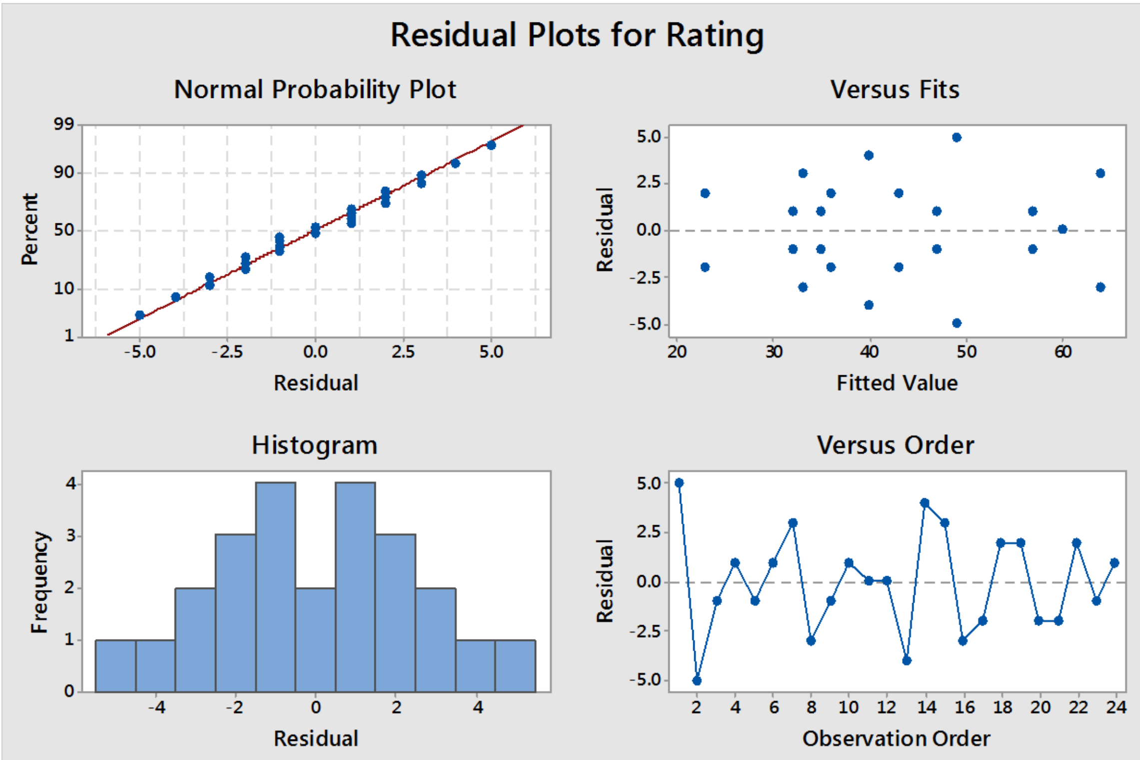
Since p-value = 0.009 < alpha = 0.025 (or 0.05), we reject H_o.

There is sufficient evidence to warrant rejection of the claim that the four brands have the same medians.

(5 marks: 1.5 marks for Hypotheses, 1 mark for the Minitab output, 1.5 marks for the right decision, 1 mark for the right managerial statement)

Question 4

Q4a) (5 marks)



Equal variance assumption
The residual plot shows the residuals exhibiting a roughly consistent spread (no patter can be detected). Thus, the equal variance assumption seems valid.

Normality assumption
There are no residuals more than 2 SD from zero (2*SD=3.54) -- no outliers. There are also similar number of data points above as below the line. The Normal Probability plot of the residuals is straight. Thus, the normality assumption seems valid.

(5 marks –1 mark for mentioning the two assumptions, 1 mark for the plot(s), 2 marks for the discussion of the normality assumption using more than one plot, 1 mark for the discussion of the equal variance assumption).

Q4b) (5 marks)

H_0: There is no interaction effect between colours and shapes
H_1: There is an interaction effect between colours and shapes

Minitab Output

```
Analysis of Variance

Source          DF   Adj SS   Adj MS  F-Value  P-Value
  Shape          2    579.0   289.50    23.16    0.000
  Colour         3   2711.2   903.72    72.30    0.000
```

```
  Shape*Colour    6    150.3    25.06      2.00     0.144
Error            12    150.0    12.50
Total            23   3590.5


Model Summary

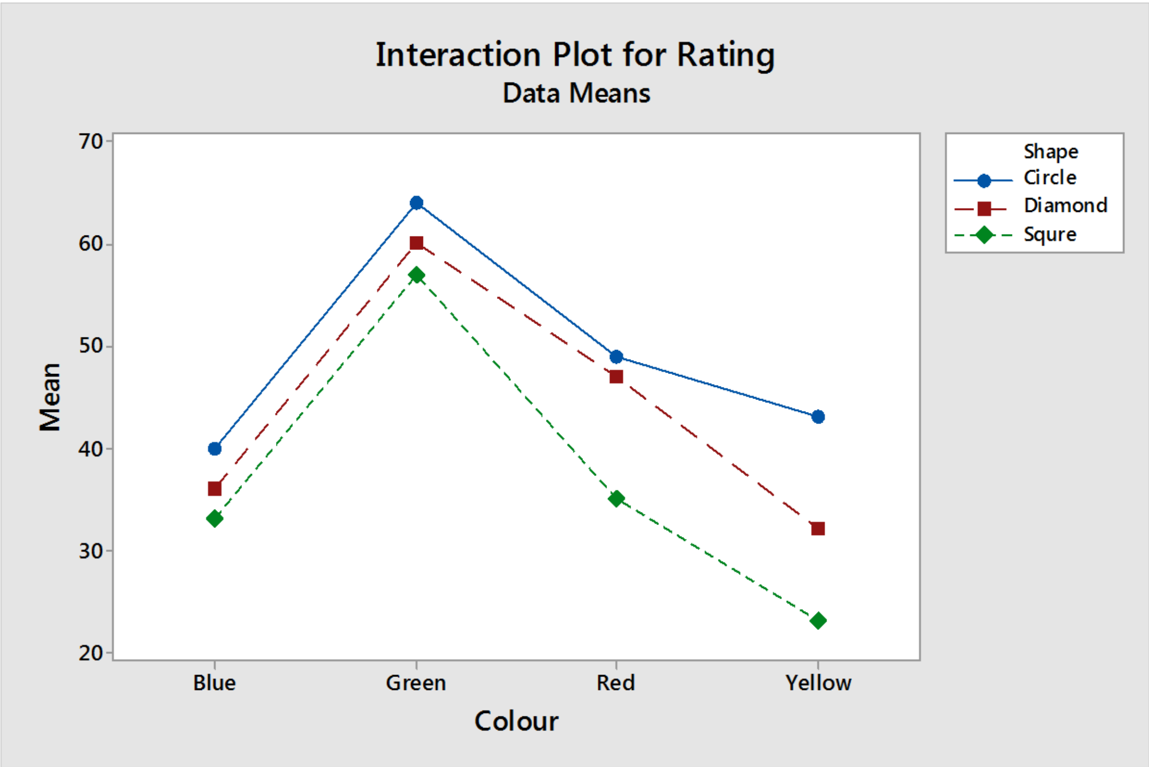
      S     R-sq  R-sq(adj)  R-sq(pred)
3.53553   95.82%     91.99%      83.29%
```

Since F = 2.00 < $F_{0.05(6, 12)}$ = 3.00 or P-value = 0.144 > alpha = 0.05, we fail to reject H_o.

There is insufficient evidence to claim that there is an interaction effect between colours and shapes.

(5 marks: 1 mark for Hypotheses, 1 mark for the Minitab output, 2 marks for the right decision based on the critical value or p-value approach (1 mark off for every mistake), 1 mark for the right managerial statement)

Q4c) (2 marks)



The plot suggests that a possibility of a week interaction exists (lines are not parallel but they don't cross) but the presence of an interaction was not confirmed by the significant interaction in the Anova table (Q1b).

Note: Unequal distance between lines indicates a weak interaction (no crossover of the mean lines).

(2 marks – 1 mark for the discussion of unequal distance between lines, 1 mark for the non-significant interaction in Anova)

Q4d) (5 marks)

H_0:The colour of the box's logo has no effect on the approval rate of the cereal
H_1:The colour of the box's logo does have an effect on the approval rate of the cereal

```
Analysis of Variance

Source          DF   Adj SS   Adj MS  F-Value  P-Value
  Shape          2    579.0   289.50    23.16    0.000
  Colour         3   2711.2   903.72    72.30    0.000
  Shape*Colour   6    150.3    25.06     2.00    0.144
Error           12    150.0    12.50
Total           23   3590.5


Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
3.53553  95.82%     91.99%      83.29%
```

Since $F = 73.30 < F_{0.05(3,\,12)} = 3.49$ or P-value = 0.0005 < alpha = 0.05, we reject $H_{o\,(colour)}$.

There is sufficient evidence to claim that the colour of the box's logo does have an effect on the approval rate of the cereal.

(5 marks: 1 mark for Hypotheses, 1 mark for the Minitab output, 2 marks for the right decision based on the critical value or p-value approach (1 mark off for every mistake), 1 mark for the right managerial statement)

Q4e) (5 marks)

Although you are dealing with Two-Way ANOVA, you are actually comparing only two column means just as you would in One-Way ANOVA.

Factor A: Colour
a = 4 (4 colours)

Factor B: Shape
b = 3 (3 shapes)

Replicates: Obs/ cell = r
r = 2

Since you are comparing Factor A,
$m = {}_{a}C_{2} = {}_{4}C_{2} = 6$ (multiple comparisons)
2 m = 2 x 6 = 12

$t^{**} = t_{\alpha/(2m)}(df_E) = t_{0.05/12}(12) = t_{0.004167}(12) = 3.15264$ (can be found with MiniTab)

$$ME = t^{**} s_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 3.15264 * 3.536 * \sqrt{\frac{1}{6} + \frac{1}{6}} = 6.4361$$

The CI can be found as follows:

$$\left|\bar{X}_{Red} - \bar{X}_{Green}\right| \pm ME = |43.667 - 60.333| \pm 6.4361 = 16.667 \pm 6.4361$$

This CI is: (10.2309, 23.1031)

Since this CI does not straddle '0', there is a difference in the two colour means. More specifically, with a Confidence Coefficient of 95%, one can state that:

$$\mu_{Red} + 10.2309 < \mu_{Green} < \mu_{Red} + 23.1031$$

(5 marks: 1 mark for the right t-value, 1 mark for the right Pooled StDev, 1 for the correct CI, 2 marks the correct decision/conclusion)

Question 5

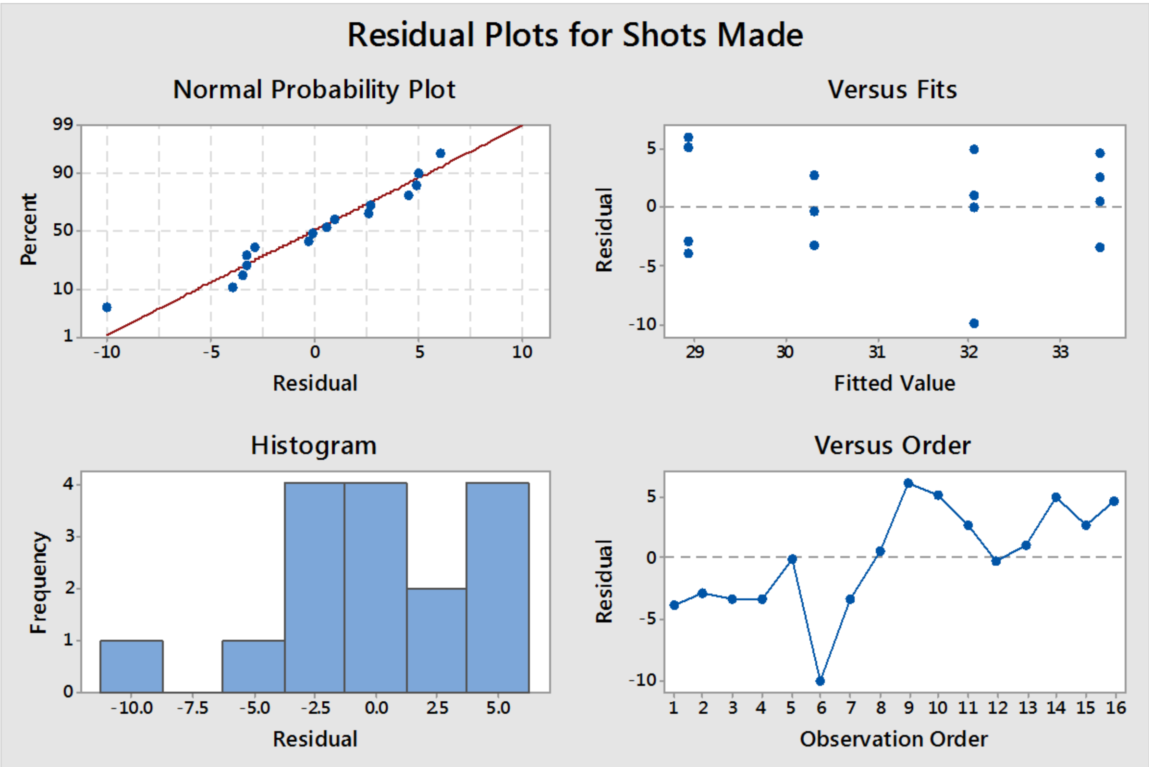
Q5a) (1 mark – Three null/alternative hypotheses)

H_0:The time of day has no effect on the number of shots made
H_1:The time of day does have an effect on the number of shots made

H_0:The type of shoes has no effect on the number of shots made
H_1:The type of shoe does have an effect on the number of shots made

H_0: There is no interaction effect between the type of shoes and the time of day
H_1: There is an interaction effect between the type of shoes and the time of day

Q5b) (5 marks)



<u>Equal variance assumption</u>
The residuals plot shows no pattern and no systematic change in spread. Thus, the equal variance assumption seems valid.

<u>Normality assumption</u>
There is only one residual lying more than two SDs from zero (2*SD=4.69) -- less than 5% of the data. The residuals look nearly Normal in the NPP plot, although, there is a possible outlier. Thus, the normality assumption would appear valid.

(5 marks –1 mark for mentioning the two assumptions, 1 mark for the plot(s), 2 marks for the discussion of the normality assumption using more than one plot, 1 mark for the discussion of the equal variance assumption).

Q5c) (6 marks)

Minitab Output

```
Analysis of Variance

Source                DF   Adj SS   Adj MS  F-Value  P-Value
  Time of Day          1    7.563    7.563     0.34    0.568
  Shoes                1   39.063   39.063     1.78    0.207
  Time of Day*Shoes    1   18.062   18.062     0.82    0.382
Error                 12  263.750   21.979
Total                 15  328.438


Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
4.68819  19.70%      0.00%       0.00%
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Since all p-values are greater than alpha = 0.05, we fail to reject all H_0.

There is insufficient evidence to conclude that there is an interaction effect between the type of shoes and the time of day. In fact, none of the effects appears to be significant. It looks as though the student cannot conclude that either shoes or time of day affect her mean free throw percentage.

(6 marks- 1 mark for the Minitab output, 2 marks for the right decision(s), 3 marks for the right managerial statements)