MAAE 2300 Fluid Mechanics I

PROBLEM SETS

Problem Set 1
Review of Dynamics; Units; Fluid Properties

Fluids have mass and respond to applied forces according to Newton's Laws like all other matter. Review the basic concepts of particle dynamics.

1.1 What acceleration would a vertically upward force of 30 lb_f give to a stone which weighs 10 lb_f, on earth? (Note that values of various physical constants are given in Appendix A).

1.2 What acceleration would that force give to the same stone on the moon where the acceleration due to gravity is 5.31ft/sec^2? (Ans. 91.2 ft/sec^2)

1.3 A mass on the end of a 1 m long string is swinging in a circle about a vertical axis. The half-angle of the cone of revolution generated by the stone is 60°. What is the speed of rotation, in revs/min (RPM), and what is the tension in the string if the mass is 1 kg? (Ans. 42.3 RPM, 19.6 N)

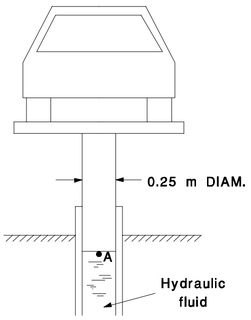
1.4 Decide whether each of the following statements is true or false.
(a) 1 lb_m always weighs 1 lb_f
(b) 1 lb_m weighs 1 lb_f (approximately) on earth
(c) the weight of a quantity of material is independent of its location
(d) the mass of a quantity of material is independent of its location.

1.5 Indicate the correct answer:
A fluid is a substance that:
(a) always expands until it fills any container
(b) is practically incompressible
(c) cannot be subjected to shear forces
(d) cannot remain at rest under action of any shear force
(e) has the same shear stress at a point regardless of its motion

1.6 Complete the following table of equivalents, assuming that the reference atmospheric pressure is 14.696 psia (the value at sea level for the standard atmosphere):

psia	psig	psfa	psfg	kPa(a)	kPa(g)
14.696					
	25				
		985			
			2116		
				101.32	
					10^5

1.7 The drawing shows an automobile held up by a hydraulic lift. The hydraulic piston and the automobile together have a mass of 3000 kg. The hydraulic piston has a diameter of 0.25 m. What is the pressure in the hydraulic fluid at point A? (Ans. 600 kPa)



1.8 An aircraft carries an oxygen cylinder to which a gauge is attached. The gauge reads 100 kPa(g). The air inside the aircraft is pressurized to 8 psi above the ambient pressure outside the aircraft, which is flying at 12,000 m in a standard atmosphere (see data sheets at back). What is the absolute pressure of the cabin air and of the oxygen in the tank?
(Ans. 74.6 kPa(a), 174.6 kPa(a))

1.9 Air is enclosed in a rigid cylinder containing a piston. A pressure gauge attached to the cylinder gives an initial reading of 20 psig. Determine the reading of the gauge after the piston has compressed the air to one third of its initial volume. Assume that the compression process is isothermal (i.e. constant temperature) and that the local atmospheric pressure is 14.7 psia.
(Ans. 89.4 psig)

1.10 As will be shown in lectures, the pressure variation in the vertical direction at any point in any fluid in a gravitational field is given by

$$\frac{dP}{dy} = -\rho g$$

where y is positive vertically upward, ρ is the local density of the fluid, and g is the local acceleration due to gravity.

Consider the change in atmospheric pressure from the bottom to the top of Mt. Tremblant. The bottom of the mountain is at an elevation of 266 m above sea level and the top is at 915 m. On a particular winter day, the atmospheric conditions at the bottom are: T = -20 C, P = 100 kPa. What is the density of the air under these conditions? (Ans. 1.377 kg/m^3)

Estimate the change in atmospheric pressure based on the following assumptions:

(a) Neglecting the variation in both ρ and g with elevation. (Ans. ΔP = -8770 Pa)
(b) Neglecting the variation in g but taking into account the change in ρ due to the change in pressure with elevation. (Ans. ΔP = -8390 Pa)
(c) In the lower part of the earth's atmosphere, the temperature is found to vary essentially linearly with elevation:

$$T = T_0 - B\left(y - y_0\right)$$

where T_0 is temperature at elevation y_0 and B is the "lapse rate" (B = 0.00650 K/m for the standard atmosphere). Estimate again the change in pressure, neglecting the variation in g but taking into account the effect of varying P and T on the density. What is the resulting density at the top of the mountain? (Ans. ΔP = -8460 Pa, 1.282 kg/m^3)

This problem illustrates the "compressibility" of gases: that is, that changes in pressure cause noticeable changes in the density. Here the change in pressure was the result of a change in elevation. As will be shown later, changes in pressure also arise in moving fluids due to changes in velocity. Therefore, caution must be used in assuming "incompressible" conditions (that is, constant density) when dealing with gas flows. On the other hand, liquids are almost always assumed incompressible. The next problem provides an opportunity to examine the validity of the latter assumption.

1.11 The pressure of the water at a depth of 10 km in the ocean is about 100 MPa (recall that sea level atmospheric pressure is about 0.1 MPa). Assuming a value of the bulk modules of elasticity K = 2100 MPa, what is the density of the water at that depth, if the S.G. is 1.03 at the surface?

(Hint: The definition of the bulk modulus is

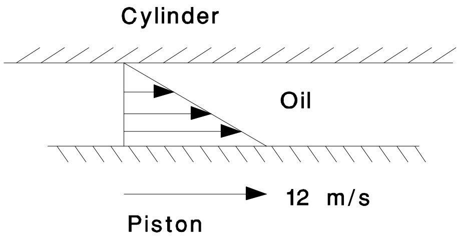
$$K = -V\left(\frac{\partial P}{\partial V}\right)$$

where V is volume. If K is assumed to remain constant as the pressure of the fluid is changed, then we can write

$$K = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

which relates the fractional change in the volume of the fluid, ΔV/V, to the change in applied pressure, ΔP) (Ans. 1081 kg/m^3)

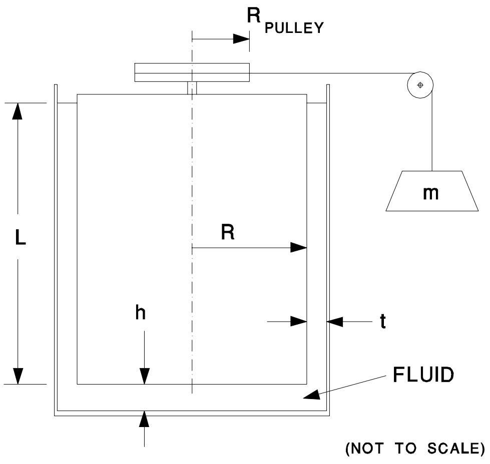
1.12 A smooth-sided piston 5 cm long and of diameter 5.000 cm moves within a cylinder of 5.010 cm diameter. The lubricating oil has a viscosity of 2 x 10^{-3} N s/m^2. What force is required to move the piston at a speed of 12 m/s? The velocity in the gap between the cylinder and the piston may be assumed to vary linearly, as indicated in the sketch. (Ans. 3.77 N)



1.13 The drawing shows a concentric-cylinder viscometer, a device for measuring the viscosity of a fluid. The fluid fills the gap between the two cylinders. The falling weight unreels a string from the pulley, thus rotating the inner cylinder. The viscosity of the fluid is then inferred from the rotational speed of the cylinder. Assume that the fluid velocity varies linearly both in the vertical gap between the cylinders and in the gap between the flat bottom of the inner cylinder and the bottom of the outer cylinder. The viscometer has the following dimensions: R = 6 cm, R_{PULLEY} = 3 cm, L = 15 cm, and t = h = 0.1 cm. When the falling weight has a mass of 30 gm, the cylinder is observed to rotate at 30 RPM.

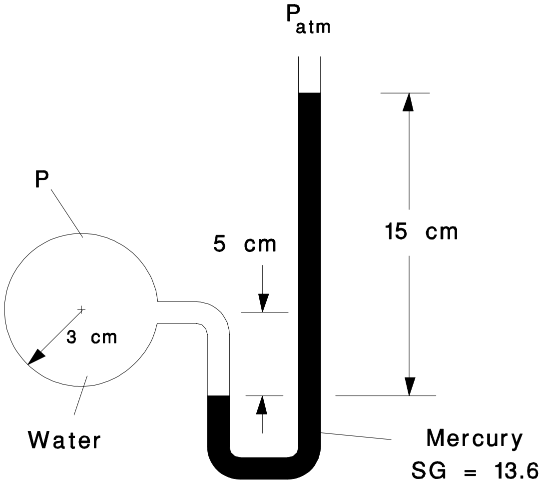
(a) Estimate the viscosity of the fluid neglecting the frictional torque due to the fluid in the bottom gap.

(b) Repeat (a) but now include the effect of the fluid in the bottom gap.
(Ans. 1.38 x 10^{-2} Ns/m^2, 1.26 x 10^{-2} Ns/m^2)

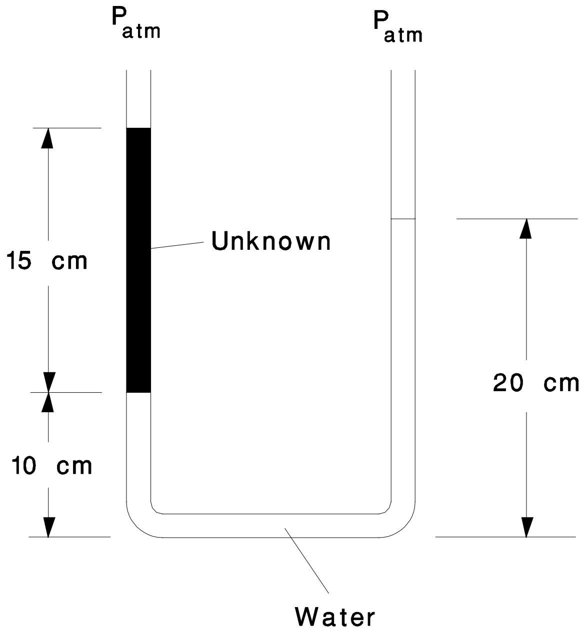


Problem Set 2
Pressure Measurement; Effects of Acceleration

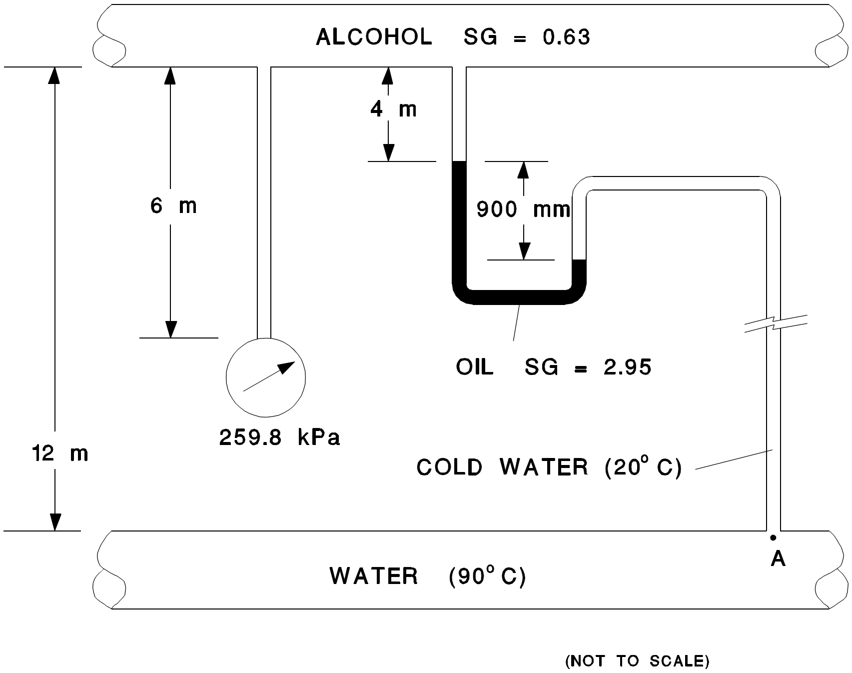
2.1 Find the pressure P in kPa(g) at the top of the water pipe shown in the drawing.
(Ans. 19.2 kPa(g))



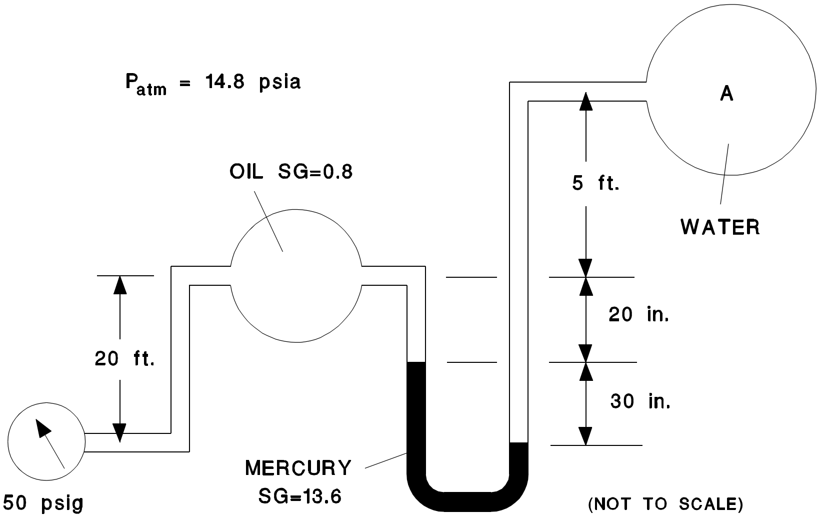
2.2 The drawing shows a simple arrangement which can be used to determine the density of an unknown liquid which is insoluble in water. Determine the specific gravity of the unknown liquid for the conditions shown.
(Ans. SG = 0.667)



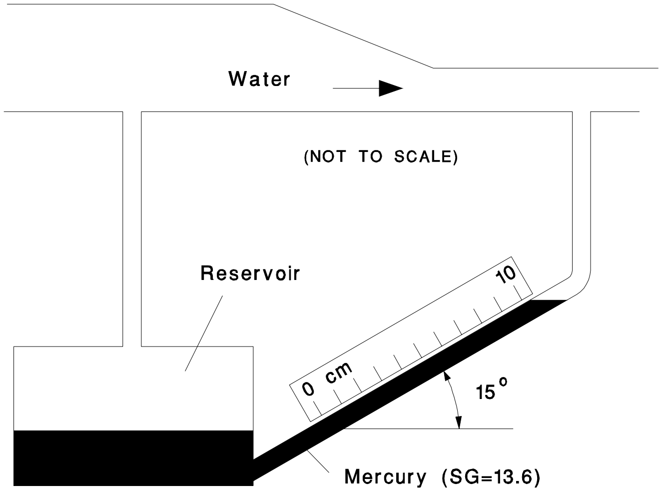
2.3 The drawing shows some piping in a chemical plant. What is the pressure at point A in the water line? Density of water: 1000 kg/m^3 at 5°C; 998.2 kg/m^3 at 20°C; 965.3 kg/m^3 at 90°C.
(Ans. 343 kPa)



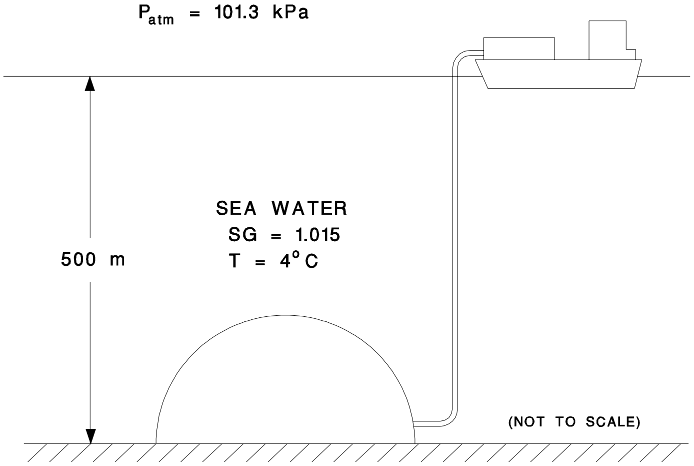
2.4 What is the pressure in pipe A in psia? The density of water is 62.4 lb$_m$/ft^3.
(Ans. 69.2 psia)



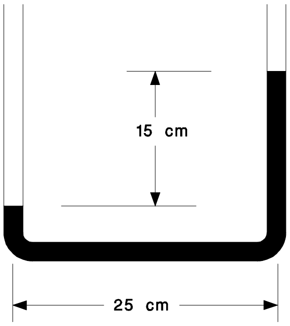
2.5 An inclined mercury manometer is used to measure the pressure difference at two points in a water stream. The two pressure taps are at the same elevation. When there is no flow, the scale on the inclined tube reads 0 cm. When the water is flowing the mercury rises in the inclined tube until the scale reads 10 cm, as measured along the tube. The cross-sectional area of the reservoir is 25 times the cross-sectional area of the reading tube. What is the pressure difference between the two taps when the water is flowing? Note that a fixed volume of mercury is present under all conditions. (Ans. 3.69 kPa)



2.6 Fresh water (SG = 1.0) and air are supplied to an experimental ocean-floor laboratory by flexible hoses. What pressure must the pump at the surface produce to supply water to the laboratory if the water hose must not collapse? What pressure must the air compressor at the surface produce?
(Ans. 73.6 kPa(g), 4680 kPa(g))



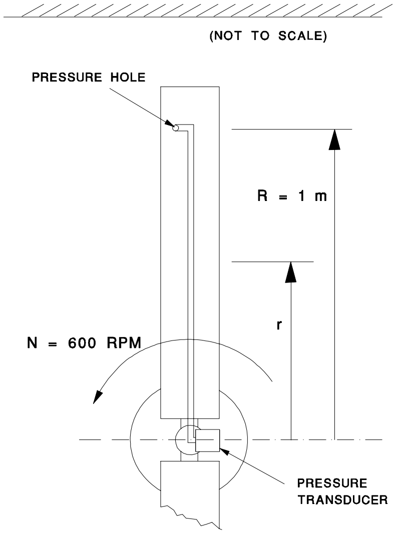
2.7 The tube in the drawing is installed in a drag racer to serve as an accelerometer during testing. What is the acceleration of the vehicle when the liquid levels are as shown?
(Ans. 5.9 m/s^2 to the left)



2.8 A model of a helicopter rotor is being tested in a wind tunnel. The rotor is in the horizontal plane. One rotor blade is instrumented with holes for measuring the pressure on the surface of the blade. Each hole is connected by a tube inside the blade to a pressure transducer, an instrument for measuring pressure. The transducer is located at the axis of rotation.

(a) What is the acceleration at a point on the rotor which is at radius r from the axis of rotation? Give the answer in symbolic form in terms of the parameters which appear in the drawing.

(b For the pressure hole shown in the drawing, the transducer shows an output of 99,125 Pa(a). What is the actual pressure on the blade at the hole? the air trapped inside the tube may be assumed to have a density of 1.2 kg/m^3.
(Ans. 101,494 Pa(a))

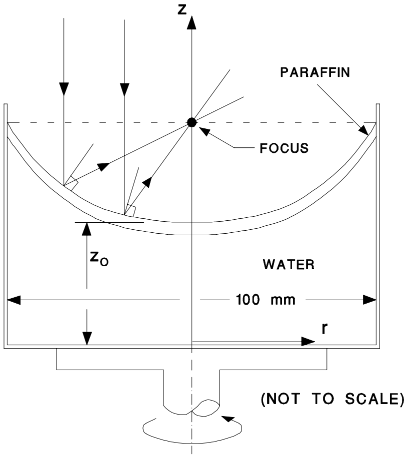


2.9 An experimenter plans to make a paraboloidal mirror by rotating a cylinder of warm water with a layer of paraffin wax floating on top. When the water cools the wax will solidify into a thin shell of the required shape. Assume that the density of the paraffin wax is essentially the same as the density of water.

(a) Show that the surface of the paraffin forms a parabola.

(Ans. $z = z_0 + \dfrac{r^2\omega^2}{2g}$)

(b) Find the rotational speed in RPM which will give a mirror with the focus as shown in the drawing. Recall that the focus is the point through which parallel light rays approaching the mirror will be reflected. (Ans. 134 RPM)



Problem Set 3
Control Volume Analysis I
Continuity; Linear Momentum

3.1 A diffuser is a length of duct which reduces the velocity of a moving fluid. For the diffuser shown in the drawing, assume that the density of the fluid does not vary. If the velocity is uniform at 10 m/s at the inlet, what is the velocity on the centreline at the outlet if:

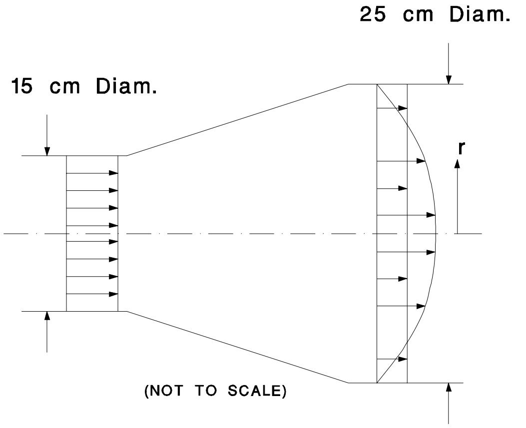
(a) The velocity is uniform at the outlet?

(b) The velocity varies parabolically with radius according to

$$\frac{V}{V_{CL}} = 1 - \left(\frac{r}{R}\right)^2$$

where V_{CL} is the velocity on the centreline and R is the radius of the duct? (This is closer to the distribution which would be expected in reality.)

(Ans. 3.6 m/s, 7.2 m/s)



3.2 An open tank is being filled with water from the top at a constant rate of 500 USgal/min (1 USgal = 0.1337 ft^3). At the same time, water is draining from the tank through a small pipe near the bottom. For reasons which will be explained in lectures, the outflow from the small pipe has a velocity V which depends on the depth of water in the tank, h, according to:

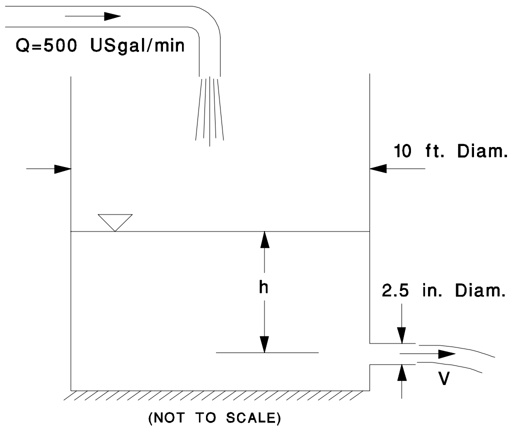
$$V = \sqrt{2gh}$$

where g is the acceleration due gravity.

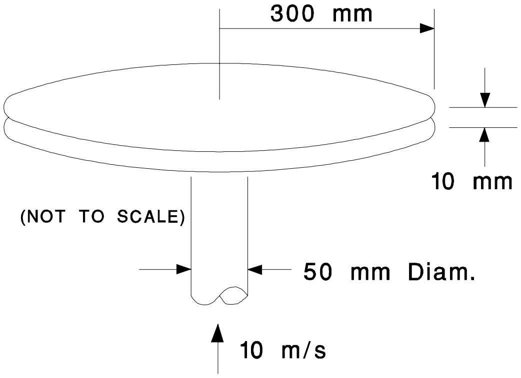
(a) What is the steady-state level of water in the tank? That is, for what value of h does the outflow match the inflow so that the level stays constant?

(b) When h = 5 ft, how fast is the level of the free surface changing, in ft/min?

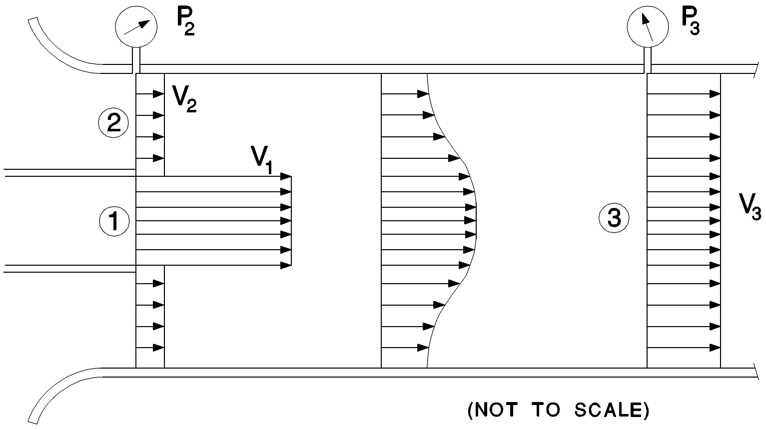
(Ans. 16.6 ft., rising at 0.384 ft/min)



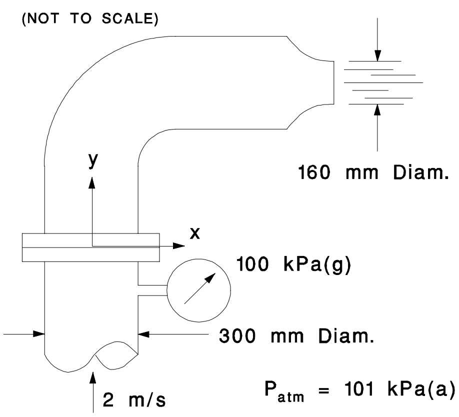
3.3 Water flows along a 50 mm diameter pipe with an average velocity of 10 m/s and then out radially between two large circular disks. The disks are parallel and spaced 10 mm apart. Determine the velocity of the water as it leaves the circular disks.
(Ans. 1.04 m/s)



3.4 The sketch shows a simple jet pump. A high-speed primary flow (1) is injected along the centreline of the tube. This stream drags the secondary fluid (2) along with it through friction and mixing. Thus, the device is effectively pumping the secondary fluid. At the completion of the mixing (3), the stream is assumed to have an essentially uniform velocity, V_3 (Note that you will be examining the validity of this assumption in Experiment 2). With an injector velocity V_1 = 100 ft/sec, the velocity at the end of the mixing tube is observed to be V_3 = 19 ft/sec. What is the velocity of the secondary flow, V_2? The corresponding mass flow rate? Assume that the working fluid is a liquid with a density of 64.4 lb_m/ft^3 and that the following areas apply: $A_1 = 0.1\ ft^2$, $A_3 = 1.0\ ft^2$.
(Ans. 10 ft/sec, 18.0 slug/sec)



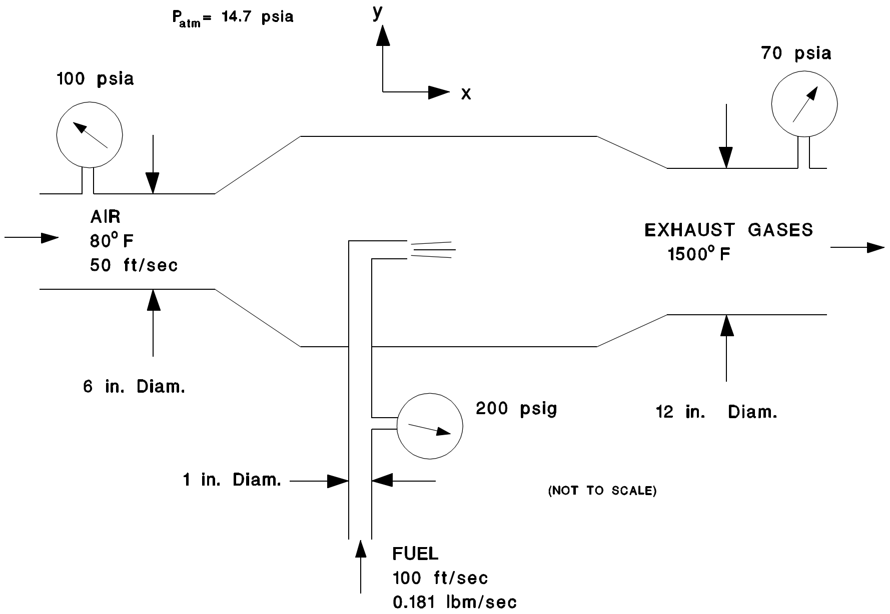
3.5 The drawing shows an elbow and a nozzle through which water is flowing. Determine the components of force in the x and y directions at the flanged joint. Is the joint in tension or compression? Neglect the weight of the pipe itself and the water inside it.
(Ans. F_X = 994 N, F_Y = -7350 N (tension))



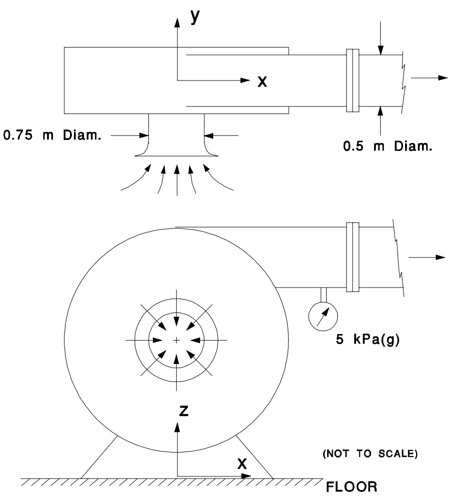
3.6 A combustion chamber is shown in the drawing. For the conditions shown determine:

(a) The mass flow rate and velocity of the exhaust gases. Assume that the exhaust gases have the same value of R as air.

(b) The forces in the x and y directions which are needed to hold the combustion chamber in place. These forces will be supplied by the three pipes to which the chamber is attached. You will not be able to determine how, for example, F_X is divided among the three connections, only what its net value must be. Neglect the weight of the pipes and the combustion chamber.
(Ans. 5.09 lb_m/sec, 67.2 ft/sec, F_X = 3850 lb_f, F_Y = -158 lb_f)



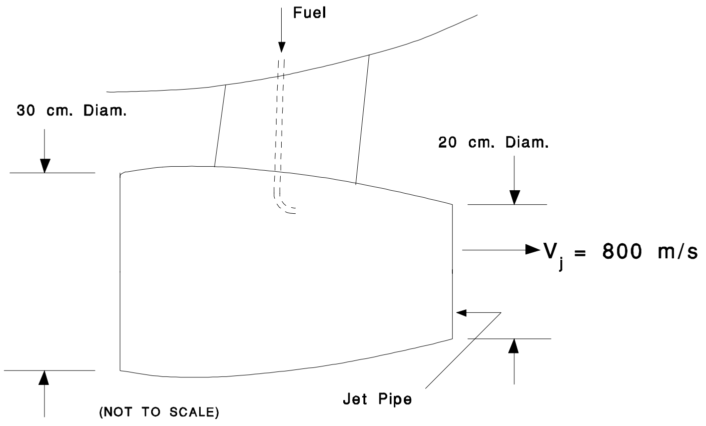
3.7 The drawing shows a centrifugal fan. The fan draws 25 m^3/s of air from the room through the 0.75 m diameter bellmouth inlet. The air is discharged into a 0.5 m diameter pipe which is connected to the downstream piping by a flexible coupling. Note that a flexible coupling does not transmit any forces or moments between the connected components. Determine the forces in the x and y directions which the floor must apply to hold the fan in place. Assume incompressible flow with $\rho = 1.3$ kg/m^3. Atmospheric pressure is 101 kPa.
(Ans. $F_X = 5120$ N, $F_Y = 0$ N)



3.8 A jet-propelled aircraft travels at 1000 km/hr. The engine takes in 20 kg/s of air and burns 1/50 kg of fuel for each kg of air. The exhaust gases leave the jet pipe at 800 m/s relative to the engine. Assume that the liquid fuel has negligible momentum.

(a) What thrust does the engine produce if the exhaust gases are at atmospheric pressure as they leave the jet pipe?

(b) For a subsonic flow discharging into a large reservoir, the pressure at the nozzle exit will be equal to the reservoir pressure. However, if the flow is sonic or supersonic the pressure can be <u>higher</u> than the reservoir pressure at the nozzle exit. The expansion down to the reservoir pressure is then completed downstream of the nozzle. What thrust will this jet engine produce if the nozzle exit pressure is +15 kPa(g)? All other parameters are unchanged.
(Ans. 10,764 N, 11,235 N)



3.9 For the jet pump in Problem 3.4, determine the pressure rise, (P_3 - P_2), which occurs along the mixing tube. Assume that P_2 and P_1 are equal: in Experiment 2, you will be able to confirm that this is a reasonable assumption. Also, neglect friction on the walls of the mixing tube.
(Ans. 10.1 psi)

Problem Set 4
Bernoulli's Equation

4.1 The drawing shows an open-circuit wind tunnel. Air is drawn from the room using a bellmouth inlet. A U-tube manometer is connected to a pressure tap in the wall of the test section and the other end is open to the room. Assume incompressible flow.

(a) What is the total or stagnation pressure of the air in the test section?

(b) For the operating point shown, what is the velocity in the test section?

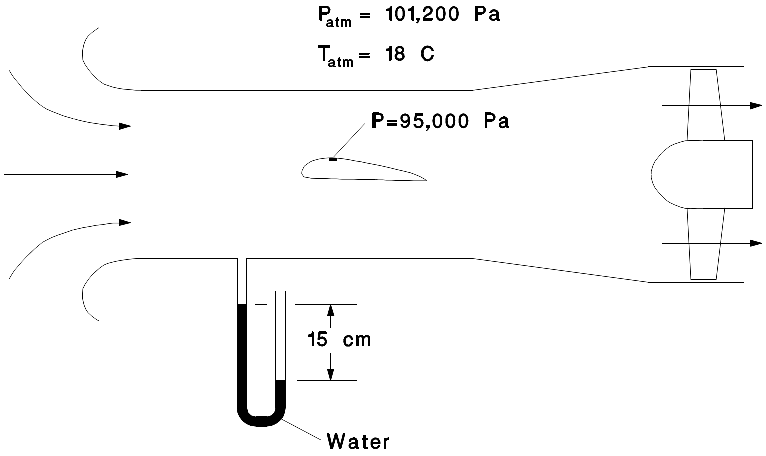
(c) A small pressure sensor mounted flush with the top surface of the airfoil indicates a local pressure of 95,000 Pa(a). What the local fluid velocity at that point on the airfoil?

(d) Assume now that the same airfoil is part of the wing on a aircraft which is flying at an altitude of 5,000 m and with the same forward speed as the velocity used in the wind tunnel. Assume standard atmospheric conditions.

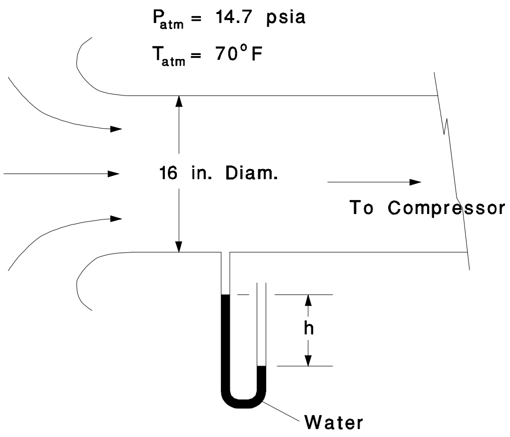
(i) What pressure would be sensed at the stagnation point on the airfoil under these conditions?

(ii) What pressure would the sensor from (c) measure at that same point on the airfoil surface? Assume that the local velocity is the same as found in (c).

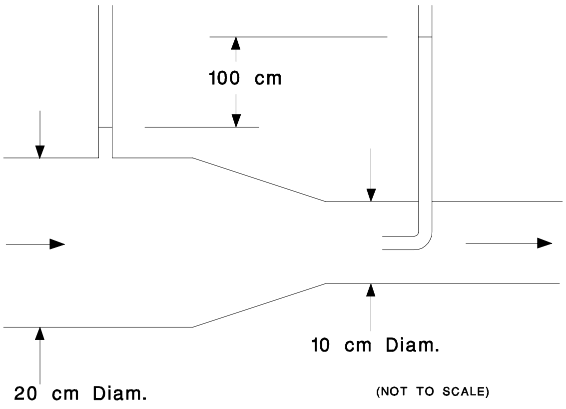
(Ans. 101,200 Pa(a), 177.4 km/hr, 101.2 m/s, 54,940 Pa(a), 51,170 Pa(a))



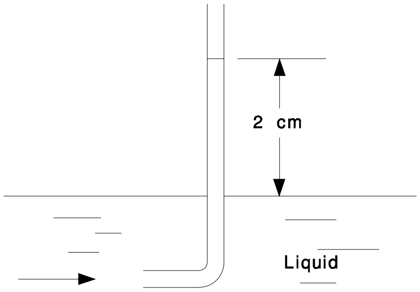
4.2 The bellmouth inlet shown in the drawing is used to measure the flow rate into an air compressor. If the flow rate is 12,000 cfm (ft^3/min), what reading h will the U-tube give in inches of water? Assume incompressible flow.
(Ans. 4.59 in.)



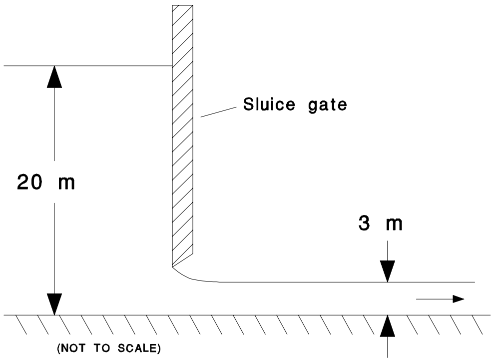
4.3 Calculate the flow rate in m^3/s for the unknown liquid in the pipeline shown. The two vertical tubes are open to the atmosphere. Assume uniform flow and neglect friction.
(Ans. 0.139 m^3/s)



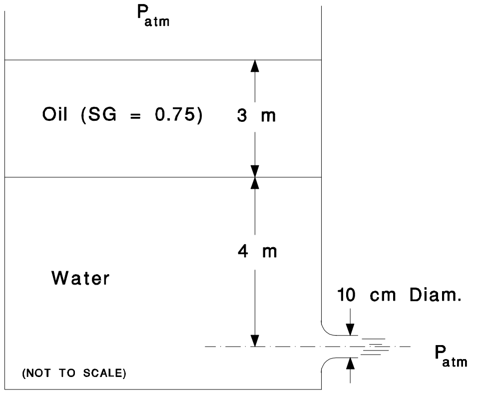
4.4 An L-shaped glass tube is lowered into a flowing liquid stream, as shown in the drawing. The liquid is observed to rise 2 cm in the tube above the free surface of the liquid. What is the velocity of the stream at the depth where the mouth of the tube is located?
(Ans. 0.63 m/s)



4.5 The drawing shows a section through a sluice gate which is used to control the flow of water in a channel. The channel has a rectangular cross section with a width of 10 m in the horizontal direction. The gate spans the full width of the channel. For the conditions shown, what is the volume flow rate of water in m^3/s through the gate? Assume uniform velocity well upstream and downstream of the gate and neglect friction.
(Ans. 554 m^3/s)

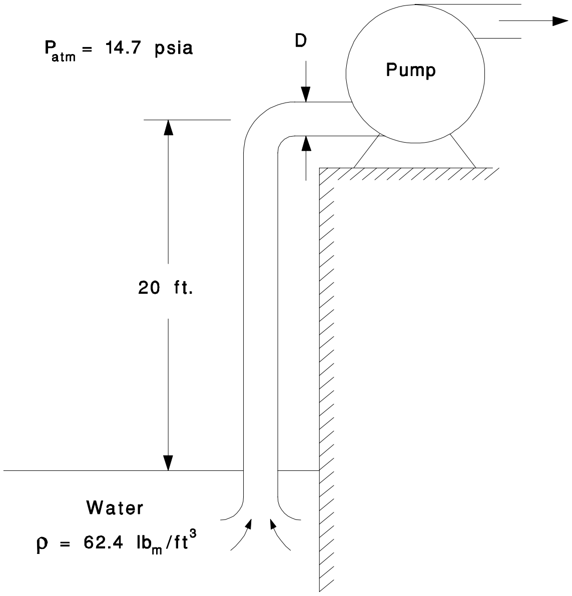


4.6 For the tank shown, determine the outlet flow rate of water in m^3/s. Neglect friction. (Ans. 0.087 m^3/s)

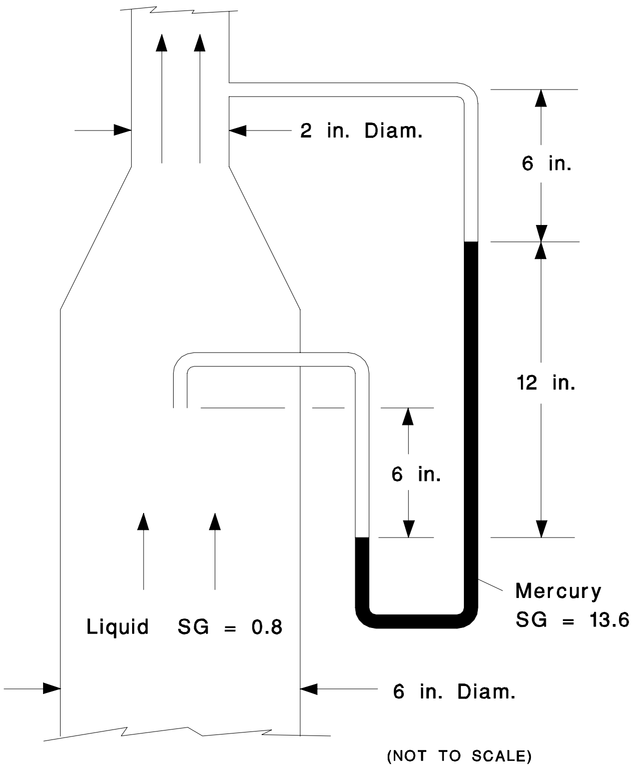


4.7 A pump is to be installed to pump 3,000 Imperial gallons per minute of water from a reservoir. For practical reasons, the inlet to the pump must be located 20 ft. above the free surface of the reservoir.

If the pressure in a liquid falls below the vapour pressure the liquid will boil. This results in the formation of pockets or bubbles of vapour. The subsequent collapse of such vapour pockets is known as cavitation. Cavitation results in very large and very destructive forces on any metal surfaces which are exposed to it. It has been determined that to avoid cavitation in the present pump it is necessary to maintain the pressure at its inlet above 2 psia. Choose a diameter D for the inlet pipe which will avoid cavitation. Assume that pipe is available in only integral diameters (i.e. 1,2, 3, ...inches diameter, etc.). By definition, 1.0 Imperial gallon contains 10 lb_m of water. (Ans. 8 in.)



4.8 Find the flow rate in ft^3/sec and lb_m/sec for the liquid in the pipeline shown. The centreline of the pipe is vertical. Assume uniform flow in the pipe and neglect friction.
(Ans. 0.70 ft^3/sec, 350 lb_m/sec)



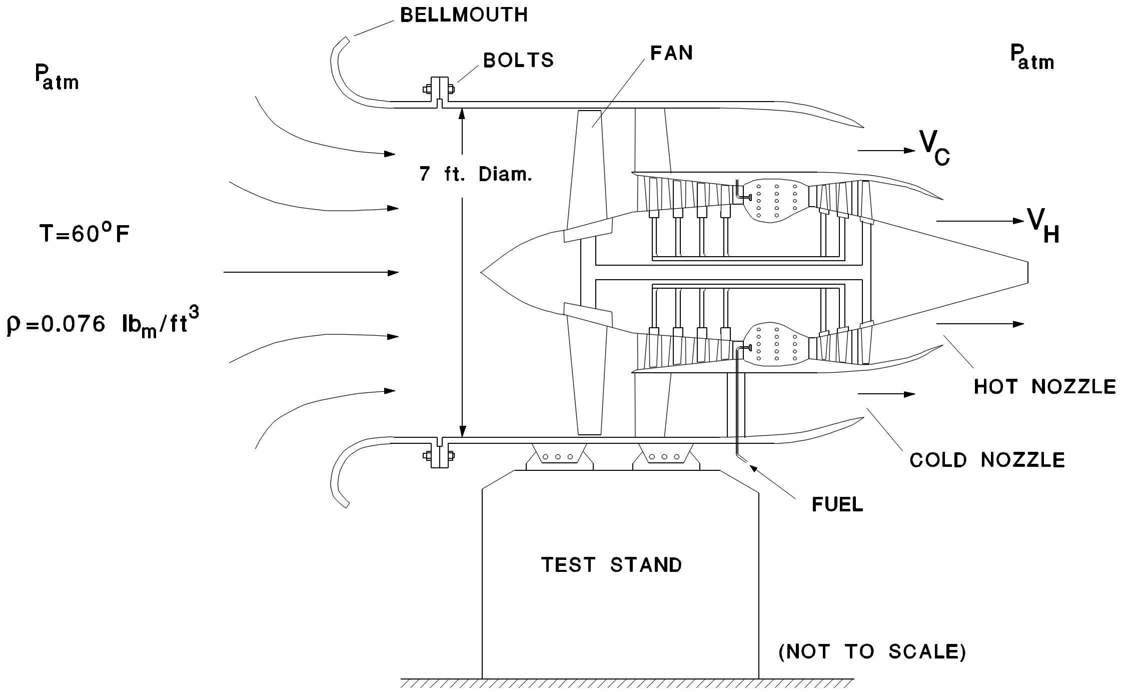
Problem Set 5
Control Volume Analysis II
Linear and Angular Momentum

5.1 The drawing shows a cross-section through a turbofan engine which is mounted on a test stand at sea level. The total mass flow rate of air through the engine is 1200 lb_m/sec. After being compressed by the fan, 83% of the air by-passes the core of the engine and exhausts through the "cold" nozzle with a velocity V_C = 900 ft/sec. The remaining 17% of the air passes through the core, including the combustion chamber, and leaves through the "hot" nozzle with a velocity V_H = 1450 ft/sec. The engine burns fuel at a rate of 22,000 lb_m/hr. Assume that the velocity of the liquid fuel in the fuel pipe is very small.

(a) Determine the horizontal force the engine exerts on the test stand.

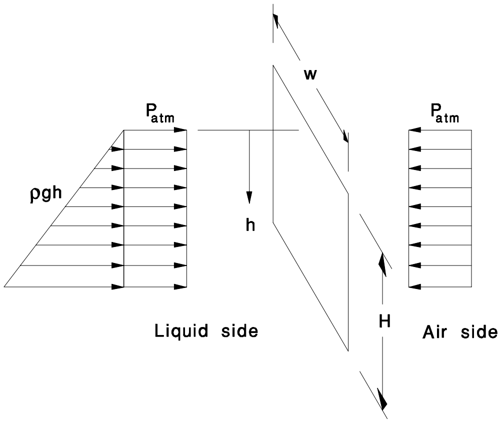
(b) Since the engine is being tested outside its normal nacelle, a bellmouth has been attached to bring the air smoothly into the engine. Determine the total force in the bolts holding the bellmouth to the engine. Are the bolts in tension or compression? You may assume incompressible flow for this analysis. Note that any fluid force on the bellmouth will be transmitted to the test stand and will show up as part of the measured "thrust" for the engine. In practice, a correction must therefore be made to the measured thrust to obtain the true engine thrust.

(Ans. 37,300 lb_f, 7660 lb_f)



5.2 Consider again the sluice gate in Problem 4.5. We want to determine the horizontal force needed to hold the gate in place, both with the gate closed and with the outflow examined in 4.5.

(a) Consider first the fluid force on a vertical rectangular surface exposed to a liquid on one side and the atmosphere on the other.



On the liquid side, the pressure on the vertical surface at any depth h (measured vertically downward) is

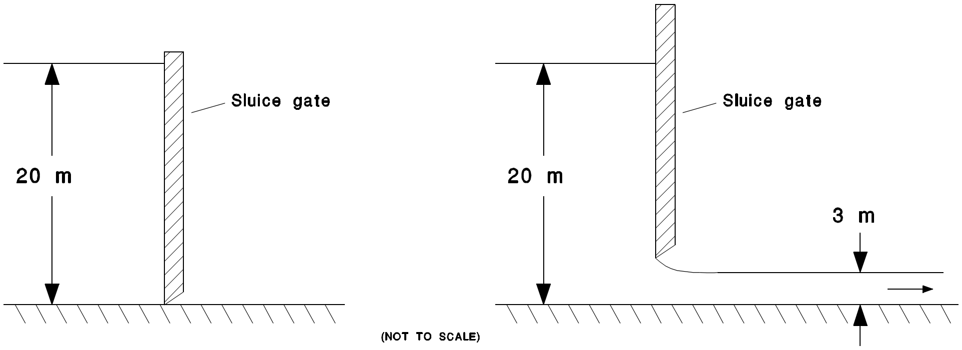
$$P = P_{atm} + \rho g h$$

where ρgh is known as the hydrostatic pressure. The contribution from P_{atm} results in a force on the liquid side of the surface is $P_{atm}A$, where A is the total area of the surface. However, P_{atm} also acts on the air side and exerts a force of $P_{atm}A$ there. Thus, there is no net force contribution in the horizontal direction due to the atmospheric pressure. Show that the resultant force from the triangular distribution of hydrostatic pressure can be calculated from

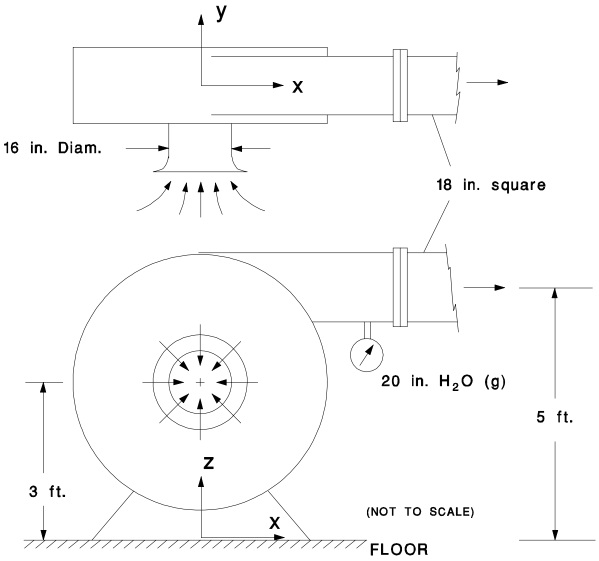
$$F_{hydrostatic} = \rho g \frac{H}{2} A$$

where ρgH/2 is the value of the hydrostatic pressure half way down the surface and A is again the total surface area. (N.B. This result is only valid for a rectangular surface).

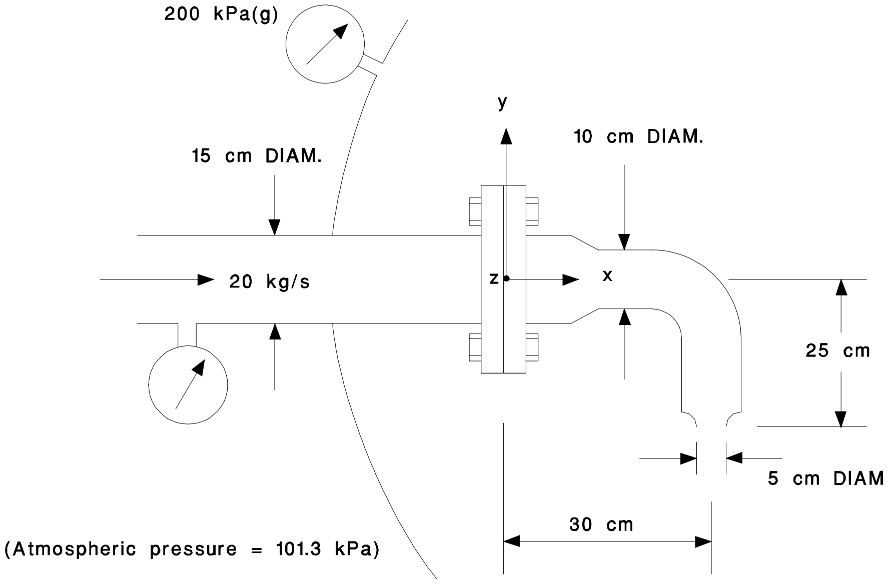
(b) Determine the net horizontal fluid force on the sluice gate for the two cases shown: (Ans. 1.96×10^7 N, 1.05×10^7 N)



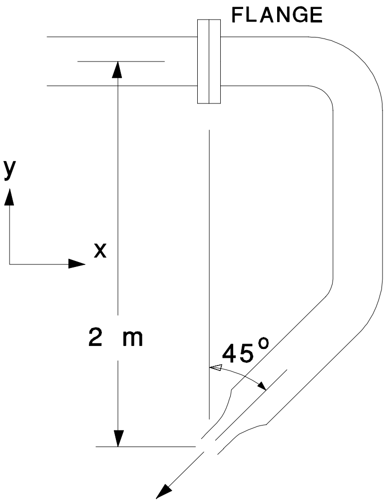
5.3 The drawing shows a centrifugal fan. It draws 12,000 cfm (ft^3/min) of air at 70^{o}F from the atmoshere and delivers it at a pressure of 20 inches of water (gauge) to an 18 in. square duct. The fan is connected to the downstream piping by a flexible coupling. Determine the forces in the x and y directions and the moment about the y axis which the mounts at the floor must apply to hold the fan in place.
(Ans. 275 lb$_f$, 0 lb$_f$, 1377 ft-lb$_f$)



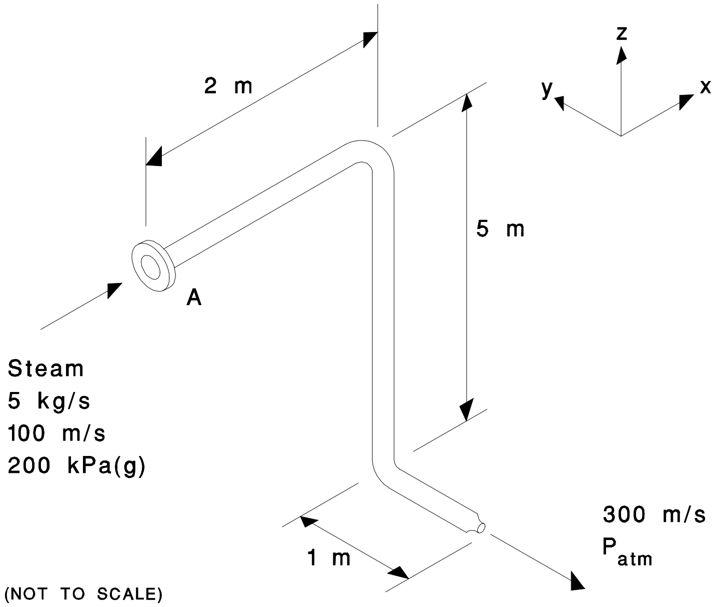
5.4 The drawing shows a nozzle which is used to inject a liquid (SG = 0.86) into a closed cylindrical tank containing a pressurized gas.
(a) What pressure will be shown by the gauge on the pipe upstream of the nozzle?
(b) Determine the forces in the x and y directions and the moment about the z axis at the flanged joint. The nozzle lies in the horizontal plane. Neglect the weight of the nozzle itself and of the liquid inside it.
(Ans. 361 kPa(a), F_X = -1079 N, F_Y = 237 N, M_Z = -71 Nm)



5.5 Water flows through the pipe shown at a rate of 250 litres/minute. It discharges to atmosphere at the nozzle outlet. The pipe has an inside diameter of 30 mm and the nozzle an inside diameter of 15 mm. The water pressure at the flage is 300 kPa(g). Determine the forces and moments the left flange exerts on the right flange. The pipe lies in the horizontal plane. Neglect the weight of the pipe itself and the water inside it.
(Ans. $F_X = 306$ N, $F_Y = 69.5$ N, $M_Z = -139$ Nm)



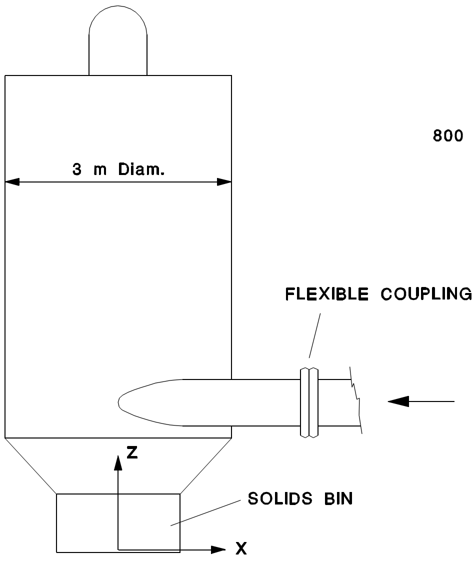
5.6 The pipe shown is connected to the upstream piping at flange A. Determine the forces and moments which the upstream piping must apply at flange A to hold the pipe in place. The pipe has an inside diameter of 150 mm, except at the nozzle outlet.
(Ans. $F_X = -4040$ N, $F_Y = -1500$ N, $F_Z = 0$ N (+ weight of the pipe and the steam inside),
$M_X = -7500$ Nm, $M_Y = 0$ Nm, $M_Z = -3000$ Nm)



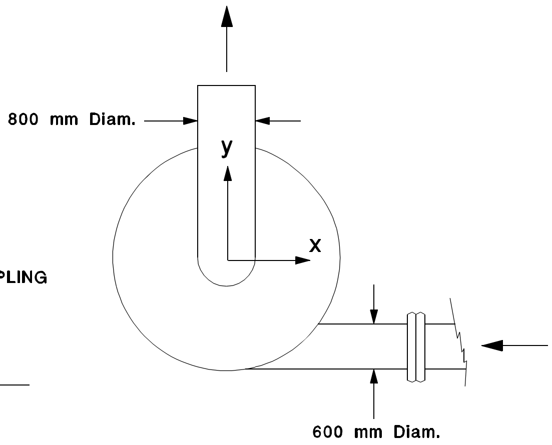
5.7 The drawing shows a cyclone separator, which is used to remove solid matter from an air stream by centrifugal action. The air enters tangentially at the outside and leaves through the duct at the top. As the air swirls up through the cylindrical tank the solid particles are slung against the walls and settle into the solids bin at the bottom.

The air enters at a pressure of 100 mm of H_2O (gauge) at 20°C at a flow rate of 1000 m^3/min. It leaves at atmospheric pressure and the same temperature. The solids collect at the rate of 6000 kg/hr. The solids enter at the same velocity as the air carrying them and they occupy negligible volume.

Find the horizontal forces and the moment about the vertical axis which must be applied to hold the cyclone separator in place.
(Ans. $F_X = 1571$ N, $F_Y = 679$ N, $M_Z = 1870$ Nm)



SIDE VIEW



TOP VIEW

(NOT TO SCALE)

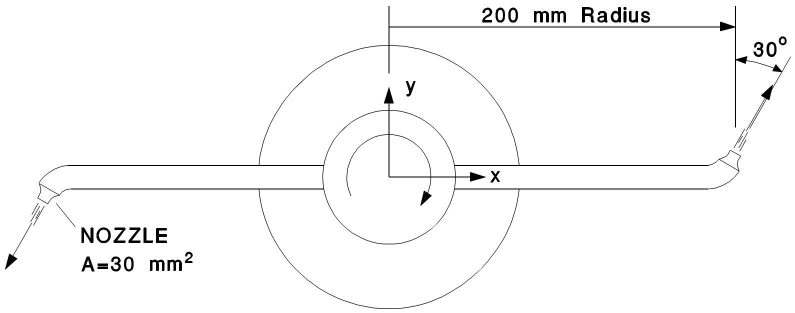
5.8 The drawing shows two views of a simple lawn sprinkler. The flow rate of water is 1 l/s and it enters the sprinkler with a velocity whose direction passes through the axis of rotation of the sprinkler head. The flow is divided evenly between the two outlet nozzles, each of which has an exit area of 30 mm^2. The nozzles are at a radius of 200 mm from the axis of rotation and their centrelines lie in the horizontal plane but at angle of 30^o from the tangential direction, as shown.

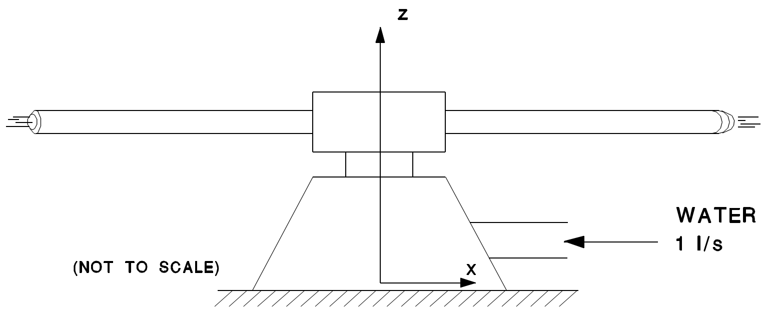
(a) Determine the moment which must be applied to hold the sprinkler head stationary.

(b) If the head is rotating at a steady 500 RPM, what resisting moment is being applied to the sprinkler head?

(c) If the head is allowed to rotate freely with no resisting moment, what is the rotational speed of the sprinkler in RPM?

(Ans. 2.89 Nm, 0.792 Nm, 689 RPM)





5.9 Figure 1 shows an axial-flow fan operating in a duct. The flow rate of air through the fan is 250 ft^3/sec. The changes in density of the air through the fan are negligible and the density is 0.077 lb$_m$/ft^3. The fan speed N is 1000 RPM. Perform the calculations for the mean radius of the fan, r$_m$, and assume that conditions there represent a reasonable average for all of the flow passing through the fan. That is, assume 1-D flow through the fan at the conditions which exist at the mean radius:

$$r_{hub} = 1.3\ ft. \qquad r_{tip} = 1.7\ ft. \qquad r_m = \frac{r_{hub} + r_{tip}}{2} = 1.5\ ft.$$

(a) Figure 2 shows a section through the rotor at the mean radius. The flow vectors shown are for the <u>absolute</u> fluid velocities. The flow enters the rotor with no tangential component of velocity (that is, zero swirl). The flow leaving the rotor has a tangential velocity component such that its swirl angle is 50°, as shown. Calculate the power needed to drive the rotor, neglecting friction.

(b) If the flow downstream of the stators again has zero swirl, what is the total moment applied by the fluid to the stators?
(Ans. 13.5 HP, 70.9 ft-lb$_f$)

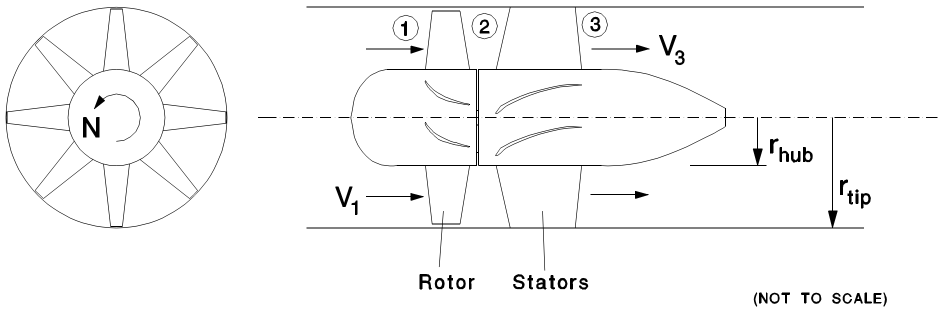


FIGURE 1

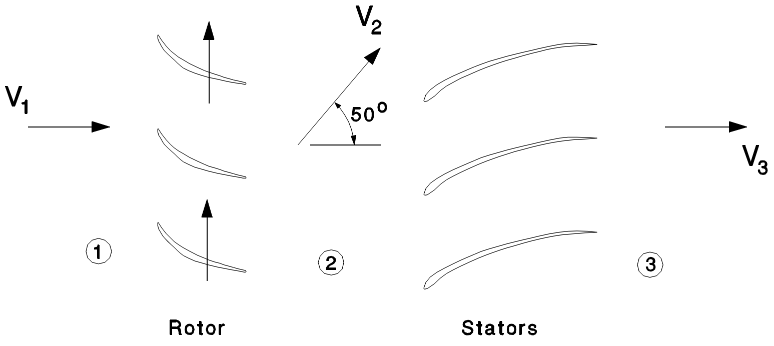


FIGURE 2

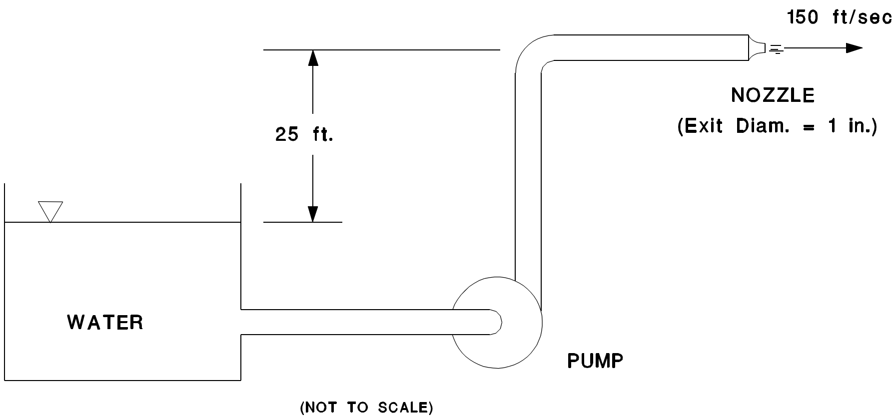
Problem Set 6
Steady Flow Energy Equation; Viscous Effects

6.1 Water is discharged from the nozzle shown with a velocity of 150 ft/sec. The efficiency of the pump is 70%. Neglect friction in the pipes and the air.

(a) What horsepower is required to drive the pump?

(b) If the water were to leave the nozzle in the vertically upward direction, what maximum height above the nozzle would it reach?

(Ans. 50 HP, 350 ft.)

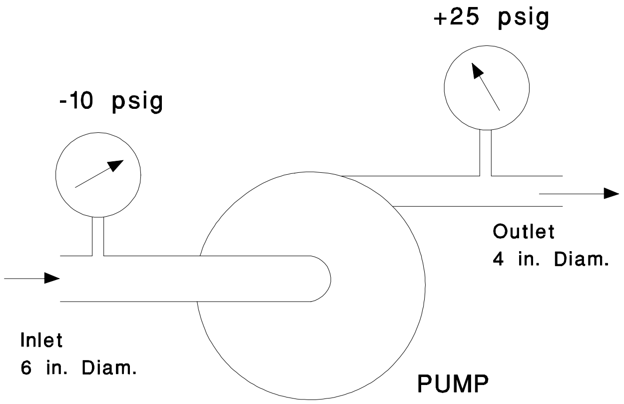


6.2 Water is being pumped at a flow rate 8 ft^3/sec by the pump shown in the drawing. The change in elevation between the inlet and outlet of the pump is negligible.

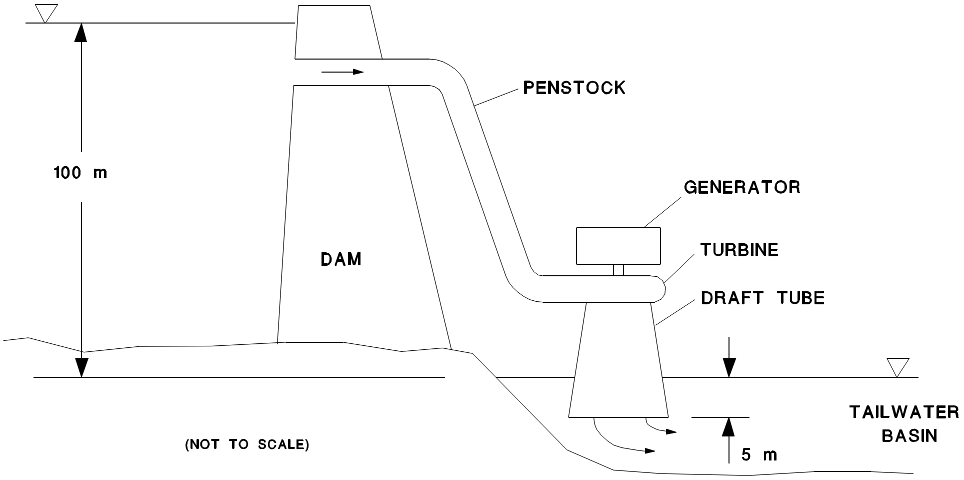
(a) What useful power does the pump transfer to the water?

(b) What power is required to drive the pump if its efficiency is 70%?

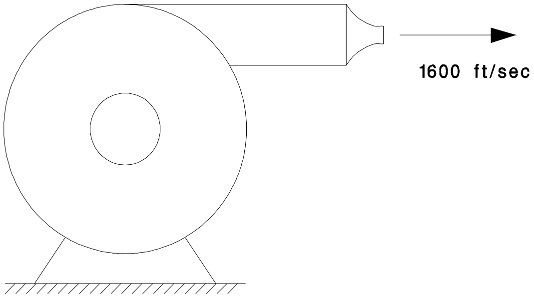
(Ans. 169 HP, 241 HP)



6.3 A large hydraulic turbine is supplied with 200 m^3/s of water from a reservoir which is created by a dam. The free surface of the reservoir is 100 m above that of the large basin to which the turbine discharges. The diameter of the discharge pipe (called the draft tube) is 7 m at its outlet. The outlet is at 5 m below the surface of the tailwater basin. The efficiency of the turbine is 94%. Neglect the losses in the penstock and the draft tube. What is the shaft power output of the turbine?
(Ans. 182 MW)



6.4 An air compressor takes 12,000 ft^3/min of atmospheric air (at 70°F and 14.7 psia) and delivers it at a pressure of 30 psig and a temperature of 315°F. The air has very low speed at the inlet to the compressor and in the pipe immediately downstream of the compressor. The air is then accelerated to a speed of 1600 ft/sec in a convergent nozzle. There is no heat transfer during both the compression and acceleration processes.

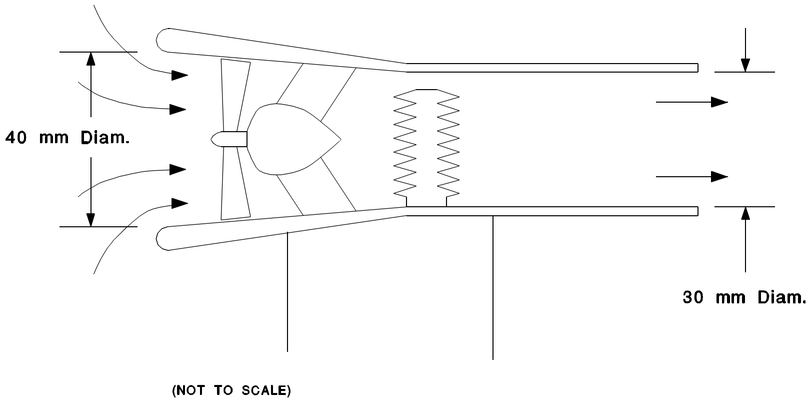


(a) What power is transferred to the air by the compressor?
(b) What is the temperature of the air leaving the nozzle?
(Ans. 1245 HP, 102°F)

Notes: $\Delta u = C_v \Delta T$ where u is the internal energy
$C_v = 0.17$ $BTU/lb_m{}^oR$ for air
$\rho = P/RT$ where ρ is the density, T is the absolute temperature and R is the gas constant, $R = 53.3$ $ft\text{-}lf_f/lb_m{}^oR$ for air

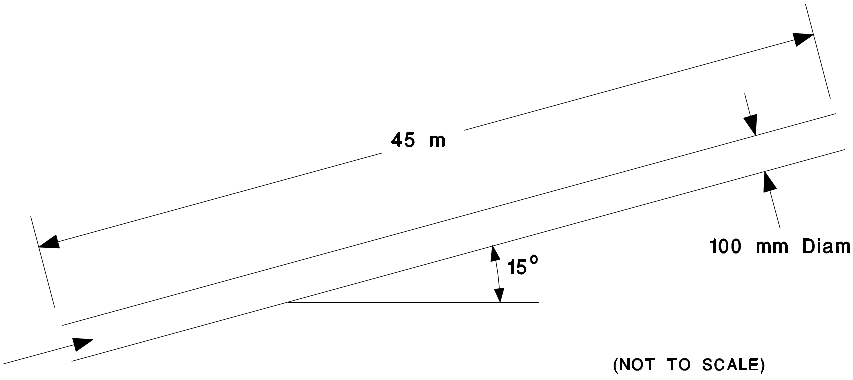
6.5 A hair drier has a 100 W motor driving the fan and a 1 kW heater. The flow rate of room air (101.3 kPa, 25°C) into the drier is 60 m^3/hr. What is the temperature of the air at the outlet? Suggestion: Ignore the kinetic energy of the air at inlet and outlet initially to get the density at these locations. Iterate if necessary to obtain the final answer.
(Ans. 80.5°C)

Data: C_{pair} = 1 kJ/kgK
R_{air} = 287 N-m/kgK



6.6 At the LaGrange 2 site at the James Bay hydroelectric power project the penstock supplying each turbine has a length L of 585 ft. and a diameter D of 26 ft. The difference in elevation between the free surface of the upstream reservoir and free surface of the tailwater basin is 450 ft. (see Problem 6.3 for the terminology used here). The maximum flow rate through each turbine is 10,000 ft3/sec. Assume that the turbine has an efficiency of 94% and that the head loss in the penstock is $0.016H_{DYN}L/D$, where H_{DYN} is the dynamic head of the water in the penstock. Assume also that the efficiency accounts for the kinetic energy of the water as it leaves the draft tube. What is the maximum shaft power delivered by the turbine?
(Ans. 478,000 HP)

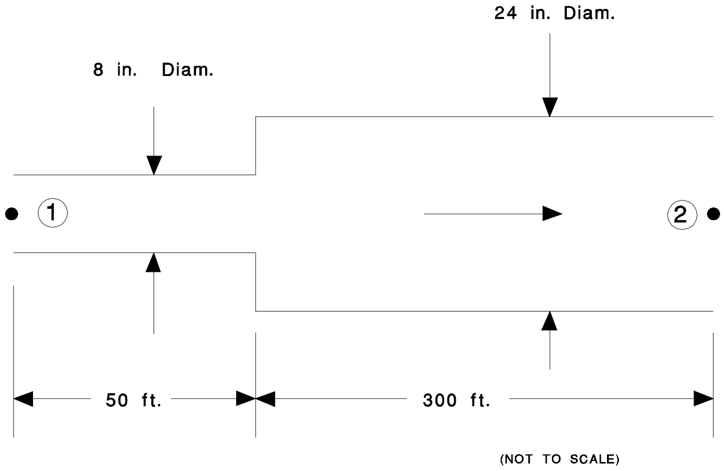
6.7 Glycerine (S.G. = 1.26, μ = 0.9 Pa s) is pumped at 20 l/s through a smooth, straight 100 mm diameter pipe which is 45 m long and inclined at 15° to the horizontal. The gauge pressure at the lower, inlet end of the pipe is 590 kPa(g). Is the flow laminar or turbulent? Calculate the pressure at the outlet end of the pipe and the average shear stress at the wall. The pipe is smooth.
(Ans. Laminar, 117 kPa(g), 183 Pa)



6.8 Oil with a kinematic viscosity v = 0.0002 ft^2/sec is to be pumped along a pipeline at 8.0 ft^3/sec. To keep the pumping power at a reasonable level, it has been decided that the frictional head loss, expressed as gh_f, must be kept below 0.4 ft-lb_f/lb_m per 100 ft. of pipe length. Select the smallest diameter of wrought iron pipe which would be suitable for this application. Assume that pipe is only available in diameters with 0.1 ft. increments (eg. D = 0.4, 0.5, 0.6 ft. etc.)
(Ans. 1.6 ft.)

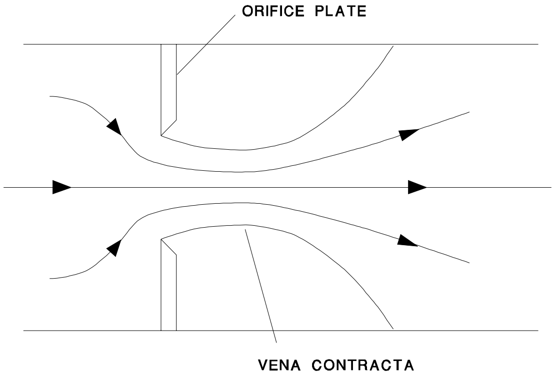
6.9 A rectangular ventilation duct has a height of 0.4 m and a width of 0.6 m. It passes 200 m^3/hr of air with a typical pressure of 101 kPa. During the heating season the air has a temperature of 35°C (μ = 1.88×10^{-5} kg/m-s) and during the air-conditioning season it is at 12°C (μ = 1.77×10^{-5} kg/m-s). Determine the frictional pressure drop in Pa which will occur over 100 m length of duct under each of these conditions. The duct is made of galvanized iron.
(Ans. 0.223 Pa, 0.231 Pa)

6.10 A pipeline consists of two sections of different diameters with an abrupt change in cross-section at the junction, as shown in the drawing. The pipe carries water at 60°F ($\rho = 62.4\ lb_m/ft^3$, $\mu = 23.7\text{x}10^{-6}\ lb_f sec/ft^2$ and the average velocity in the smaller diameter pipe is 18 ft/sec. Determine the head loss between stations 1 and 2. Both pipes are smooth.
(Ans. 8.5 ft. of water)

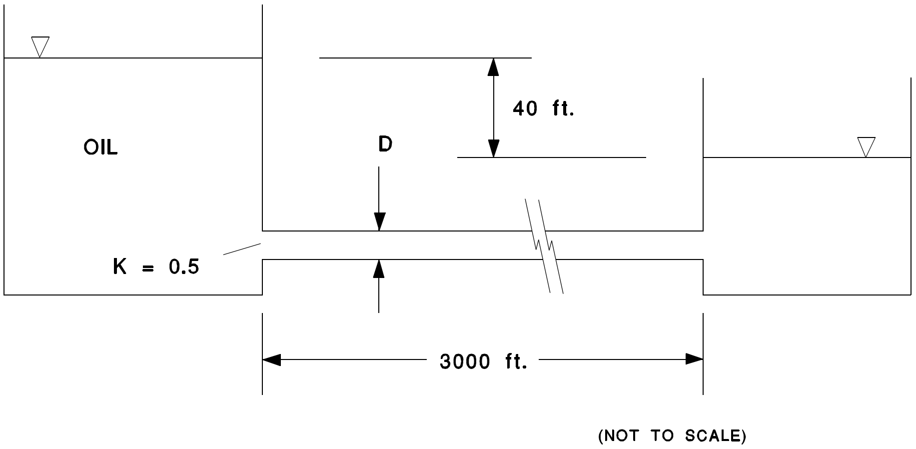


6.11 A flow rate of 1 ft^3/sec of water having a viscosity of $2.11\text{x}10^{-5}\ lb_f\text{-}sec/ft^2$ is being pumped through a 3 in. diameter pipe of length 200 ft. The intake pressure is atmospheric (14.7 psia) and the water is to be delivered at the outlet at a pressure of 20 psia. The pipe outlet is at the same elevation as the inlet. The pipe has an average roughness height of 0.006 in. Assume that the only source of losses is friction in the pipeline. What power must be transferred to the fluid?
(Ans. 15.4 HP)

6.12 Repeat 6.11 with the following changes. The pipeline has two standard 90° elbows (K = 0.54) and an orifice-plate flow meter. The drawing shows the flow through the orifice plate. The orifice has a diameter of 2 in. and has a sharp edge. Since the fluid cannot turn a sharp corner, the jet emerging from the orifice continues to contract for some distance downstream. The minimum-area section of the jet is known as the vena contracta. The ratio C_c = cross-sectional area at vena contracta/orifice area is known at the contraction coefficient and it typically has a value of about 0.6. To obtain the head loss through the orifice plate assume that it is the same as if the water experienced a sudden increase in area from the area at the vena contracta to the downstream pipe area. The pump has an efficiency of 70%. What shaft power must be supplied to the pump?
(Ans. 23.7 HP)



6.13 What diameter of wrought iron pipe is required to convey oil (S.G. = 0.9, $\mu = 0.0008\ lb_f\text{-sec/ft}^2$) at a rate of 1.0 ft^3/sec between the two tanks shown in the drawing? The pipeline is 3000 ft. long and the difference in elevation between the free surfaces is 40 ft. The pipe inlet is sharp-edged and has a loss coefficient K = 0.5. Assume that the pipe is available only in diameter increments of 0.1 ft. (eg. D = 0.2, 0.3, 0.4 ft. etc.).
(Ans. 0.6 ft.)

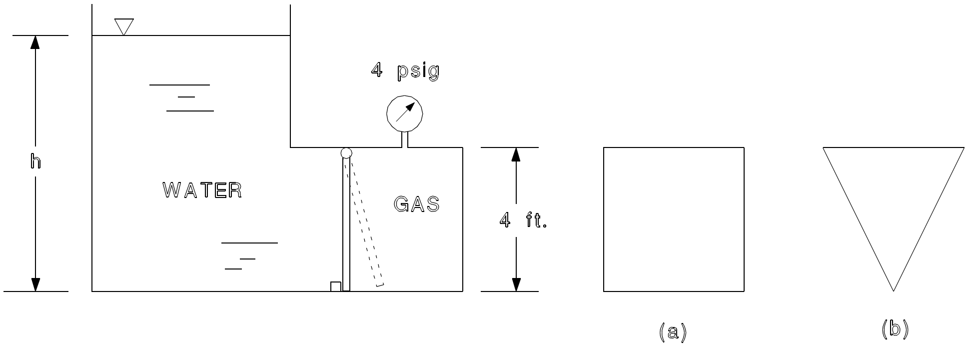


6.14 A swimming pool of 100 m^3 capacity is to be filled using 50 m of 2 cm diameter rubber hose. The hose is connected to a water main in which the pressure is 400 kPa(g). Assume that the roughness of the hose is similar to that of drawn tubing. The hose empties the water (at 15°C, $\mu = 1.136\text{x}10^{-3}$ kg/m-s) at the top of the pool and there is negligible change in elevation between the inlet and outlet of the hose. Assume an atmospheric pressure of 101 kPa(a). Estimate the time required to fill the pool.
(Ans. About 22 hrs.)

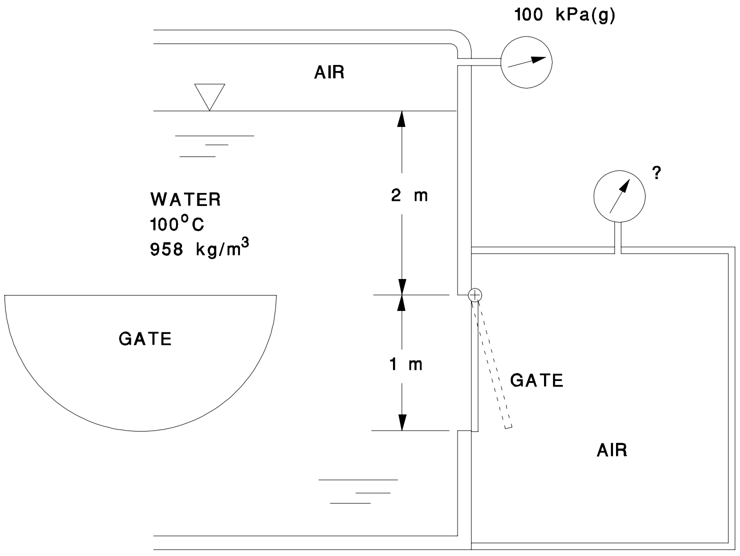
Problem Set 7
Fluid Statics

7.1 Find the minimum value of h for which the gate in the drawing will rotate counterclockwise if the gate is:
- (a) Rectangular, 4 ft. x 4 ft.
- (b) Triangular, with a 4 ft. base a 4 ft. height.

Neglect friction in the hinge.
(Ans. 10.6 ft., 11.2 ft.)

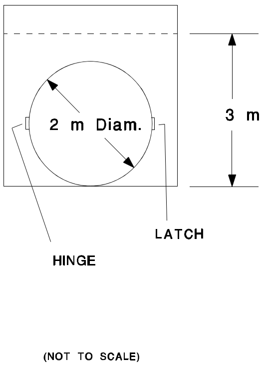


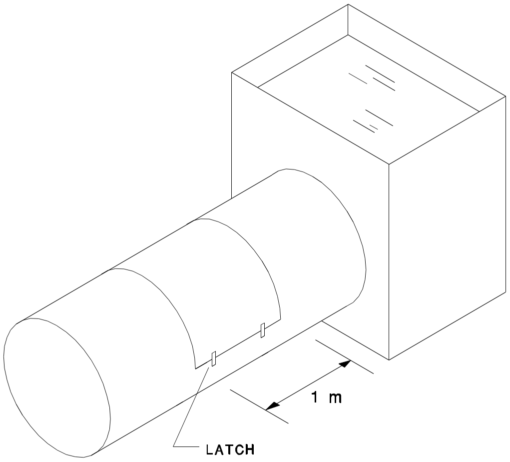
7.2 Determine the air pressure needed to hold the semi-circular gate closed in the emergency dump system shown in the drawing.
(Ans. 124.3 kPa(g))



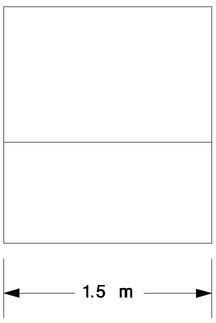
(NOT TO SCALE)

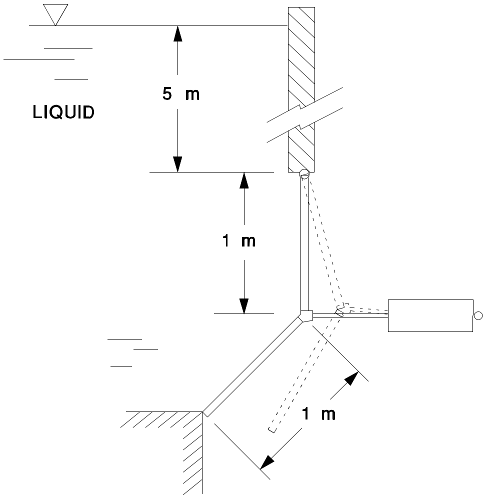
7.3 A mixing tank consists of a horizontal cylindrical drum of 2 m diameter with a tall rectangular hopper at one end for filling. An access hatch on the drum consists of a semi-cylindrical section, 1 m long, at the top of the drum. The hatch is hinged on one side and has latches on the other. The mixture has a specific gravity of 1.5 and is loaded to a depth of 3 m in the hopper. The top of the hopper is open to atmosphere. Find the total vertical force on the access hatch and the total force in the two latches.
(Ans. 35.8 kN, 17.9 kN)



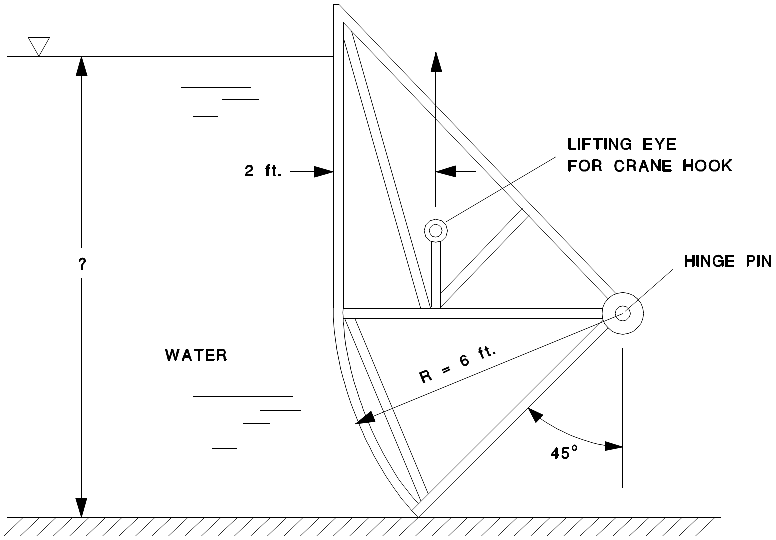


7.4 The drawing shows the actuating mechanism of a dump door on a storage tank. The door consists of two steel plates, 1 m x 1.5 m, welded together at a 45^o angle along the 1.5 m side, as shown. The door is hinged at the top and is operated by a hydraulic jack pushing on the joint between the plates. What force must the jack exert to hold the door shut? The liquid in the tank has a specific weight $\gamma = 8$ kN/m^3.
(Ans. 127 kN)





7.5 A canal gate is shown in the drawing. The gate is designed to maintain a constant depth of water in the canal by rotating open automatically under the action of the hydrostatic forces on it when the depth exceeds some value. The canal is 30 ft. wide and the gate spans the full width of the canal. When the canal is empty it requires a force of 30,000 lb_f, applied at the lifting eye as shown, to rotate the gate open. At what depth of water will the gate start to open automatically?
(Ans. 11.5 ft.)



7.6 The hydraulic gate shown in the drawing is known as a Tainter gate.

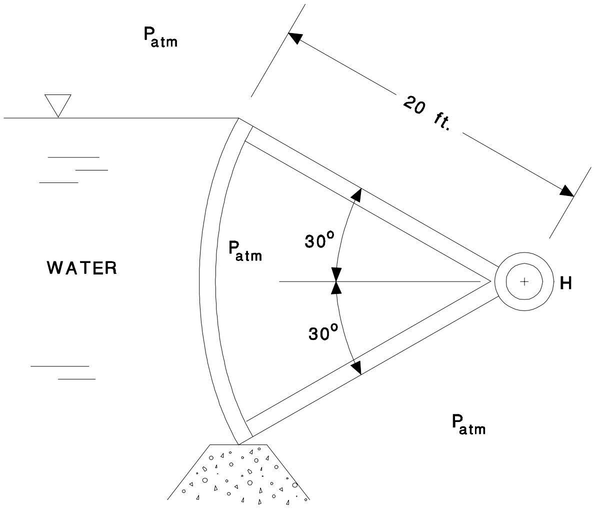
(a) Find the horizontal component of the hydraulic force acting on 1 ft. width of the gate.

(b) Find the vertical component of the hydraulic force acting on 1 ft. width of the gate and indicate its direction.

(c) Find the moment about the hinge H due to the hydraulic forces for 1 ft. width of the gate.

(d) Explain from first principles why the moment in (c) had to be zero.

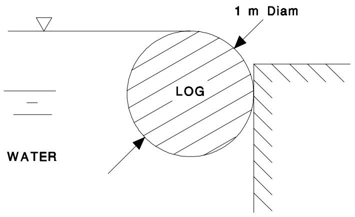
(Ans. 12,480 lb_f, 2260 lb_f upward, 0)



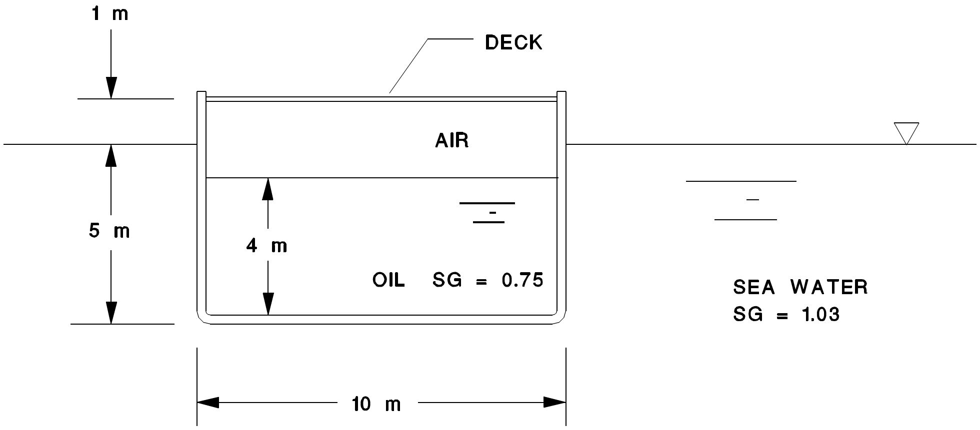
7.7 A cylindrical steel silo (storage tank) 20 m high and 6 m in diameter holds silage with S.G. = 1.5. If the steel of the tank has an allowable working stress level of 100 Mpa, what thickness would you recommend for the top, middle and bottom rings if the tank is fabricated from rings of steel 2 m high. Use some some engineering judgment in selecting the thicknesses, in addition to looking at the strict minimum allowable thicknesses obtained from the calculations.
(Ans. 2-3 mm, 5 mm, 10 mm (minimum 8.82 mm))

7.8 A balloon has a volume of 4000 m^3. If it is at an altitude of 2000 m, what weight can it lift if it is filled with:
(i) hydrogen at the same temperature as the surrounding air?
(ii) helium at the same temperature as the surrounding air?
(iii) hot air, at 40°C?
Note that since the balloon material is very flexible, the gas inside it must be at essentially the same pressure as the surrounding air. The following values of the gas constant may be used: for H_2, 4125 J/kgK; for He, 2077 J/kgK; for air, 287 J/kgK.
(Ans. 36.7 kN, 34 kN, 4.8 kN)

7.9 Ignoring the friction between the cylindrical log and the wall, find the density of the log. What is the force between the log and the wall, per unit length?
(Ans. 1068 kg/m^3, 1226 N/m)

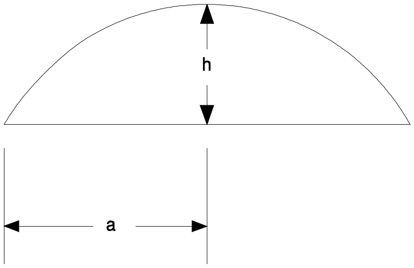


7.10 A filled oil barge is shown in cross-section. The air space above the oil is at atmospheric prressure. How deep will the barge float when it is empty? Assume that the top deck is used to hold the sides of the barge together or apart at the top. Assume also that the lower corners of the hull cannot transmit a moment (ie. they effectively act as hinges). What is the force in the top deck per unit length of the barge (that is, in the direction normal to the page) for the conditions shown in the diagram?
(Ans. 2.09 m, 22 kN/m compression)



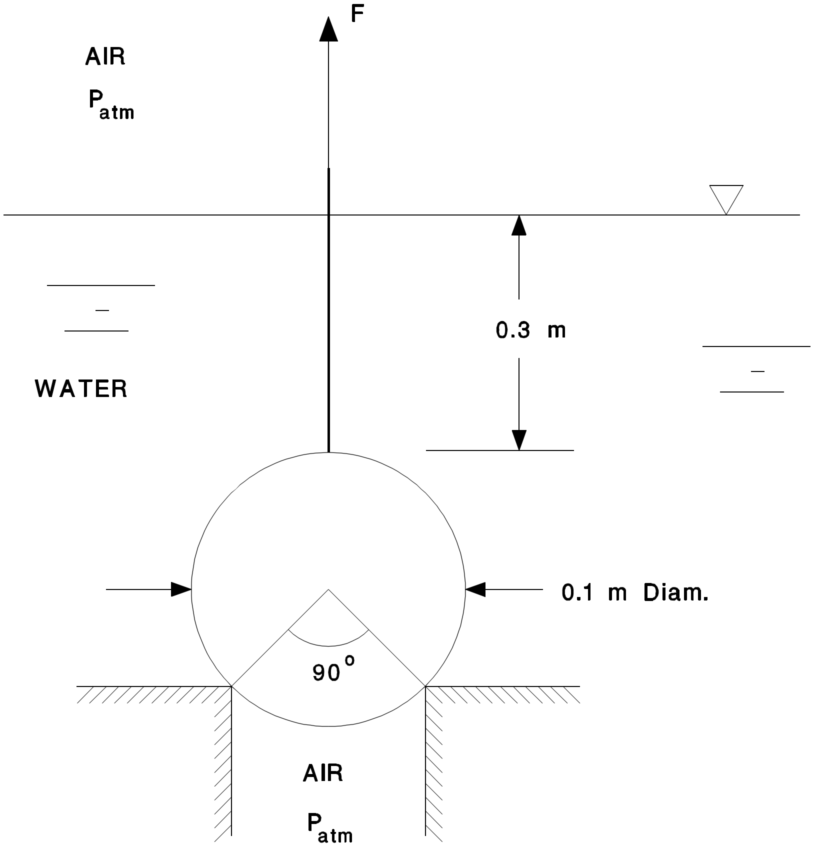
7.11 The drawing shows an idealized ball valve. What force F must be applied to lift the ball off its seat if the ball has negligible weight?
(Ans. 10 N)

Volume of a sphere: $V = \dfrac{4\pi r^3}{3}$



For the spherical cap: $a^2 = h(2r - h)$

and thus the volume: $V = \dfrac{\pi}{6} h\left(3a^2 + h^2\right) = \dfrac{\pi}{3} h^2 (3r - h)$



APPENDIX A

Data and Formulae

Units, Conversion Factors and Physical Constants

	English System	S.I.
length	foot (ft)	meter (m)
mass	lb_m or slug 1 slug = 32.174 lb_m	kilogram (kg)
time	second (sec)	second (s)
temperature	°R (Rankine) °R = °F + 460	K (Kelvin) K = °C + 273.15
force	lb_f	N (Newton)
pressure	lb_f /in^2 (psi) lb_f /ft^2 (psf)	Pa (Pascal) 1 Pa = 1 N/m^2
work, energy	ft-lb_f or BTU 1 BTU = 778 ft-lb_f	J (Joule) 1 J = 1 Nm
power	ft-lb_f /sec, BTU/sec, HP 1 HP = 550 ft lb_f /sec	W (Watt) 1 W = 1 J/s

Conversion Factors

1 ft = 0.3048 m

1 lb$_f$ = 4.4482 N

1 ft-lb$_f$ = 1.3558 J

1 Imp. gal. = 4.546 litre

1 °R = 1.8 K

1 BTU = 1055 J

1 lb$_m$ = 0.4536 kg

1 psi = 6894.8 Pa

1 HP = 745.7 W

Physical Constants

$g = 9.81\ m/s^2 = 32.174\ ft/sec^2$

Water at 20°C:

$\mu = 10^{-3}$ N sm^2 = 2.1 x 10^5 lb_f sec/ft^2
$\rho = 1000$ kg/m^3 = 62.4 lb_m/ft^3
C_p = 4.187 kJ/kg K = 1 BTU/lb_m °R

Air at Standard Sea-Level Conditions:

$\mu = 1.7 \times 10^{-5}$ N s/m^2 = 3.8 x 10^{-7} lb_f sec/ft^2
$\rho = 1.23$ kg/m^3 = 0.00238 slug/ft^3
C_p = 1.005 kJ/kg K = 0.24 BTU/lb_m °R
$\gamma = C_p/C_v = 1.4$

Perfect gas constant: R = 287 N m/kgK = 53.3 ft-lb_f/lb_m °R

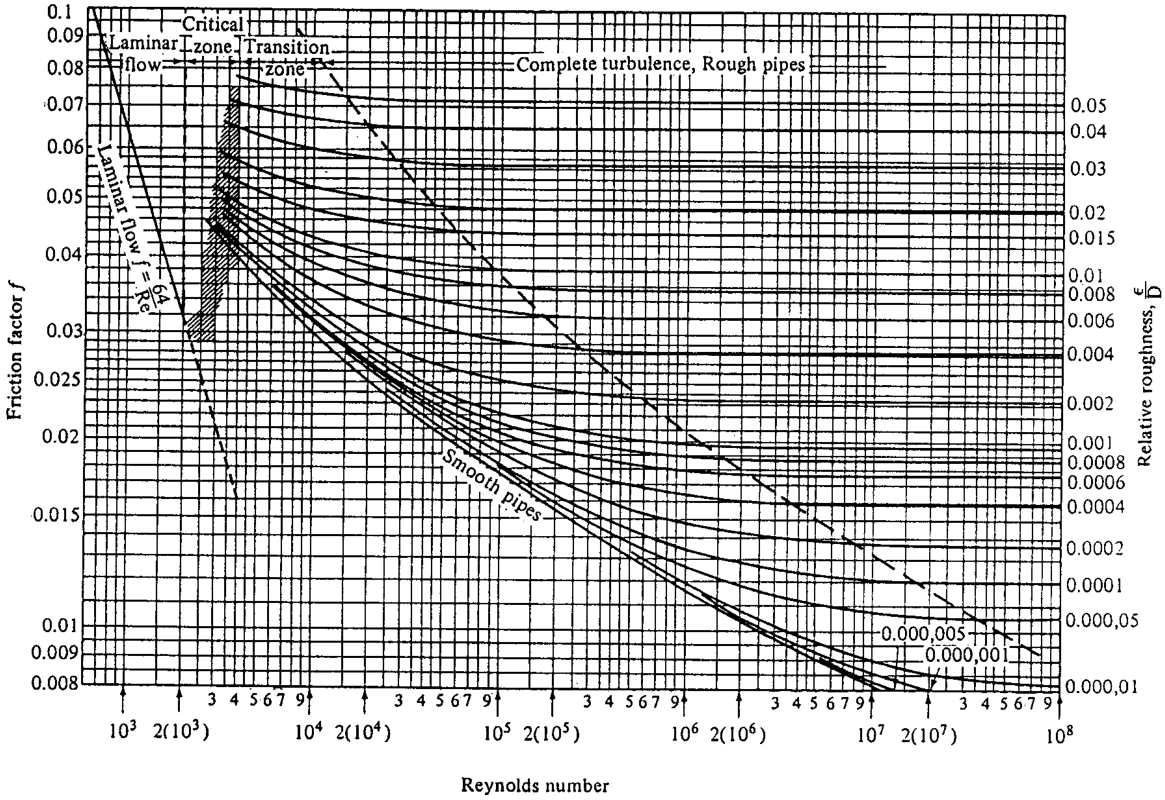
Standard atmospheric pressure: P_{atm} = 101.325 kPa = 14.7 psia

Standard Atmosphere (British Units)

Altitude (ft)	Temperature, T (^oR)	Pressure, P (lb_f/ft^2)	Density, ρ ($slugs/ft^3$)
0	518.69	2116.2	0.0023769
1000	515.12	2040.9	0.0023081
2000	511.56	1967.7	0.0022409
3000	507.99	1896.7	0.0021752
4000	504.43	1827.7	0.0021110
5000	500.86	1760.9	0.0020482
6000	497.30	1696.0	0.0019869
7000	493.73	1633.1	0.0019270
8000	490.17	1572.1	0.0018685
9000	486.61	1512.9	0.0018113
10000	483.04	1455.6	0.0017556
12000	475.92	1346.2	0.0016480
14000	468.80	1243.6	0.0015455
16000	461.67	1147.5	0.0014480
18000	454.55	1057.5	0.0013553
20000	447.43	973.27	0.0012673
22000	440.32	894.59	0.0011836
24000	433.20	821.16	0.0011043
26000	426.08	752.71	0.0010292
28000	418.97	688.96	0.00095801
30000	411.86	629.66	0.00089068
32000	404.75	574.58	0.00082704
34000	397.64	523.47	0.00076696
36000	390.53	476.12	0.00071028
38000	389.99	432.63	0.00064629
40000	389.99	393.12	0.00058727
50000	389.99	243.61	0.00036391
60000	389.99	151.03	0.00022561
70000	389.99	93.672	0.00013993
80000	389.99	58.125	0.000086831

Standard Atmosphere (SI Units)

Altitude (m)	Temperature, T (K)	Pressure, P (kPa)	Density, ρ (kg/m^3)
0	288.16	101.325	1.2250
500	284.91	95.461	1.1673
1000	281.66	89.876	1.1117
1500	278.41	84.560	1.0581
2000	275.16	79.510	1.0066
2500	271.92	74.692	0.95696
3000	268.67	70.121	0.90926
3500	265.42	65.780	0.86341
4000	262.18	61.660	0.81935
4500	258.93	57.752	0.77704
5000	255.69	54.048	0.73643
6000	249.20	47.217	0.66011
7000	242.71	41.105	0.59002
8000	236.23	35.651	0.52578
9000	229.74	30.800	0.46706
10000	223.26	26.500	0.41351
11000	216.78	22.700	0.36480
12000	216.66	19.399	0.31194
13000	216.66	16.579	0.26659
14000	216.66	14.170	0.22785
15000	216.66	12.112	0.19475
16000	216.66	10.353	0.16647
17000	216.66	8.8496	0.14230
18000	216.66	7.5652	0.12165
19000	216.66	6.4674	0.10399
20000	216.66	5.5293	0.088909
25000	216.66	2.5273	0.040639
30000	231.24	1.1855	0.017861
40000	260.91	0.29972	0.0040028
50000	282.66	0.087858	0.0010829



Moody Diagram - Frictional Head Loss in Fully-Developed Pipe Flow

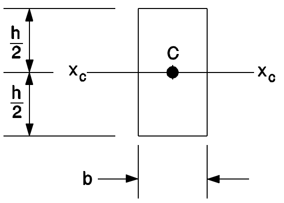
For turbulent flow, the diagram was plotted for an earlier empirical formula developed by Colebrook:

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{where} \qquad f = \frac{h_f}{\frac{L}{D}\frac{V^2}{2g}}$$

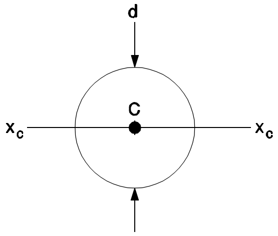
Roughness Heights for Some Common Pipe Materials

Material	Roughness Height, ϵ	
	ft.	mm
Rivetted steel	0.003 - 0.03	0.9 - 9.0
Concrete	0.001 - 0.01	0.3 - 3.0
Wood stave	0.0006 - 0.003	0.18 - 0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015
Glass, Plastic	Smooth	Smooth

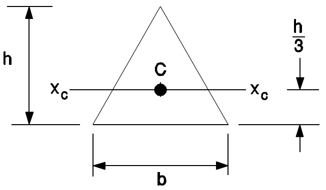
Second Moments of Area for Some Common Shapes



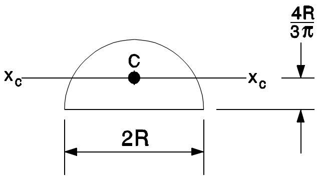
$$I_{x_c x_c} = \frac{bh^3}{12}$$



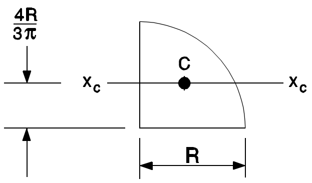
$$I_{x_c x_c} = \frac{\pi d^4}{64}$$



$$I_{x_c x_c} = \frac{bh^3}{36}$$



$$I_{x_c x_c} = 0.1098R^4$$

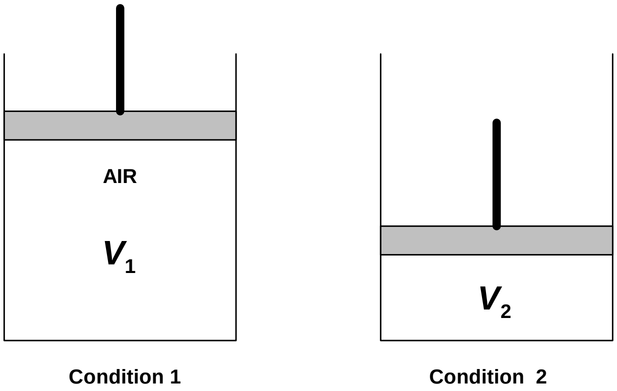


$$I_{x_c x_c} = 0.05488R^4$$

APPENDIX B
Solutions to Selected Problems

SOLUTIONS TO SELECTED PROBLEMS

1.9 Change in pressure due to change in volume:



$P_{atm} := 14.7$ psia

Given: Initially (condition 1) $P_{1g} := 20$ psig

$P_{1a} := P_{1g} + P_{atm}$ $P_{1a} = 34.7$ psia

After compression $V_2 = \frac{V_1}{3}$ (V = volume)

$T_2 = T_1$ (isothermal process specified)

At both conditions, the pressure, temperature and density on the air in the cylinder are related through the Perfect Gas Law

eg. at condition 1 $P_1 = \rho \cdot R \cdot T_1 = \frac{m}{V_1} \cdot R \cdot T_1$ (m = mass of trapped air)

Since the same mass of air is present in the cylinder at the two conditions, we can write

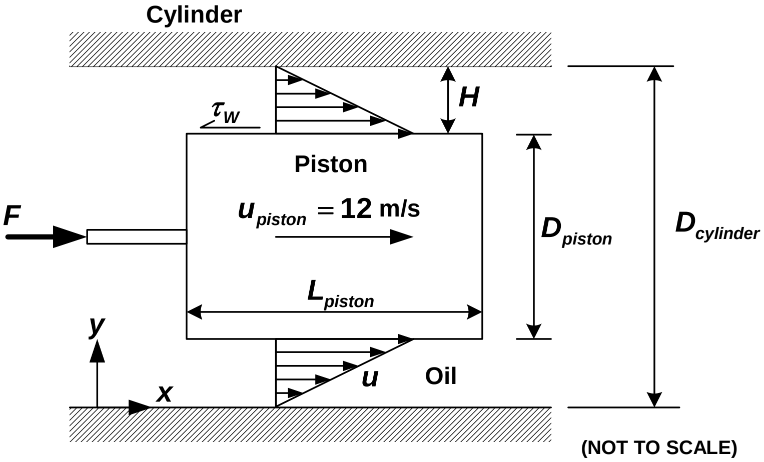
$$P_2 = \frac{m}{V_2} \cdot R \cdot T_2 = \frac{m}{\left(\frac{V_1}{3}\right)} \cdot R \cdot T_1 = 3 \cdot \left(\frac{m}{V_1} \cdot R \cdot T_1\right) = 3 \cdot P_1$$

thus $P_{2a} := 3 \cdot P_{1a}$ $P_{2a} = 104.1$ psia

or $P_{2a} - P_{atm} = 89.4$ psig

Note: (1) In this course, whenever you need the density of a gas and it is not specified, you should always look for information which allows it to be calculated from the Perfect Gas Law. Only if there is insufficient information to calculate it should an approximate value (such as the density at sea level on a standard day) be used.
(2) N.B. The P and T used in the Perfect Gas Law must always be <u>absolute</u> values.

1.12 Force to move piston in cylinder



$D_{piston} := \frac{5.000}{100}$ $D_{piston} = 0.05$ m $D_{cylinder} := \frac{5.010}{100}$ $D_{cylinder} = 0.0501$ m

Then gap width $H := \frac{1}{2}\cdot\left(D_{cylinder} - D_{piston}\right)$ $H = 0.00005$ m

$L_{piston} := \frac{5}{100}$ $L_{piston} = 0.05$ m

The piston feels a resisting force because of the fluid friction at its surface. The shear stress (tangential force per unit area) at any plane in a viscous (Newtonian) fluid is given by

$\tau = \mu\cdot\frac{du}{dy}$ here $\mu := 2\cdot10^{-3}$ Ns/m2 (given)

or at the surface of the piston $\tau_w = \mu\cdot\left(\frac{du}{dy}\right)$ with du/dy evaluated at the wall

The velocity is assumed to vary linearly between the piston and cylinder walls. If the gap height H is small compared to the length of the piston L_{piston} and the diameter (D_{piston} or $D_{cylinder}$), then the exact solution for this flow does in fact give a velocity that varies in a straight line across the gap. Thus, the slope of the velocity distribution du/dy is the same everywhere in the gap and is easily calculated since we know the velocities at the two walls from the no-slip condition:

$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{u_{piston} - u_{cylinder}}{H}$$

where $u_{piston} := 12$ m/s $u_{cylinder} := 0$ m/s

Thus $\tau_w := \mu\cdot\frac{u_{piston} - u_{cylinder}}{H}$ $\tau_w = 480$ N/m2

This stress is applied to the cylindrical surface of the piston

$$A := 2\cdot\pi\cdot\frac{D_{piston}}{2}\cdot L_{piston} \qquad A = 7.853982\times10^{-3} \quad \text{m}^2$$

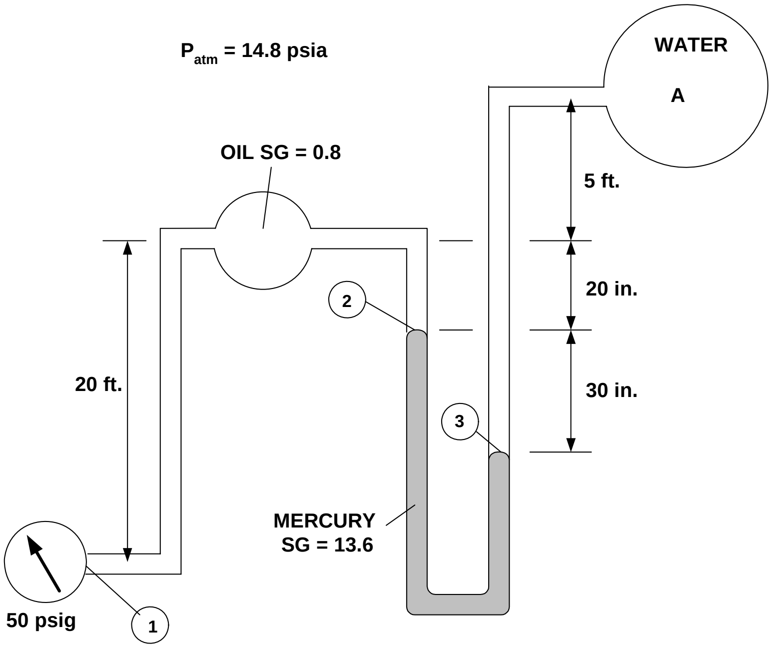
Then the total shear force is

$$F_S := \tau_w\cdot A \qquad F_S = 3.77 \quad \text{N}$$

This is the frictional force resisting the motion which arises when the cylinder moving at 12 m/s. Thus to keep it moving at this velocity we must supply an equal and opposite force:

$$F := F_S \qquad F = 3.77 \quad \text{N}$$

2.4 Determining the pressure in the water pipe at A. Express the pressure in psia.



$P_{atm} := 14.8$ psia

By definition

$$SG = \frac{\rho}{\rho_{water}}$$

As recommended in lectures, when working in british units begin by converting all masses from lb_m to slugs:

By definition $\quad slug = \dfrac{lb_m}{32.174}$ $\qquad$ For water $\quad \rho_w := 62.4$ lb_m/ft^3 (given)

then $\quad \rho_w := \dfrac{\rho_w}{32.174}$ $\qquad \rho_w = 1.939$ $slug/ft^3$

and for oil $\quad SG_{oil} := 0.8$ $\qquad \rho_{oil} := SG_{oil} \cdot \rho_w$ $\qquad \rho_{oil} = 1.552$ $slug/ft^3$

for mercury $\quad SG_{Hg} := 13.6$ $\qquad \rho_{Hg} := SG_{Hg} \cdot \rho_w$ $\qquad \rho_{Hg} = 26.377$ $slug/ft^3$

The hydrostatic pressure equation derived in lectures only applies in a continuous fluid of constant density. At the interface between two stationary fluids in contact, they exert the same pressure on each other. We can then work our way through system one fluid at a time.

The pressure at 1 is known: $\quad P_{1g} := 50$ psig

then $\quad P_{1a} := P_{1g} + P_{atm}$ $\qquad P_{1a} = 64.8$ psia

We must be careful to work in compatible units. In the British system it is very common to quote pressures in lb_f/in^2 (psi). To be compatible with the units of the other quantities, we must convert this from psi to psf (lb_f/ft^2), using the fact that 1 ft = 12 in.:

then $\quad P_{1a} := P_{1a} \cdot 12^2$ $\qquad P_{1a} = 9331$ psfa

Then applying the hydrostatic pressure from 1 to 2:

$P_1 + \rho_{oil} \cdot g \cdot y_1 = P_2 + \rho_{oil} \cdot g \cdot y_2$ (1) $g := 32.174$ ft/sec^2

Once we have chosen a datum for y, we can use (1) to calculate the pressure at point 2. Equivalently we could have written "Pressure at 2 is the pressure at 1 minus the change in pressure in going upward from the elevation at 1 to the in elevation at 2":

$P_2 = P_1 - \rho_{oil} \cdot g \cdot \Delta y_{12}$ $\Delta y_{12} := 20 - \frac{20}{12}$ $\Delta y_{12} = 18.333$ ft

thus $P_{2a} := P_{1a} - \rho_{oil} \cdot g \cdot \Delta y_{12}$ $P_{2a} = 8416$ psfa

Through the mercury

$P_3 = P_2 + \rho_{Hg} \cdot g \cdot \Delta y_{23}$ (positive since level 3 is lower than level 2 and the pressure will therefore increase)

$\Delta y_{23} := \frac{30}{12}$ $\Delta y_{23} = 2.5$ ft

$P_{3a} := P_{2a} + \rho_{Hg} \cdot g \cdot \Delta y_{23}$ $P_{3a} = 10538$ psfa

and finally through the water from 3 to A $\Delta y_{3A} := \frac{30}{12} + \frac{20}{12} + 5$ $\Delta y_{3A} = 9.167$ ft

$P_{Aa} := P_{3a} - \rho_w \cdot g \cdot \Delta y_{3A}$ $P_{Aa} = 9966$ psfa

We were asked to quote the pressure at A in psia. Therefore converting

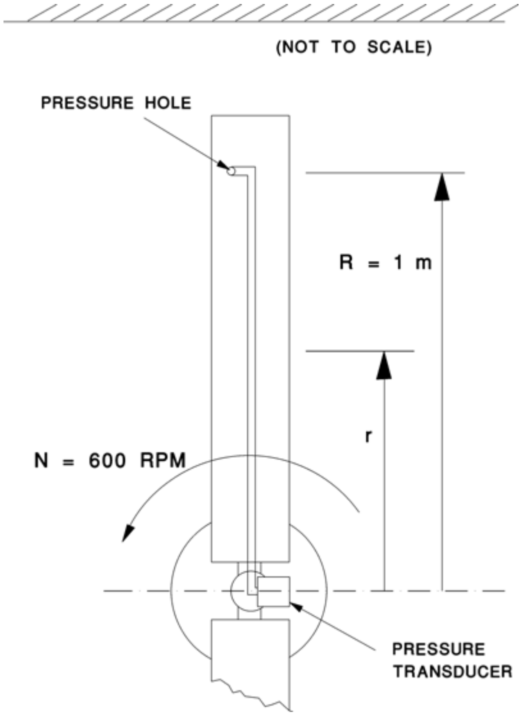
$P_{Aa} := \frac{P_{Aa}}{144}$ $P_{Aa} = 69.2$ psia

Notice that if British units are based on slug-ft-sec (rather than the "standard" lb$_m$-ft-sec), then the British system behaves exactly the same as SI units (kg-m-s).

Finally, note that since the form of the hydrostatic pressure equation we are using gives us changes in pressure through the fluid, we could have calculated the gauge pressure (in psfg not psig) at each point through the system starting from the gauge pressure at 1. We would then convert the gauge pressure at A to the absolute value as the last step. As will become clearer later in the course, gauge pressures must be used very carefully. **If in doubt, work in absolute pressures. Using absolute pressure is always valid.**

2.8 Pressure on helicopter rotor:

When the rotor is rotating, the air trapped inside the pressure tube, between the pressure tap and the pressure transducer, experiences an acceleration back towards the centre of rotation. Since there is acceleration, there must be a force on each fluid particle towards the centre of rotation. For an arbritrary particle in the middle of the fluid, the only possible source of this force is a pressure variation in the fluid, with higher pressures at larger radii to provide the net inward force. Note that since the tube is in the horizontal plane, the acceleration due to gravity does not contribute to this pressure variation.



(a) Acceleration

The acceleration of the rotor at radius r (and therefore the acceleration experienced at that radius by the air trapped inside the tube) is directed back towards the axis of rotation and has magnitude

$$a_r = -\frac{V^2}{r} = -r\cdot\omega^2$$ where ω is in rads/s and is related to N by $$\omega = \frac{2\cdot\pi\cdot N}{60}$$

then $$a_r = -r\cdot\left(\frac{2\cdot\pi\cdot N}{60}\right)^2$$

The acceleration is negative since it is back towards the axis of rotation while the radial co-ordinate is measured positive outwards from the axis of rotation.

(b) Pressure at tap

As shown in lectures, the pressure variation in the x direction in a fluid which is experiencing an acceleration in that direction is given by

$$\frac{dP}{dx} = -\rho\cdot a_x$$ or, in terms of the radial direction $$\frac{dP}{dr} = -\rho\cdot a_r$$

Separating variables and integrating from the axis of rotation out to the pressure tap (at radius R), the pressure change along the tube is then given by

$$\Delta P = \int_0^R -\rho\cdot a_r\,dr$$

Substituting the expression for the acceleration from (a)

$\Delta P = \int_0^R \rho \cdot r \cdot \omega^2 \, dr$ thus $\Delta P = \rho \cdot \omega^2 \cdot \frac{R^2}{2}$ since ρ is constant along the tube (assumed)

here $\rho := 1.2$ kg/m3 $R := 1$ m

$N := 600$ RPM then $\omega := \frac{2 \cdot \pi \cdot N}{60}$ $\omega = 62.83$ rads/s

and the pressure increase out to radius R $\Delta P := \rho \cdot \omega^2 \cdot \frac{R^2}{2}$

$\Delta P = 2368.7$ Pa

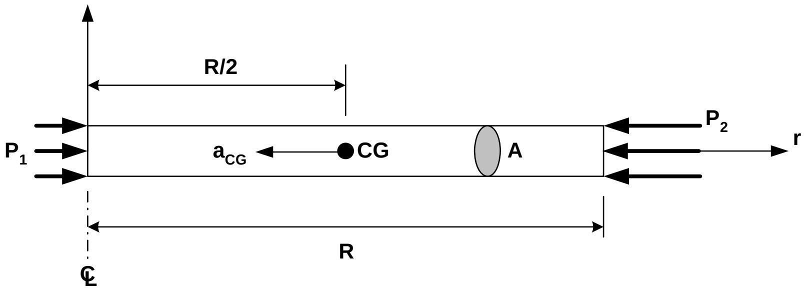
Thus the pressure at the pressure hole must be $P := 99125 + \Delta P$

$P = 101494$ Pa

For the present case, since the density is asssumed constant, the air trapped in the radial tube can be viewed as a solid body with centre of gravity located at R/2. Therefore, the solution can also be obtained by a "rigid body" analysis of the sort that one would perform in solid body dynamics.

$\Sigma F = ma$ (1) where m = mass of "body", a = acceleration of its centre of gravity

Free-body diagram for the "body" consisting of the fluid between the pressure transducer and the tap:



$a_{CG} = r \cdot \omega^2 = \frac{R}{2} \cdot \omega^2$ directed in negative r direction

$m = \rho \cdot A \cdot R$ where A = cross-sectional area of pressure tube (we do not know what the area is and therefore just give it a symbol for now)

Then for the force balance, substitutuing into (1)

$$P_1 \cdot A - P_2 \cdot A = (\rho \cdot A \cdot R) \cdot \left(-\frac{R}{2} \cdot \omega^2\right)$$ and thus for $\Delta P = P_2 - P_1$ $$\Delta P = \rho \cdot \omega^2 \cdot \frac{R^2}{2}$$ as obtained earlier

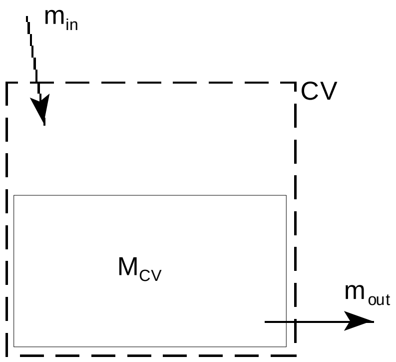
Note that the cross-sectional area appeared on both sides and therefore cancelled out.

This "solid body" approach would be much less convenient if the effect of the pressure variation on the density is taken into account. The centre of gravity is then not at R/2 and an analysis would have to be made to locate it. The first approach would then be much more straight-forward

Exercise: Examine whether the variation in density with pressure should be taken into account in this analysis. Recall that the Perfect Gas Law relates density to pressure and temperature. Assume a reasonable value of temperature in the wind tunnel (eg. 20° C) and determine the new blade pressure.

3.2 Analysis of tank with inflow and outlflow

Consider a control volume enclosing the interior of the tank:



$m_{in} = \rho Q_{in}$ = mass flow rate into the CV

(Q = volume flow rate)

$m_{out} = \rho Q_{out}$ = mass flow rate out of the CV

M_{CV} = mass currently in the CV

From conservation of mass ("continuity") then

$$m_{in} - m_{out} = \frac{dM_{CV}}{dt} \qquad (1)$$

For the present case

$D_{tank} := 10$ ft $\qquad D_{pipe} := \frac{2.5}{12} \qquad D_{pipe} = 0.208$ ft

Inlet volume flow rate $\qquad Q_{in} := 500$ USgal/min $\qquad$ (conversion: 1 USgal = 0.1337 ft^3)

$Q_{in} := \frac{Q_{in} \cdot 0.1337}{60} \qquad Q_{in} = 1.114$ ft^3/sec

(a) Level of water in tank at steady state conditions (which will occur when the flow rate out just balance the inflow rate):

For steady state, (1) reduces to $\qquad m_{in} = m_{out}$

or $\qquad Q_{in} = Q_{out} \qquad$ since ρ is the same at inlet and outlet

Outlet volume flow rate: $\qquad Q_{out} = A_{pipe} \cdot V_{pipe}$

where $\qquad V_{pipe} = \sqrt{2 \cdot g \cdot h} \qquad$ (given) $\qquad g := 32.174$ ft/sec2

thus $\qquad Q_{out} = A_{pipe} \cdot \sqrt{2 \cdot g \cdot h}$

Then for steady state $\qquad Q_{out} = Q_{in} \qquad$ or $\qquad A_{pipe} \cdot \sqrt{2 \cdot g \cdot h} = Q_{in}$

solving for h $h = \frac{1}{2} \cdot \frac{Q_{in}^{2}}{\left(g \cdot A_{pipe}^{2}\right)}$

and $A_{pipe} := \frac{\pi}{4} \cdot D_{pipe}^{2}$ $A_{pipe} = 0.03409$ ft2

Thus, for steady state conditions $h := \frac{1}{2} \cdot \frac{Q_{in}^{2}}{\left(g \cdot A_{pipe}^{2}\right)}$ $h = 16.602$ ft

(b) Rate at which depth is changing when h = 5 ft

Since the depth is different from the steady state value, the depth will be changing (clearly, it will be increasing since the flow rate out will be lower than the flow rate in) and the full unsteady continuity equation must be used

$$m_{in} - m_{out} = \frac{dM_{CV}}{dt}$$

$m_{in} = \rho \cdot Q_{in}$ $m_{out} = \rho \cdot Q_{out}$ where $Q_{out} = V_{pipe} \cdot A_{pipe}$

$M_{CV} = \rho \cdot Vol$ where Vol is the volume of water currently in the tank

$Vol = A_{tank} \cdot h$ $A_{tank} := \frac{\pi}{4} \cdot D_{tank}^{2}$ $A_{tank} = 78.54$ ft2

thus $\frac{dM_{CV}}{dt} = \rho \cdot A_{tank} \cdot \frac{dh}{dt}$

and substituting into the continuity equation $\rho \cdot Q_{in} - \rho \cdot Q_{out} = \rho \cdot A_{tank} \cdot \frac{dh}{dt}$

rearranging $\frac{dh}{dt} = \frac{Q_{in} - Q_{out}}{A_{tank}}$

Inlet volume flow (given): $Q_{in} = 1.114$ ft3/sec

For Q_{out} $h := 5$ ft $V_{pipe} := \sqrt{2 \cdot g \cdot h}$ $V_{pipe} = 17.937$ ft/sec

thus $Q_{out} := V_{pipe} \cdot A_{pipe}$ $Q_{out} = 0.611$ ft3/sec

Finally, the surface level will be changing at $\frac{Q_{in} - Q_{out}}{A_{tank}} = 6.400806 \times 10^{-3}$ ft/sec

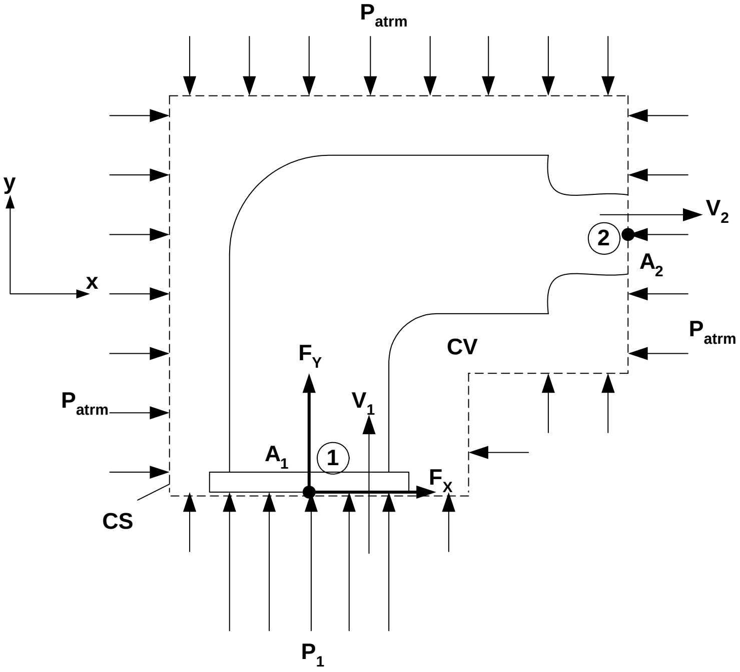
or $0.006401 \cdot 60 = 0.384$ ft/min (positive, indicating h is increasing, as expected)

3.5 Forces in flanged joint:

The key to any force (or moment) problem involving moving fluids is an appropriate control volume

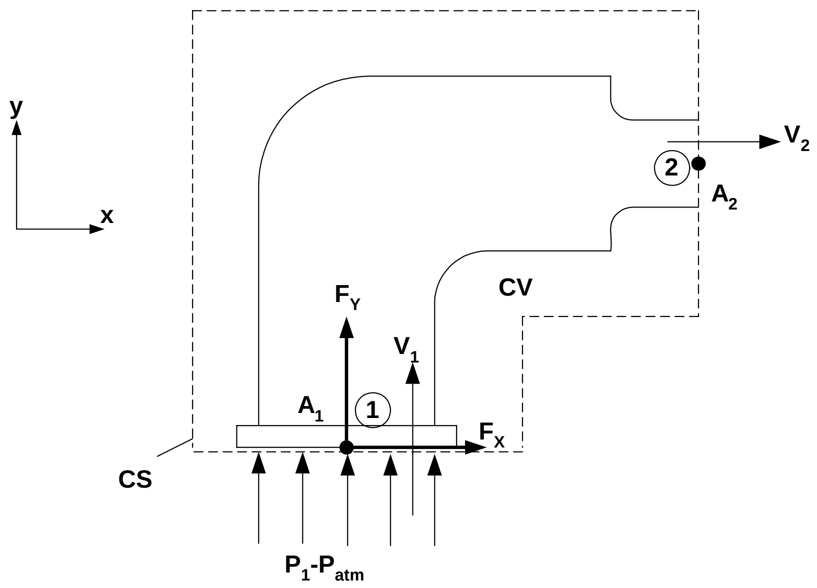
Place the control surface where:
(i) you are trying to determine information
(ii) you have as much known information as possible

The following is an appropriate CV for this problem:



Notes:
- wherever the CS cuts through a solid body it exposes any internal forces and moments in the bod at that point
- thus, we put the control surface through the flanged joint in order to expose the joint forces, which are the quantities we are trying to determine (any moments in the joint will also be exposed; you wil learn later in the course how to determine these moments)
- at steady state for a subsonic flow, the pressure at the outlet of a nozzle must be equal to the pressure in the reservoir the fluid is being discharged to (in this case the "reservoir" is simply the atmosphere)
- since we also know the velocity at this plane, this is another good location for the CS
- we then complete the CV by completely closing the control surface, identifying any additional forces etc. which appear, such as the pressure forces acting on the CV from the surrounding fluid

The next step is to apply the linear momentum equation to the CV. However, we can simplify the determination of the pressure forces a little by using a modified but equivalent CV. If we subtract a constant pressure from the pressures applied to <u>all</u> surfaces of the CV, then we do not change the net forces (or moments for that matter) acting on the control volume. P_{atm} acts on most sections of the CS of our original CV. If we therefore subtract P_{atm} from all pressures acting on the CV we obtain the following CV which is statically and dynamically equivalent to the original one:



Then apply the linear momentum equation to the modified control volume:

(i) In the x direction: $\Sigma F_x = m_{out} \cdot u_{out} - m_{in} \cdot u_{in}$ (1)

$u_{in} := 0$ since the fluid enters the CV in the y direction

For steady flow, from continuity

$m_{in} = m_{out} = m$ where from conditions at plane 1 $m = \rho \cdot A_1 \cdot V_1$

- for water $\rho := 1000$ kg/m3 $V_1 := 2$ m/s (given)

$A_1 = \frac{\pi}{4} \cdot D_1^2$ $D_1 := \frac{300}{1000}$ $D_1 = 0.3$ m

$A_1 := \frac{\pi}{4} \cdot D_1^2$ $A_1 = 0.07069$ m2

then $m := \rho \cdot A_1 \cdot V_1$ $m = 141.37$ kg/s

also $m = \rho \cdot A_2 \cdot V_2$ which can then be used to determine V_2

$D_2 := \frac{160}{1000}$ $D_2 = 0.16$ m thus $A_2 := \frac{\pi}{4} \cdot D_2^2$ $A_2 = 0.02011$ m2

and $V_2 := \frac{m}{\rho \cdot A_2}$ $V_2 = 7.031$ m/s

then $u_{out} := V_2$ (u_{out} is positive since the flow is in the positive x direction - note that we must be careful to give the correct signs to the velocities in the momentum equation since they are components of the vector velocity)

For the forces:
- for the modified CV all faces of the control surface are now exposed to a pressure of zero
- thus there is no pressure force contribution in the x direction
- the only remaining x-wise force is the F_X exposed at the flange

thus $\Sigma F_x = F_X$ and (1) reduces to $F_X := m \cdot V_2$

$F_X = 994$ N

(ii) In the y direction: $\Sigma F_y = m_{out} \cdot v_{out} - m_{in} \cdot v_{in}$ (2) $m_{out} = m_{in} = m$

$v_{out} := 0$ since the fluid leaves the CV in the x direction

and $v_{in} := V_1$ (positive since v_{in} is in the positive y direction)

- consider then the pressure forces in the y direction

  - there are two y-wise forces on the modified CV:
    (a) the pressure force at plane 1, $(P_1\text{-}P_{atm})A_1$ and it is in the positive y direction
    (b) the force exposed at the flange, F_Y (which was assumed to be in the positive y direction)
  - substituing into (2)

$$F_Y + \left(P_1 - P_{atm}\right) \cdot A_1 = m \cdot 0 - m_{in} \cdot V_1$$

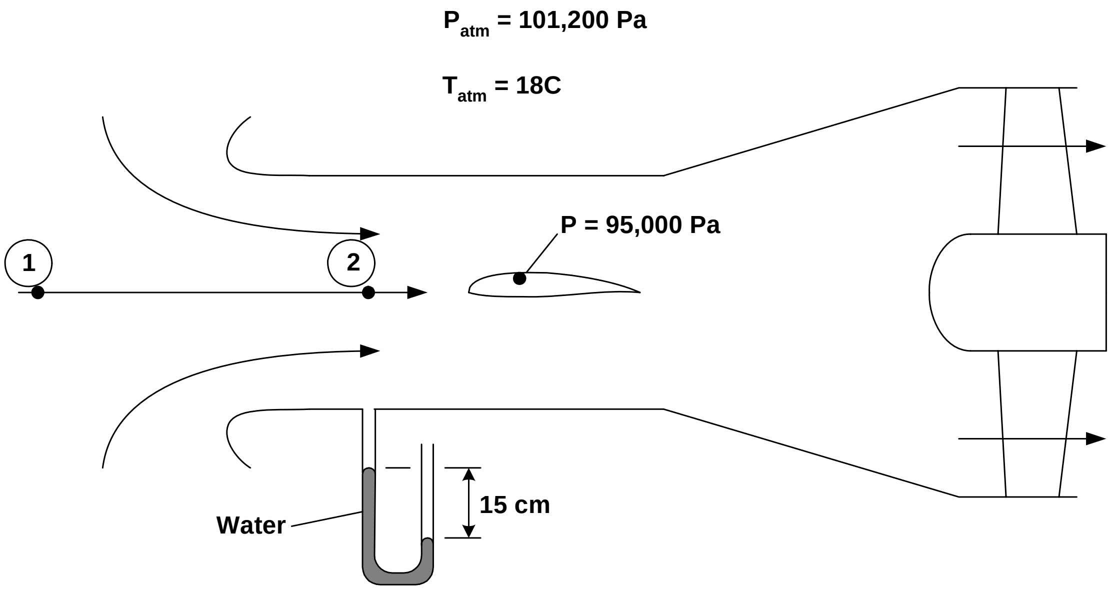
and $P_1\text{-}P_{atm}$ is just the gauge value of P_1 $P_{1g} := 100000$ Pa(g) (given)

- finally, solving for F_Y

$F_Y := -m \cdot V_1 - P_{1g} \cdot A_1$ $F_Y = -7351.3$ N

- F_Y is the force exerted on the control volume by the lower flange
- it was assumed upward (in the positive y direction), which would correspond to the joint being in compression
- since F_Y came out with the a negative sign, it is in the opposite direction to the one assumed, and the joint is in tension

4.1 Flow in open circuit wind tunnel:



(a) Total pressure in the test section:

- consider the streamline from 1 to 2
- Bernoulli is applicable (steady, incompressible flow with negligible friction)

$$P_1 + \frac{1}{2}\cdot\rho\cdot V_1^2 + \rho\cdot g\cdot y_1 = P_2 + \frac{1}{2}\cdot\rho\cdot V_2^2 + \rho\cdot g\cdot y_2 \qquad (1) \qquad g := 9.81 \ \text{m/s}^2$$

taking $y_1 = y_2$ (since the working fluid is a gas, the potential energy change would be small even if elevation changed between 1 and 2)

- thus, (1) reduces to

$$P_1 + \frac{1}{2}\cdot\rho\cdot V_1^2 = P_2 + \frac{1}{2}\cdot\rho\cdot V_2^2 \qquad \text{or} \qquad P_{01} = P_{02}$$

- therefore, the total pressure in the test section is the same as the total pressure in the room
- but for 1 well away from the bellmouth, $V_1 = 0$ and $P_1 = P_{atm}$
- thus,

$$P_{01} = P_{atm} + \frac{1}{2}\cdot\rho\cdot 0^2 = P_{atm} \qquad P_{atm} := 101200 \ \text{Pa}$$

and hence $P_{02} := P_{atm}$ $\quad P_{02} = 101200$ Pa

Note: As derived in lectures, the continuity and linear momentum equations apply to a finite control volume. Therefore, in any analysis in which these equations are used the corresponding CV must be clearly identified. However, Bernoulli's equation applies along a streamline (or an infinitesimally thin streamtube). Therefore, if only Bernoulli's equation is being used in an analysis, it is neither necessary nor appropriate to define a CV. However, the streamline along which it is being applied must be clearly indicated.

(b) Velocity in wind tunnel test section:

- the left side of the U-tube manometer senses the test section static pressure at 2, P_2, and the right side is open to P_{atm}

thus, from the manometer $P_2 + \rho_w \cdot g \cdot \Delta h = P_{atm}$ $\rho_w := 1000$ kg/m3

$\Delta h := \frac{15}{100}$ m

- but from (a) $P_{02} = P_{atm}$

and thus $P_{02} - P_2 = \rho_w \cdot g \cdot \Delta h$ $\rho_w \cdot g \cdot \Delta h = 1471.5$ Pa

- from the definition of total pressure, for the test section air $P_{02} = P_2 + \frac{1}{2} \cdot \rho \cdot V_2^{\,2}$

$\frac{1}{2} \cdot \rho \cdot V_2^{\,2} = \rho_w \cdot g \cdot \Delta h$ or $V_2 = \sqrt{\frac{2 \cdot \rho_w \cdot g \cdot \Delta h}{\rho}}$ where ρ = density of the air

- thus, we need the density of the air
- since we know the pressure and temperature in the room and we can apply the Perfect Gas Law

$$P = \rho \cdot R \cdot T$$

here $P := 101200$ Pa $T := 18 + 273$ $T = 291$ K $R_{air} := 287$ J/kgK

then $\rho := \frac{P}{R_{air} \cdot T}$ $\rho = 1.212$ kg/m3

finally, $V_2 := \sqrt{\frac{2 \cdot (\rho_w \cdot g \cdot \Delta h)}{\rho}}$ $V_2 = 49.28$ m/s

or $V_2 \cdot \frac{3600}{1000} = 177.4$ km/hr

(c) Local velocity on the airfoil:

- apply Bernoulli along the streamline that passes from far upstream to near the pressure sensor on the airfoil upper surface (call that the new point 2)

$$P_1 + \frac{1}{2} \cdot \rho \cdot V_1^{\,2} + \rho \cdot g \cdot y_1 = P_2 + \frac{1}{2} \cdot \rho \cdot V_2^{\,2} + \rho \cdot g \cdot y_2$$

- neglect elevation changes again

then $P_1 + \frac{1}{2}\cdot\rho\cdot V_1^{\,2} = P_2 + \frac{1}{2}\cdot\rho\cdot V_2^{\,2}$ where $V_1 := 0$ $P_1 := P_{atm}$

and $P_2 := 95000$ Pa (given)

thus $V_2 := \sqrt{\frac{2\cdot(P_{atm} - P_2)}{\rho}}$ $V_2 = 101.16$ m/s

(Equivalently, we could have recognized that since every streamline originates in the room, the total pressure is the same (= P_{atm}) for every streamline passing through the test section. From the known P and P_0 at the sensor, the corresponding V is then quickly calculated from the definition of total pressure.)

(d) Airfoil as part of aircraft wing, flying at 5000 m:

- for standard atmosphere, at 5000 m $T_{atm} := 255.69$ K $P_{atm} := 54048$ Pa

then $\rho := \frac{P_{atm}}{R_{air}\cdot T_{atm}}$ $\rho = 0.7365$ kg/m^3

and $V_{ac} := 49.28$ m/s (from (b))

(i) Pressure at stagnation point:

- the atmospheric pressure is the undisturbed static pressure in the air through which the aircraft flying, whether viewed from a stationary or moving frame of reference

- using a frame of reference moving with the aircraft and considering a streamline from an undisturbed point well upstream to the stagnation point (neglecting elevation changes)

$P_{stag} := P_{atm} + \frac{1}{2}\cdot\rho\cdot V_{ac}^{\,2}$ $P_{stag} = 54942$ Pa

(ii) Static pressure at reference point on upper surface:

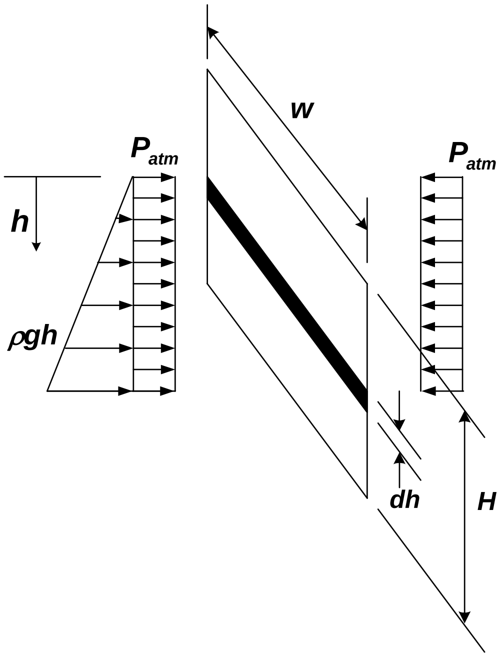
- the stagnation pressure from (i) applies to all streamlines approaching the wing

- local velocity is same as in wind tunnel: $V_{wing} := 101.16$ m/s

thus $P := P_{stag} - \frac{1}{2}\cdot\rho\cdot V_{wing}^{\,2}$ $P = 51174$ Pa

5.2 Forces on sluice gate:

(a) Force on a vertical, rectangular surface due to hydrostatic pressure distribution:



The force contribution on a small area dA on which a pressure P acts is

$$dF = P \cdot dA$$

- here, P = ρgh is constant on horizontal strips such as the strip of width dh shown on the drawing

- therefore, taking $dA = w \cdot dh$ the total force on the surface is

$$F = \int_0^H \rho \cdot g \cdot h \cdot w \, dh$$ and integrating $$F = \rho \cdot g \cdot \frac{H^2}{2} \cdot w$$

but $w \cdot H = A$ the total area of the surface

thus $F = \rho \cdot g \cdot \frac{H}{2} \cdot A$ and $\rho \cdot g \cdot \frac{H}{2}$ is just the hydrostatic pressure at depth H/2

(b) Horizontal forces on the sluice gate:

(i) For closed gate: - the net fluid force on the gate is just the hydrostatic pressure force (P_{atm} contributes equal and opposite forces on the two sides of the gate)

Then from (a) $F = \rho\cdot g\cdot\frac{H}{2}\cdot A$ here $\rho := 1000$ kg/m^3 $g := 9.81$ m/s^2

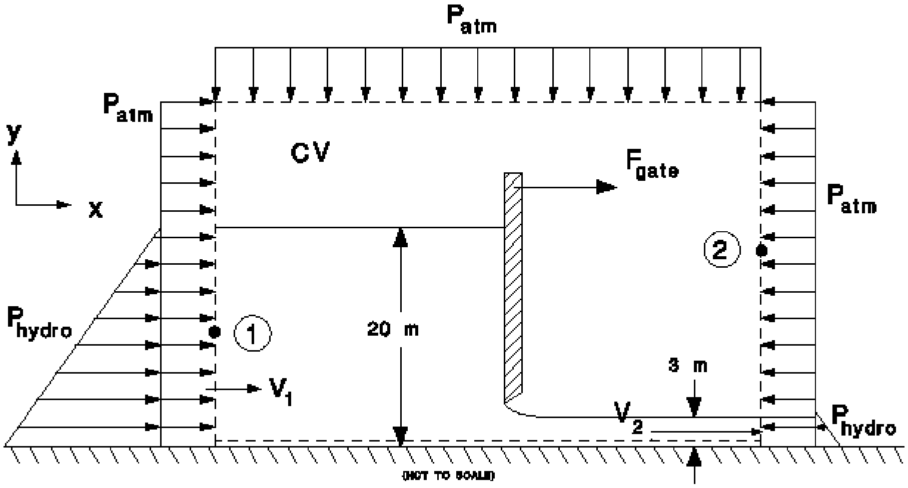
$H := 20$ m $w := 10$ m

$A := H\cdot w$ $A = 200$ m^2

thus from $F := \rho\cdot g\cdot\frac{H}{2}\cdot A$ $F = 1.962\times 10^{7}$ N

(ii) With the gate partly open, this becomes a linear momentum problem:

Define a control volume which encloses the gate and extends upstream and downstream far enough so that the velocity is uniform (constant) across the inflow and outflow portions of the control surface:



Since the CV completely encloses the gate, it will at some point cut through the attachment points for the gate and expose the horizontal force on the gate, F_{gate}, the quantity of interest. P_{atm} acts on equal areas on the left and right faces of the control volume and the resulting pressure forces $P_{atm}A$ thus cancel each other. The remaining pressure forces in the x direction are due to the hydrostatic pressure forces in the layers of liquid at the inlet and outlet.

Then applying the linear momentum equation to the CV in the x direction:

$$\Sigma F_x = m_{out}\cdot u_{out} - m_{in}\cdot u_{in} \qquad (1)$$

letting h_1 and h_2 be the depths of liquid at the inlet and outlet respectively, then

$$\Sigma F_x = \rho \cdot g \cdot \frac{h_1}{2} \cdot w \cdot h_1 + F_{gate} - \rho \cdot g \cdot \frac{h_2}{2} \cdot w \cdot h_2$$

where $h_1 := 20$ m $h_2 := 3$ m

In Problem 4.5 it was determined that $Q := 554$ m3/s

$m := \rho \cdot Q$ and thus $m = 5.54 \times 10^5$ kg/s

therefore $u_{in} := \frac{Q}{w \cdot h_1}$ $u_{in} = 2.77$ m/s

$u_{out} := \frac{Q}{w \cdot h_2}$ $u_{out} = 18.467$ m/s (both in the positive x direction)

Substituting into (1) and solving for F_{gate}

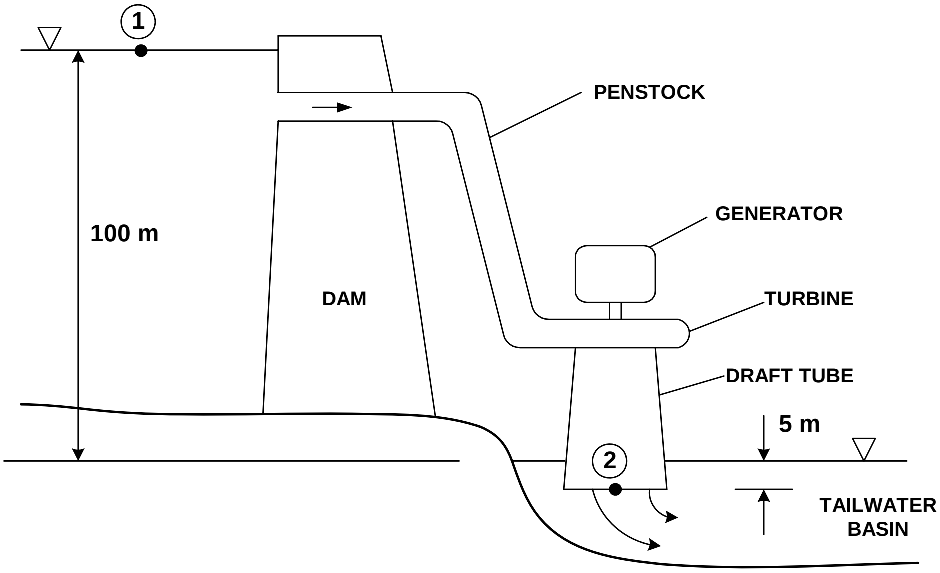
$$F_{gate} := \rho \cdot g \cdot \frac{h_2}{2} \cdot w \cdot h_2 - \rho \cdot g \cdot \frac{h_1}{2} \cdot w \cdot h_1 + m \cdot u_{out} - m \cdot u_{in}$$

$F_{gate} = -1.048 \times 10^7$ N

The result is negative indicating that the direction of the force is opposite to the one assumed. Recall that the forces in ΣF_x are the forces on the CV (and thus on the gate). Clearly, if the gate were not held in place, it would be swept downstream. Therefore, the force needed to hold it in place must be in the upstream direction, as the solution has shown.

Note that the worst case occurs when there is no flow: it requires less force to hold the gate in plac
when there is outflow.

6.3 Power output from hydraulic turbine:



Apply the Steady Flow Energy Equation along a streamline from 1 to 2: this streamline passes through the turbine and therefore the energy chages along this streamline must be a function of the energy the turbine extracts from the water

Recall that the fluid has the same energy at every point in a stationary body of liquid. Thus, any convenient point in the reservoir can be used, such as the point on the free surface for which

$V_1 := 0$ (provided 1 is not too close to the penstock inlet)

$P_1 = P_{atm}$

There is also more than one possible choice for the location of the downstream point. A point at the free surface of the tailwater basin is a possibility: the velocity there is again zero and the pressure is atmospheric. However, it must be realized that between the outlet of the draft tube and this point the water emerging from the draft tube mixes in a disorderly way with the water in tailwater basin and thereby loses the kinetic energy it has at the outlet of the draft tube. Therefore, the energy equation would have to include include a corresponding head loss term. Alternatively, the point at the outlet of the draft tube can be used. Now there are no loss terms in the energy equation since it was given that the losses in the penstock and the draft tube could be neglected (there are losses inside the turbine itself but these are accounted for by the efficiency).

Writing the SFEE in head form, since this is a hydraulic flow

$$m\cdot\left(\frac{P_1}{\rho\cdot g}+\frac{V_1^{\ 2}}{2\cdot g}+y_1\right)+\frac{1}{g}\cdot W_{turb}-h_L = m\cdot\left(\frac{P_2}{\rho\cdot g}+\frac{V_2^{\ 2}}{2\cdot g}+y_2\right) \qquad (1)$$

Thus $\frac{1}{g \cdot m} \cdot W_{turb} = \frac{P_2 - P_1}{\rho \cdot g} + \frac{V_2^{\,2}}{2 \cdot g} + (y_2 - y_1)$ $g := 9.81$ m/s2 $\rho := 1000$ kg/m3

$Q := 200$ m3/s then $m := \rho \cdot Q$ $m = 200000$ kg/s

$P_2 = P_{atm} + \rho \cdot g \cdot h_2$ (hydrostatic pressure in the tailwater basin at the depth outlet of draft tube)

$h_2 := 5$ m

$P_1 = P_{atm}$

From continuity $V_2 = \frac{Q}{A_2}$ $A_2 = \frac{\pi}{4} \cdot D_2^{\,2}$ where $D_2 := 7$ m

thus $A_2 := \frac{\pi}{4} \cdot D_2^{\,2}$ $A_2 = 38.485$ m2

$V_2 := \frac{Q}{A_2}$ $V_2 = 5.197$ m/s

Using the outlet of the draft tube as the datum for elevation

then $\Delta y_{21} := -105$ m ($\Delta y_{21} = y_2 - y_1$)

Substituting into eqn. (1), then

$$W_{turb} := m \cdot \left(\frac{\rho \cdot g \cdot h_2}{\rho} + \frac{V_2^{\,2}}{2} + g \cdot \Delta y_{21} \right)$$

$W_{turb} = -1.935 \times 10^{8}$ W (negative since power in is positive for eqn.(1))

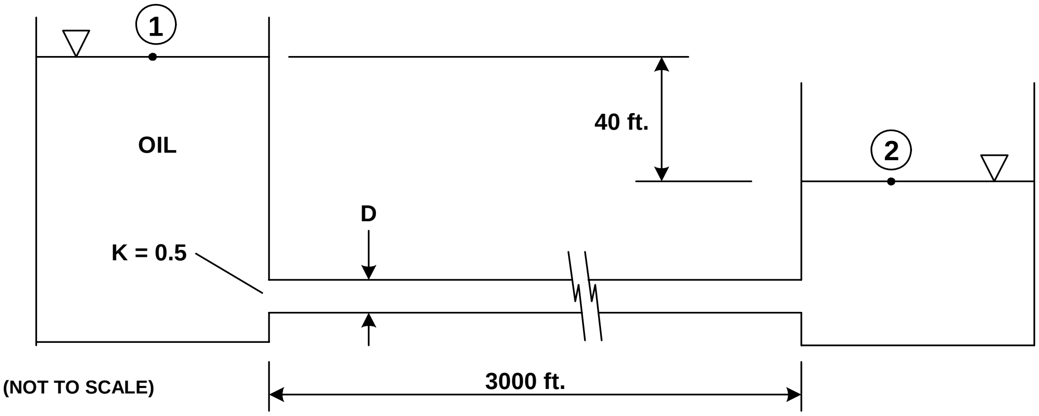
This is the power released by the water as it passes from 1 to 2. However, some of this power is used to overcome the frictional losses inside the turbine. The output power which will appear at the shaft is given by:

$W_{shaft} = \eta_{turb} \cdot W_{turb}$ where $\eta_{turb} := 0.94$

thus $W_{shaft} := \eta_{turb} \cdot W_{turb}$ $W_{shaft} = -1.819 \times 10^{8}$ W

(ie. shaft output power is 182 MW)

6.13 Choosing diameter of pipeline to convey 1 ft³/sec of oil between two tanks:



Apply the Steady Flow Energy Equation between 1 and 2.

As is often the case with application of the SFEE, there are a number of start and end points which could be used. For example, 2 could be located at the outlet of the pipe. Choosing 2 at the downstream free surface is convenient since we know that the velocity there is zero and the pressure is atmospheric. However, it must then be realized that between the pipe outlet and 2 the oil loses all its kinetic energy as it mixes with the stationary oil in the tank: thus, we must include a local loss term, with K = 1, to account for this.

Then $$\frac{P_1}{\rho\cdot g}+\frac{V_1{}^2}{2\cdot g}+y_1-f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}-\Sigma K\cdot\frac{V^2}{2\cdot g}=\frac{P_2}{\rho\cdot g}+\frac{V_2{}^2}{2\cdot g}+y_2 \qquad (1)$$

but $P_1 = P_2 = P_{atm}$ $\quad V_1 = V_2 = 0$

and (1) reduces to $$f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+\Sigma K\cdot\frac{V^2}{2\cdot g}+\left(y_2-y_1\right)=0 \qquad (2)$$

Using the downstream free surface as the datum for elevation

$y_1 := 40$ ft. $\quad y_2 := 0$ ft. $\quad L := 3000$ ft. $\quad g := 32.174$ ft/sec²

also $\rho_{water} := 1.94$ slug/ft³ $\quad SG := 0.9$ (given)

then $\rho_{oil} := SG\cdot\rho_{water}$ $\quad \rho_{oil} = 1.746$ slug/ft³

Since the volume flow rate is specified, the flow velocity V is a function of pipe diameter D

$Q := 1$ ft³/sec $\quad V = \dfrac{Q}{A} = \dfrac{Q}{\left(\dfrac{\pi}{4}\cdot D^2\right)}$

If the flow is laminar, f can also be expressed in terms of D since

$$f = \frac{64}{Re} = \frac{64}{\left(\frac{\rho_{oil} \cdot V \cdot D}{\mu}\right)}$$ here $\mu := 0.0008$ lb$_f$-sec/ft^2

However, if the flow is turbulent we will have to obtain f from the Moody chart. In order to do this we will have to guess a value of D (or V), obtain f from the chart, see whether these values satisfy equation (2), and iterate until they do.

[There is another possibility: if you have a software package such as Mathcad, Matlab etc. which includes an equation solver, it is possible to formulate a solution which does not require iteration. This is because the Moody chart was in fact generated from an equation which expresses f as a function of Re and the roughness size (see Appendix A). Unfortunately, the equation is not explicit in f and therefore if you tried to solve it manually you would again have to resort to iteration. The present solution will use the Moody chart.]

We then need to devise an iteration strategy (there are probably several different approaches which could be used successfully). In this case we know that we have a fixed head of 40 ft. available and that the pipe diameter must be a multiple of 0.1 ft. For the specified flow rate, we can then try a number of diameters until we find the smallest one which requires a head 40 ft. or less to drive 1 ft^3/sec through the pipeline.

Letting $\Delta y_{12} := y_1 - y_2$ where Δy_{12} = 40 ft. is the available head

Rearranging (2) then $$\Delta y_{12} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \Sigma K \cdot \frac{V^2}{2 \cdot g} \qquad (3)$$

Consider the loss terms

Local losses:

$K_{inlet} := 0.5$ $K_{outlet} := 1.0$ thus $\Sigma K := 1.5$ (applied to the dynamic head in the pipe)

Frictional loss:

f will be a function of the roughness $\varepsilon := 0.00015$ ft. for wrought iron (see Appendix A)

(i) First iteration: try $D := 0.1$ ft

Then $A := \frac{\pi}{4} \cdot D^2$ $A = 7.854 \times 10^{-3}$ ft^2 $V := \frac{Q}{A}$ $V = 127.324$ ft/sec (looks high)

$$Re := \frac{\rho_{oil} \cdot V \cdot D}{\mu} \qquad Re = 2.779 \times 10^4$$

$\varepsilon_{rel} := \frac{\varepsilon}{D}$ $\varepsilon_{rel} = 0.0015$ then from Moody chart $f := 0.0275$

From (3)

$$\Delta y_{12} := f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \Sigma K \cdot \frac{V^2}{2 \cdot g} \qquad \Delta y_{12} = 208223 \quad \text{ft.}$$

- this is much higher than the available head and the diameter must be increased to lower the dynamic head in the pipe

(ii) Second iteration: Try $D := 0.5$ ft.

Then $A := \frac{\pi}{4} \cdot D^2$ $A = 0.196$ ft^2 $V := \frac{Q}{A}$ $V = 5.093$ ft/sec

$$Re := \frac{\rho_{oil} \cdot V \cdot D}{\mu} \qquad Re = 5.558 \times 10^3$$

$\varepsilon_{rel} := \frac{\varepsilon}{D}$ $\varepsilon_{rel} = 0.0003$ then from Moody chart $f := 0.0365$

From (3) $\Delta y_{12} := f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \Sigma K \cdot \frac{V^2}{2 \cdot g}$ $\Delta y_{12} = 88.9$ ft.

which is much closer to the available head

(iii) Third iteration: Try next available diameter $D := 0.6$ ft.

Then $A := \frac{\pi}{4} \cdot D^2$ $A = 0.283$ ft^2 $V := \frac{Q}{A}$ $V = 3.537$ ft/sec

$Re := \frac{\rho_{oil} \cdot V \cdot D}{\mu}$ $Re = 4.631 \times 10^3$ (transitional range, but flow will probably be turbulent)

$\varepsilon_{rel} := \frac{\varepsilon}{D}$ $\varepsilon_{rel} = 0.00025$ then from Moody chart $f := 0.0385$

From (3) $\Delta y_{12} := f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + \Sigma K \cdot \frac{V^2}{2 \cdot g}$ $\Delta y_{12} = 37.7$ ft.

Thus D = 0.6 ft. is the smallest available diameter which will result in the required flow rate. Since the available head is slightly larger, the flow rate will be a little larger than the required 1 ft^3/sec. If exactly 1 ft^3/sec is needed, a valve can be installed to adjust the flow rate.

APPENDIX C

Excerpts from Laboratory Health and Safety Manual

Laboratory Health and Safety Manual

Section 1: Introduction and Objectives

This manual provides a set of minimum standards and practices for the safe and healthy operation of a laboratory. Following the requirements set out in the manual will help meet the requirements of the Occupational Health and Safety Act of Ontario (OHSA) for the purposes of the operation of a laboratory. It is required reading for all laboratory supervisors, researchers, staff, and students working in research and teaching laboratories at Carleton University.

The manual was developed by Environmental Health and Safety Services (EHSS) based on the Laboratory Health and Safety Manual from the University of Western Ontario, and in consultation with faculty and staff in the Faculties of Science and Engineering, the Department of Physical Plant, and the Joint Occupational Health and Safety Committee. Revisions and updates will continue to be made. Please contact EHSS with any comments or suggestions you have about the manual.

This manual is intended to:

- Define health and safety responsibilities within the University community;
- Outline specific policy application for laboratory operation;
- Explain basic emergency procedures; and
- Provide information and standards for the healthy and safe operation of a laboratory.

The manual is not all encompassing. There are many special procedures conducted within our laboratories, which require unique health and safety precautions.

Departments will have additional procedures that apply to their own situations and work. In all cases the laboratory supervisor is ultimately responsible for teaching safe work practices and must insist upon the use of proper procedures to eliminate unnecessary hazards.

If you have any questions about how to safely undertake a task or project, ask your supervisor before you begin.

The objectives of this manual are to:

1. Define who is a supervisor and who is a laboratory worker;
2. Define the responsibilities of the supervisor and the laboratory worker for the safe operation of a laboratory;
3. Highlight sections of the OHSA which affect the operation of a laboratory;
4. Provide a standard of good laboratory safety practices which also allows the University to meet the requirements of Section 25(2)h of the OHSA, *An employer shall take every precaution reasonable in the circumstances for the protection of a worker.*
5. Provide the general guidelines and basic rules considered the minimum for the safe operation of a laboratory at Carleton University;
6. Protect all laboratory users from health and safety hazards;

Our goal is a healthy and safe environment for everyone.

Richard Van Loon, President

G. Stuart Adam, Vice-President
(Academic)

John ApSimon, Vice-President
(Research and External)

Duncan Watt, Vice-President
(Finance and Administration)

Peter Watson, Dean
Faculty of Science

Samy Mahmoud, Dean
Faculty of Engineering & Design

May 2000

Section 2: Definitions

2.1 Teaching Laboratory

A laboratory where a group of students simultaneously receive instruction in, and perform, experimental procedures associated with a formally approved Carleton University academic course.

2.2 Research Laboratory

A laboratory set up primarily to conduct research.

2.3 Supervisor

A supervisor is a person who has charge of a workplace or authority over a worker. (OHSA Section1(1)) At the University, this includes all faculty and staff who supervise a laboratory. Deans, Directors, Chairs and other department heads and researchers are supervisors. The department head will appoint a supervisor for each lab. This supervisor is responsible for all matters of health and safety in the lab and will keep the records pertaining to health and safety for the lab. The department head will ensure that on appointment each supervisor attends a Health and Safety for Supervisors training session provided by EHSS.

2.4 Laboratory Worker/User

A laboratory worker or user is anyone, student, staff or faculty, who works as a student or for pay in a laboratory, including those who have supervisory responsibilities.

2.5 Unattended Procedures/Equipment

A procedure or piece of equipment that is left operating when no one is in the lab.

2.6 Hazardous Agent

Any physical, chemical, radioactive, or biological agent that may pose a health or safety hazard to those exposed.

Section 3: Responsibilities of Supervisors

The supervisor of a laboratory has overall responsibility for safety.

Prior to any work being performed by a new laboratory worker it is the supervisor's responsibility to ensure that workers are aware of safety rules and follow them and that the following training has been provided:

a) An appropriate safety orientation when individuals are first assigned to a laboratory space;

b) Generic WHMIS training, which may be provided by the Department WHMIS Coordinator, and specific WHMIS training provided by the supervisor;

c) Radiation Safety Training, provided by the Radiation Safety Officer, if applicable;

d) Training on special or unusual hazards in the lab;

e) Training in the use of laboratory specific emergency equipment and emergency response;

Records of the training must be kept on file in the department and a copy sent to EHSS. (See Appendix 1, Record of Training)

In addition the supervisor is responsible for the following:

a) That adequate emergency equipment in proper working order is readily available;

b) That an incident investigation report is completed for every incident or injury that occurs in his/her lab. (See Appendix 2, Incident/Injury Report Form) Examples include incidents requiring first aid or other medical attention and incidents resulting in property damage, such as, spills, fires, explosions as well as near misses in either category. Incidents resulting in personal injury to a worker require completion of a Workplace Safety Insurance Board, Form 7. The WSIB forms are available from Human Resources.

c) That every two weeks safety and housekeeping inspections of the lab are conducted with a record of the inspection kept on file in the lab.

d) That an appropriate alternate is appointed as supervisor when the laboratory supervisor is absent. In a teaching lab where safety is a concern, the supervisor or alternate will always be present. In a research lab, an alternate will be appointed when the supervisor is away from the campus.

Section 4: Responsibilities of Laboratory Workers

All laboratory workers are responsible for:

a) following all applicable safety rules and practices as outlined in this manual and by the supervisor;

b) using and wearing personal protective equipment according to instructions;

c) reporting all incidents to the laboratory supervisor;

d) reporting all unsafe conditions to the laboratory supervisor;

e) completion of recommended occupational health screening programs when applicable; and

f) attending all training courses as directed by the supervisor.

Section 5: General Health and Safety Principles

Good laboratory practice requires that every laboratory worker and supervisor observe the following:

a) Food and beverages are not permitted in the lab. Consume food and beverages only in properly designated areas. (Ontario Regulation 851 Section 131)

b) Use appropriate personal protective equipment at all times. (OHSA Section 28(1))

c) Use laboratory equipment for its designed purpose.

d) Confine long hair and loose clothing. (Ontario Regulation 851 Section 83)

e) Use a proper pipeting device. Absolutely no pipeting by mouth.

f) Avoid exposure to gases, vapours, aerosols and particulates by using a properly functioning laboratory fumehood.

g) Wash hands upon completion of laboratory procedures and remove all protective equipment including gloves and lab coats.

h) Ensure that the laboratory supervisor is informed of any unsafe condition. (OHSA Section 28 (1)(d))

i) Know the location and correct use of all available safety equipment.

j) Determine potential hazards and appropriate safety precautions before beginning new operations and confirm that existing safety equipment is sufficient for this new procedure. (See Appendix 3, Laboratory Risk Assessment)

k) Avoid disturbing or distracting other workers while they are performing laboratory tasks.

l) Ensure visitors to the laboratory are equipped with appropriate safety equipment.

m) Be certain all hazardous agents are stored correctly and labelled correctly according to Workplace Hazardous Materials Information Systems (WHMIS) requirements. (Ontario Regulation 860)

n) Consult the material safety data sheet prior to using an unfamiliar chemical and follow the proper procedures when handling or manipulating all hazardous agents.

o) Follow proper waste disposal procedures. (See Appendix 4, Disposal of Hazardous Waste)

Section 6: Basic Safety Procedures

6.1 Procedures for Unattended Work

a) Unattended procedures should be kept to a minimum.

b) An unattended procedure must be visited periodically and a sign posted on the door of the lab outlining the procedure with the name and phone number of a contact person. The sign will indicate the date and time the procedure was started, when it is expected to be completed, and when it was last checked. (See Appendix 13)

c) Unattended procedures using cooling water must have the hoses securely attached and the water adjusted to the minimum flow necessary. Ensure plumbing drains are clear before leaving the procedure.

6.2 Working Alone

a) For safety reasons working alone should be avoided. Someone should always be within call when a laboratory procedure is being performed.

b) For work with hazardous materials or procedures the supervisor has the right to require that at least one other person be present.

6.3 Housekeeping

a) Work areas must be kept clean and free of obstructions.

b) Stairways and halls must not be used for storage. This applies to both equipment and personal property. Bicycles are not allowed in buildings.

c) Walkways and aisles in laboratories must be kept clear.

d) Access to emergency equipment or exits must never be blocked. (Ontario Regulation 851 Section123 (2))

e) Equipment and chemicals must be stored properly.

f) Spilled chemicals must be dealt with immediately and if safe cleaned up by the chemical user. (See Section 11.4 of this manual) Spills must be reported to the supervisor.

g) Wastes must be placed in appropriate, labelled containers.

h) Materials no longer used must not be allowed to accumulate and must be disposed of following proper procedures. (See Appendix 4, Disposal of Hazardous Waste)

6.4 Laboratory Equipment Maintenance

Laboratory equipment must be inspected and maintained by a qualified person. The frequency of the inspection depends on the hazard posed by the equipment, the manufacturer's instructions, or as required by regulations. Records of the maintenance must be kept on file by the laboratory supervisor and be available at all times.

6.5 Guarding

a) All mechanical equipment must be adequately guarded to prevent access to electrical connections or moving parts. (Ontario Regulation 851 Section 25)

b) All centrifuges must be fitted with an interlock so that they cannot be accessed while moving or started while open. (Ontario Regulation 851 Section 31)

6.6 Shielding

a) Appropriate shielding must be used whenever an operation involves chemicals with the potential for explosion or severe splashing. Examples include:
   - when a reaction is attempted for the first time;
   - when a familiar reaction is carried out on a larger scale than usual;
   - whenever operations are carried out under non-ambient conditions; or
   - whenever a severe splashing potential exists for corrosive materials. (Ontario Regulation 851 Section 89)

b) Shielding or equivalent precautions are to be used when working with non-ionizing radiation sources, magnetic and other fields. Examples include:
   - Lasers
   - Infrared radiation
   - Ultraviolet radiation
   - Microwave radiation

Refer to the Radiation Safety Manual (available from EHSS) for shielding of ionizing radiation sources.

c) Appropriate shielding is required when using equipment with thermal hazards.

6.7 Glassware

a) Repair or dispose of any damaged glassware. Follow proper disposal procedures for damaged glassware. (See Appendix 4)
b) Ensure adequate hand protection is used when working with glass tubing.

c) Tape permanent vacuum glassware which presents an implosion risk with either electrical or duct tape or use appropriate shielding. (Ontario Regulation 851 Section 84 (b & f))
d) Wear appropriate hand protection when picking up broken glass.
e) Ensure proper instruction is given for the use of specialized glassware.
f) Specific procedures may apply for contaminated glassware.

6.8 Flammable and Combustible Material Hazards

a) Use an open flame only as long as necessary and extinguish it when done.
b) Do not use an open flame to heat flammable or combustible materials. It is generally not recommended to perform a distillation at reduced pressure using an open flame due to the possibility of local superheating.
c) Remove all flammable and combustible materials from the work area before lighting a flame.
d) Notify all others in the lab and note any procedure using flammable and combustible gases and liquids before lighting a flame.
e) Store all flammable and combustible materials properly as required by the Ontario Fire Code. (See Appendix 5, Storage and Handling of Flammable and Combustible Liquids)
f) Avoid open flames, use non-sparking equipment and adequate ventilation if a flammable atmosphere may be generated, for example when dispensing flammable or combustible solvents. (Ontario Regulation 851 Section 63)

6.9 Cryogenic Materials and Cold Traps

a) Wear proper gloves and a face shield when preparing or using a cold trap below –70 degrees C or cryogenic liquids.
b) Never use liquid nitrogen or liquid air as a cold trap to collect a flammable or combustible material mixed with air. Oxygen may condense from the air and lead to an explosion hazard.
c) Always ensure the flammable or combustible material is collected under vacuum. Use a Dewar vessel designed for cryogenic liquids not a regular domestic vacuum flask.
d) When returning the cooled material back to atmospheric pressure, ensure the cryogenic coolant has been removed to prevent liquid air condensation.
e) Use appropriate gloves when handling cryogenic materials, e.g. dry ice, etc.
f) Dry ice/solvent cooling baths should be prepared carefully by the slow addition of small amounts of the solid dry ice to the solvent to avoid excessive frothing and overflow of the solvent.
g) Never lower your head into a dry ice chest since a high level of CO2 may accumulate there posing an asphyxiation hazard.

6.10 Systems under Pressure

a) Never heat or carry out a reaction in a closed vessel unless it is designed or tested to withstand the expected pressure of the reaction.
b) Pressurized equipment must have an appropriate pressure release valve.
c) Pressurized equipment must be shielded, guarded, or designed to protect the operator against potential explosions.

6.11 Back Flow Preventers

a) All water faucets to which a hose is attached in a laboratory must be equipped with an appropriate backflow preventer. (Ontario Building Code) This prevents the contamination of the drinking water system. Contact the Manager, Maintenance Services at extension 8821 for an evaluation.

6.12 Electrical Equipment and Apparatus

a) All electrical installations must conform to the provisions of the Power Commission Act of Ontario.
b) All electrical equipment must be CSA approved or inspected by Ontario Hydro.
c) Extension cords must not be used for permanent installations. Contact Physical Plant at 3668 to install or relocate outlets in close proximity to the equipment.
d) Use ground fault circuit interrupters where there is a risk of an operator coming in contact with water and electrical equipment simultaneously.
e) Only trained, qualified personnel may repair or modify electrical or electronic equipment.
f) Power bars should not be located beneath work benches where chemicals are handled.

6.13 Compressed Gas Cylinders

All compressed gases have potential health and safety hazards related to the chemical properties of the gas, as well as pressure hazards. Take precautions to protect personnel from these potential hazards

a) All gas cylinders, empty or full, must be properly secured so they cannot be knocked over. Cylinders with safety caps in place may be secured together. All others must be secured separately. (Ontario Regulation 851 Section 49)
b) Compressed gas cylinders should be transported capped and chained on appropriate carts.
c) Always wear eye protection when working with compressed gases.
d) Always use the appropriate regulator for the gas being used. The regulator should be inspected each time before use, as recommended by the manufacturer.

d) Respiratory Protection

Under normal circumstances respirators should not be required for laboratory situations. Use of fumehoods should generally eliminate respiratory hazards. If a respirator is required, the selection should be based on the CSA Standard, Selection, Use and Care of Respirators CSA – Z94.4-93. It is essential the wearer be properly instructed for fit and safe use of a respirator.

e) Hearing Protection

Hearing protection is required for noise levels above 90 dBA. (Ontario Regulation 851 Section 139) The supervisor will determine the appropriate type of hearing protection to be worn. (Hearing Protectors CSA-Z94.2-M1984) Measuring can be done by EHSS.

f) Foot Protection

Safety footwear is designed to protect feet against a variety of injuries. Impact, compression, chemical splashes and puncture are the most common types of injuries. Footwear should be chosen according to the hazard and should be properly rated. (Protective Footwear CSA-Z195-M92)

g) Head Protection

Head protection is required when working where there is a risk of injury from moving, falling, or flying objects or when working near high-voltage equipment. Hard hats are designed to protect from the impact and penetration caused by objects hitting the head or from limited electrical shock or burns.

CARLETON UNIVERSITY
Department of Mechanical and Aerospace Engineering

MAAE 2300
Fluid Mechanics I

REQUEST FOR EXEMPTION FROM LABORATORY WORK

NAME: ____________________________________

STUDENT NUMBER: _______________________________

I have satisfactorily completed the lab portion of MAAE 2300 in the academic year _______ - ________

Term: Fall ____ Winter ____

Course Section: A _____ B _____ C _____

Course Instructor: Professor _______________________

Grade Obtained in labs if known: ______________

DATE: _____________________

SIGNATURE:______________________________