

MCG2130 – THERMODYNAMICS I  
 Chapter 5 – The Second Law of Thermodynamics for Cycles  
 Solutions to Suggested Problems

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5.22

A large coal fired power plant has an efficiency of 45% and produces net 1,500 MW of electricity. Coal releases 25 000 kJ/kg as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\begin{aligned} \dot{m} &= \frac{\dot{W}}{\eta \times HV} = \frac{1500 \text{ MW}}{0.45 \times 25000 \text{ kJ/kg}} = \frac{1500 \times 1000 \text{ kJ/s}}{0.45 \times 25000 \text{ kJ/kg}} \\ &= 133.33 \text{ kg/s} = \mathbf{480\,000 \text{ kg/h}} \end{aligned}$$

5.29

R-410A enters the evaporator (the cold heat exchanger) in an A/C unit at  $-20^\circ\text{C}$ ,  $x = 28\%$  and leaves at  $-20^\circ\text{C}$ ,  $x = 1$ . The COP of the refrigerator is 1.5 and the mass flow rate is 0.003 kg/s. Find the net work input to the cycle.

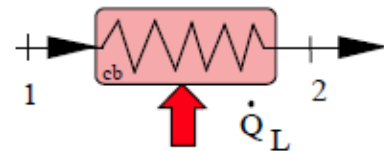
Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1 h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1 h_{fg}] = \dot{m} (1 - x_1)h_{fg}$$

$$= 0.003 \text{ kg/s} \times 0.72 \times 243.65 \text{ kJ/kg} = 0.5263 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.5263 / 1.5 = \mathbf{0.35 \text{ kW}}$$



## 5.31

An experimental power plant generates 130 MW of electrical power. It uses a supply of 1200 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 12°C.

Solution:

C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1200 - 130 = 1070 \text{ MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{\text{air}} \Delta h = \dot{m}_{\text{air}} C_p \Delta T$$

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_L}{C_p \Delta T} = \frac{1070 \times 1000}{1.004 \times 12} \frac{\text{MW}}{(\text{kJ/kg-K}) \times \text{K}} = 88\,811 \text{ kg/s}$$

This is too large to make, so some cooling by liquid water or evaporative cooling should be used, see chapter 11.

## 5.32

A water cooler for drinking water should cool 25 L/h water from 18°C to 10°C while the water reservoir also gains 60 W from heat transfer. Assume a small refrigeration unit with a COP of 2.5 does the cooling. Find the total rate of cooling required and the power input to the unit.

The mass flow rate is

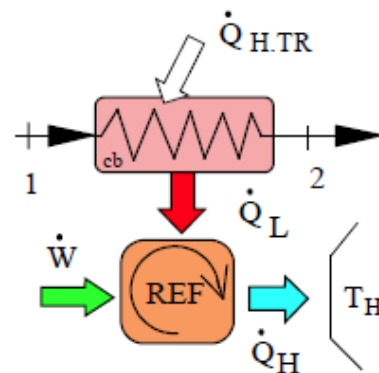
$$\dot{m} = \rho \dot{V} = \frac{25 \times 10^{-3}}{0.001002} \frac{1}{3600} \text{ kg/s} = 6.93 \text{ g/s}$$

Energy equation for heat exchanger

$$\begin{aligned} \dot{Q}_L &= \dot{m}(h_1 - h_2) + \dot{Q}_{H \text{ TR}} \\ &= \dot{m} C_p (T_1 - T_2) + \dot{Q}_{H \text{ TR}} \end{aligned}$$

$$\begin{aligned} &= 6.93 \times 10^{-3} \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times (18 - 10) \text{ K} + 60 \text{ W} \\ &= 291.8 \text{ W} \end{aligned}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \Rightarrow \dot{W} = \dot{Q}_L / \beta = 291.8 / 2.5 = 116.7 \text{ W}$$



5.36

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10°C. Assume the refrigerator has  $\beta = 3.5$  and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

C.V. Water in tray. We neglect tray mass.

Energy Eq.:  $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process :  $P = \text{constant} = P_0$

$${}_1W_2 = \int P \, dV = P_0 m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 :  $h_1 = 41.99 \text{ kJ/kg}$ , Tbl. B.1.5 :  $h_2 = -333.6 \text{ kJ/kg}$

$${}_1Q_2 = 0.25(-333.4 - 41.99) = -93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -{}_1Q_2/\beta = 93.848/3.5 = 26.81 \text{ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W} \, dt = \dot{W} \, \Delta t$$

$$\Delta t = W/\dot{W} = (26.81 \times 1000) \text{ J} / 750 \text{ W} = 35.75 \text{ s}$$



Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

## 5.45

An ideal (Carnot) heat engine has an efficiency of 40%. If the high temperature is raised 15% what is the new efficiency keeping the same low temperature?

Solution:

$$\eta_{\text{TH}} = W_{\text{net}} / Q_{\text{H}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 0.4 \quad \Rightarrow \quad \frac{T_{\text{L}}}{T_{\text{H}}} = 0.6$$

so if  $T_{\text{H}}$  is raised 15% the new ratio becomes

$$\frac{T_{\text{L}}}{T_{\text{H new}}} = 0.6 / 1.15 = 0.5217 \quad \Rightarrow \quad \eta_{\text{TH new}} = 1 - 0.5217 = \mathbf{0.478}$$

## 5.49

A large heat pump should upgrade 4 MW of heat at 65°C to be delivered as heat at 145°C. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and  $\dot{Q}_{\text{L}} = 4 \text{ MW}$ .

$$\beta_{\text{HP}} = \frac{\dot{Q}_{\text{H}}}{\dot{W}_{\text{in}}} = \frac{T_{\text{H}}}{T_{\text{H}} - T_{\text{L}}} = \frac{273.15 + 145}{145 - 65} = 5.227$$

$$\beta_{\text{REF}} = \beta_{\text{HP}} - 1 = \frac{\dot{Q}_{\text{L}}}{\dot{W}_{\text{in}}} = 4.227$$

Now we can solve for the work

$$\dot{W}_{\text{in}} = \dot{Q}_{\text{L}} / \beta_{\text{REF}} = 4 / 4.227 = \mathbf{0.946 \text{ MW}}$$

5.56

A refrigerator should remove 400 kJ from some food. Assume the refrigerator works in a Carnot cycle between  $-15^{\circ}\text{C}$  and  $45^{\circ}\text{C}$  with a motor-compressor of 400 W. How much time does it take if this is the only cooling load?

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 - 15}{45 - (-15)} = 4.3$$

This gives the relation between the low T heat transfer and the work as

$$\dot{Q}_L = \frac{Q}{t} = \beta \dot{W} = 4.3 \dot{W}$$

$$t = \frac{Q}{\beta \dot{W}} = \frac{400 \times 1000 \text{ J}}{4.3 \times 400 \text{ W}} = 233 \text{ s}$$

### 5.61

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P5.61). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{\text{NET}} = 10^6 \text{ kW}, \quad T_{\text{H}} = 550^\circ\text{C} = 823.3 \text{ K}$$

$$P_{\text{COND}} = 10 \text{ kPa} \rightarrow T_{\text{L}} = T_{\text{G}} (P = 10 \text{ kPa}) = 45.8^\circ\text{C} = 319 \text{ K}$$

$$\eta_{\text{TH CARNOT}} = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{\text{L MIN}} = 10^6 \left( \frac{1 - 0.6125}{0.6125} \right) = 0.6327 \times 10^6 \text{ kW}$$

$$\text{But } \dot{m}_{\text{H}_2\text{O}} = \frac{60 \times 8 \times 10/60}{0.001} = 80\,000 \text{ kg/s} \text{ having an energy flow of}$$

$$\dot{Q}_{\text{L MIN}} = \dot{m}_{\text{H}_2\text{O}} \Delta h = \dot{m}_{\text{H}_2\text{O}} C_{\text{P LIQ H}_2\text{O}} \Delta T_{\text{H}_2\text{O MIN}}$$

$$\begin{aligned} \Rightarrow \Delta T_{\text{H}_2\text{O MIN}} &= \frac{\dot{Q}_{\text{L MIN}}}{\dot{m}_{\text{H}_2\text{O}} C_{\text{P LIQ H}_2\text{O}}} = \frac{0.6327 \times 10^6 \text{ kW}}{80000 \times 4.184 \text{ kg/s} \times \text{kJ/kg-K}} \\ &= \mathbf{1.9^\circ\text{C}} \end{aligned}$$

## 5.62

A certain solar-energy collector produces a maximum temperature of 100°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the maximum thermal efficiency? If the collector is redesigned to focus the incoming light, what should the maximum temperature be to produce a 25% improvement in engine efficiency?

Solution:

$$\text{For } T_H = 100^\circ\text{C} = 373.2 \text{ K} \quad \& \quad T_L = 283.2 \text{ K}$$

$$\eta_{\text{th max}} = \frac{T_H - T_L}{T_H} = \frac{90}{373.2} = 0.241$$

The improved efficiency is

$$\eta_{\text{th max}} = 0.241 \times 1.25 = 0.301$$

With the Carnot cycle efficiency

$$\eta_{\text{th max}} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H} = 0.301$$

Then

$$T_H = T_L / (1 - 0.301) = 405 \text{ K} = 132^\circ\text{C}$$

## 5.79

A car engine with a thermal efficiency of 33% drives the air-conditioner unit (a refrigerator) besides powering the car and other auxiliary equipment. On a hot (35°C) summer day the A/C takes outside air in and cools it to 5°C sending it into a duct using 2 kW of power input and it is assumed to be half as good as a Carnot refrigeration unit. Find the rate of fuel (kW) being burned extra just to drive the A/C unit and its COP. Find the flow rate of cold air the A/C unit can provide.

$$\dot{W}_{\text{extra}} = \eta \dot{Q}_{\text{H extra}} \quad \Rightarrow \quad \dot{Q}_{\text{H extra}} = \dot{W}_{\text{extra}} / \eta = 2 \text{ kW} / 0.33 = 6 \text{ kW}$$

$$\beta = \frac{Q_L}{W_{\text{IN}}} = 0.5 \beta_{\text{Carnot}} = 0.5 \frac{T_L}{T_H - T_L} = 0.5 \frac{5 + 273.15}{35 - 5} = 4.636$$

$$\dot{Q}_L = \beta \dot{W} = 4.636 \times 2 \text{ kW} = 9.272 \text{ kW} = \dot{m}_{\text{air}} C_{\text{P air}} \Delta T_{\text{air}}$$

$$\dot{m}_{\text{air}} = \dot{Q}_L / [C_{\text{P air}} \Delta T_{\text{air}}] = \frac{9.272 \text{ kW}}{1.004 \text{ kJ/kg-K} \times (35 - 5) \text{ K}} = 0.308 \text{ kg/s}$$