Questions for Midterm Preparation

1) Calculate the following limits. If no limit exists, state the reason

$$\lim_{x \longrightarrow 1^-} \sqrt{x^2 - 1}$$

Solution: This limit does not exist. The left-hand limit cannot be evaluated because for all $x < 1$ $x^2 - 1 = (x-1)(x+1)$ is negative.

$$\lim_{x \longrightarrow 5} x^2 + 6x + 1$$

Solution:

$$= 5^2 + 6(5) + 1 = 56$$

$$\lim_{x \longrightarrow 2} \frac{x-2}{x^2+2}$$

Solution:

$$\lim_{x \longrightarrow 2} \frac{x-2}{x^2+2} = \frac{0}{6} = 0$$

$$\lim_{x \longrightarrow -3} \frac{x+3}{x^2+8x+15}$$

Solution: Note that pluggin in $x = -3$ will result in an indeterminate form.

$$\lim_{x \longrightarrow -3} \frac{x+3}{x^2+8x+15} = \lim_{x \longrightarrow -3} \frac{x+3}{(x+3)(x+5)} = \lim_{x \longrightarrow -3} \frac{1}{x+5} = \frac{1}{2}$$

2. Provide two answers for Assignment 1, Question 3. (Note that in order to complete this problem you must know the criteria for continuity)

Solution: See assignment 1 solutions files for an example

3. Calculate the average rate of change of the function $\frac{x+3}{x-5}$ over the interval $1 \leq x \leq 3$

Solution:

$$\frac{\frac{3+3}{3-5} - \frac{1+3}{1-5}}{3-1} = \frac{-3-(-1)}{2} = -1$$

4a)Let $f(x) = g(x)(h(x))^3$. State $f'(x)$ in terms of $g(x)$, $h(x)$, $g'(x)$, and $h'(x)$.

Solution: (This uses the product rule and the chain rule)

$$f'(x) = g'(x)(h(x))^3 + g(x)(3(h(x))^2 h'(x)$$

b)Calculate the first derivatives of the following functions:

$$f(x) = \sqrt[3]{x} - \frac{4}{x^2} + 5x^2$$

Solution: Note that

$$f(x) = x^{1/3} - 4x^{-2} + 5x^2$$

It follows from the power rule that

$$f'(x) = (1/3)x^{-2/3} + 8x^{-3} + 10x$$

$$g(x) = \frac{(x+5)(x^2-2)}{x+3}$$

Solution: This solution uses botht the product rule and the quotient rule.

$$g'(x) = \frac{[(x^2-2)+(2x)(x+5)](x+3)-(x+5)(x^2-2)}{(x+3)^2}$$

$$h(x) = (x+2)^7(x-5)^2$$

Solution: This requires the use both the chain rule and the product rule:

$$h'(x) = 7(x+2)^6(x-5)^2 + 2(x+2)^7(x-5)$$

5. Using only the limit definition of the derivative calculate the derivative of $f(x) = x^3 + 5$

Solution:

$$\lim_{h \longrightarrow 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \longrightarrow 0} \frac{(x+h)^3+5-x^3-5}{h} =$$

$$\lim_{h \longrightarrow 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim_{h \longrightarrow 0} \frac{h(3x^2+3xh+h^2)}{h} =$$

$$\lim_{h \longrightarrow 0} 3x^2+3xh+h^2 = 3x^2$$

6. Calculate the marginal revenue at $x = 2$ when the demand function for a commodity is $p(x) = -x^2 + 5x + 200$

Solution: Revenue is
$R(x) = xp(x) = x(-x^2 + 5x + 200) = -x^3 + 5x^2 + 200x$. Marginal revenue is $R'(x) = -3x^2 + 10x + 200$, hence $R'(2) = -3(2)^2 + 10(2) + 200 = 208$