

# COMP 1805A - Winter 2016 Supplemental Sheet

## Logic and Set Rules

$T = \text{True}$		$\mathcal{U} = \text{Universe}$
$F = \text{False}$		$\emptyset = \{\}$ (empty set)
$A \wedge T \equiv A$	Identity Rules	$A \cap \mathcal{U} \equiv A$
$A \vee F \equiv A$		$A \cup \emptyset \equiv A$
$A \wedge F \equiv F$	Domination Rules	$A \cap \emptyset \equiv \emptyset$
$A \vee T \equiv T$		$A \cup \mathcal{U} \equiv \mathcal{U}$
$A \vee A \equiv A$	Idempotent	$A \cup A \equiv A$
$A \wedge A \equiv A$		$A \cap A \equiv A$
$\neg(\neg A) \equiv A$	Double Negation	$\overline{\overline{A}} \equiv A$
$A \vee \neg A \equiv T$	Law of excluded middle	$A \cup \overline{A} \equiv \mathcal{U}$
$A \wedge \neg A \equiv F$	Contradiction	$A \cap \overline{A} \equiv \emptyset$
$\neg(A \vee B) \equiv \neg A \wedge \neg B$	De Morgan's Laws	$\overline{(A \cup B)} \equiv \overline{A} \cap \overline{B}$
$\neg(A \wedge B) \equiv \neg A \vee \neg B$		$\overline{(A \cap B)} \equiv \overline{A} \cup \overline{B}$
$A \vee B \equiv B \vee A$	Commutative Laws	$A \cup B \equiv B \cup A$
$A \wedge B \equiv B \wedge A$		$A \cap B \equiv B \cap A$
$((A \vee B) \vee C) \equiv (A \vee (B \vee C))$	Associativity Laws	$((A \cup B) \cup C) \equiv (A \cup (B \cup C))$
$((A \wedge B) \wedge C) \equiv (A \wedge (B \wedge C))$		$((A \cap B) \cap C) \equiv (A \cap (B \cap C))$
$(A \wedge (B \vee C)) \equiv (A \wedge B) \vee (A \wedge C)$	Distributivity Laws	$(A \cap (B \cup C)) \equiv (A \cap B) \cup (A \cap C)$
$(A \vee (B \wedge C)) \equiv (A \vee B) \wedge (A \vee C)$		$(A \cup (B \cap C)) \equiv (A \cup B) \cap (A \cup C)$
$A \rightarrow B \equiv \neg A \vee B$	Implication Relation	
$A \rightarrow B \equiv \neg B \rightarrow \neg A$	Contrapositive	
$A \vee (A \wedge B) \equiv A$	Absorption Laws	$(A \cap (A \cup B)) \equiv A$
$A \wedge (A \vee B) \equiv A$		$(A \cup (A \cap B)) \equiv A$
$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$	Bidirection	

# Inference Rules

$$\frac{A}{\therefore A \vee Q} \left. \vphantom{\frac{A}{\therefore A \vee Q}} \right\} \text{Addition} \qquad \frac{A \wedge B}{\therefore A} \quad \frac{A \wedge B}{\therefore B} \left. \vphantom{\frac{A \wedge B}{\therefore B}} \right\} \text{Simplification}$$

$$\frac{A \rightarrow B}{\frac{A}{\therefore B}} \left. \vphantom{\frac{A \rightarrow B}{\therefore B}} \right\} \text{Modus ponens} \qquad \frac{A \rightarrow B}{\frac{\neg B}{\therefore \neg A}} \left. \vphantom{\frac{A \rightarrow B}{\therefore \neg A}} \right\} \text{Modus tollens}$$

$$\frac{A \vee B}{\frac{\neg A}{\therefore B}} \left. \vphantom{\frac{A \vee B}{\therefore B}} \right\} \text{Disjunctive Syllogism} \qquad \frac{A}{\frac{B}{\therefore A \wedge B}} \left. \vphantom{\frac{A}{\therefore A \wedge B}} \right\} \text{Conjunction}$$

$$\frac{A \rightarrow B}{\frac{B \rightarrow C}{\therefore C}} \left. \vphantom{\frac{A \rightarrow B}{\therefore C}} \right\} \text{Transitivity} \qquad \frac{A \vee B}{\frac{\neg A \vee C}{\therefore B \vee C}} \left. \vphantom{\frac{A \vee B}{\therefore B \vee C}} \right\} \text{Resolution}$$

$$\frac{\forall x P(x)}{\frac{\therefore P(c)}{\text{for any } c \in U}} \left. \vphantom{\frac{\forall x P(x)}{\therefore P(c)}} \right\} \text{Universal instantiation} \qquad \frac{\exists x P(x)}{\frac{\therefore P(c)}{\text{for some } c \in U}} \left. \vphantom{\frac{\exists x P(x)}{\therefore P(c)}} \right\} \text{Existential instantiation}$$

$$\frac{P(c) \text{ for any } c \in U}{\therefore \forall x P(x)} \left. \vphantom{\frac{P(c) \text{ for any } c \in U}{\therefore \forall x P(x)}} \right\} \text{Universal generalization} \qquad \frac{P(c) \text{ for some } c \in U}{\therefore \exists x P(x)} \left. \vphantom{\frac{P(c) \text{ for some } c \in U}{\therefore \exists x P(x)}} \right\} \text{Existential generalization}$$