

MAT 2322 Assignment # 3: Solutions

1. the area is $A = \iint_0^4 \sqrt{1 + (f_x)^2 + (f_y)^2} \, dx \, dy$

$$= \int_0^4 \int_0^4 \sqrt{1 + (9x^{1/2})^2 + (6y^{1/2})^2} \, dx \, dy$$

$$= \int_0^4 \int_0^4 (1 + 81x + 36y)^{1/2} \, dx \, dy$$

$$= \left(\frac{1}{81}\right)\left(\frac{2}{3}\right) \int_0^4 \left((1 + 81x + 36y)^{3/2} \Big|_0^4 \right) dy$$

$$= \frac{2}{243} \int_0^4 \left((325 + 36y)^{3/2} - (1 + 36y)^{3/2} \right) dy$$

$$= \left(\frac{2}{243}\right)\left(\frac{1}{36}\right)\left(\frac{2}{5}\right) \left[(325 + 36y)^{5/2} - (1 + 36y)^{5/2} \Big|_0^4 \right]$$

$$= \frac{1}{10935} \left(469^{5/2} - 325^{5/2} - 145^{5/2} + 1 \right) \approx \boxed{238.3}$$

2. $\int_0^\pi \int_0^\pi \int_0^\pi \cos(x+y+z) \, dx \, dy \, dz$

$$= \int_0^\pi \int_0^\pi \left(\sin(x+y+z) \Big|_0^\pi \right) dy \, dz$$

$$= \int_0^\pi \int_0^\pi \left(\sin(\pi+y+z) - \sin(y+z) \right) dy \, dz$$

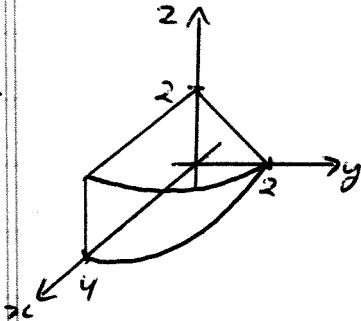
$$= \int_0^\pi \left(-\cos(\pi+y+z) + \cos(y+z) \Big|_0^\pi \right) dz$$

$$= \int_0^\pi \left(-\cos(2\pi+z) + \cos(\pi+z) + \cos(\pi+z) - \cos(z) \right) dz$$

$$= \int_0^{\pi} (2 \cos(\pi+z) - 2 \cos z) dz$$

$$= 2 \sin(\pi+z) - 2 \sin z \Big|_0^{\pi} = \boxed{0}$$

3.

top is given by $z = 2 - y$ so z goes 0 to $2 - y$

 then x goes 0 to $4 - y^2$
and y from 0 to 2

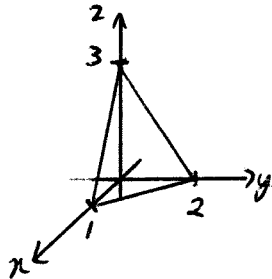
$$\text{then } V = \int_0^2 \int_0^{4-y^2} \int_0^{2-y} dz dx dy$$

$$= \int_0^2 \int_0^{4-y^2} (2-y) dx dy = \int_0^2 (2-y)(4-y^2) dy$$

$$= \int_0^2 (8 - 4y - 2y^2 + y^3) dy = 8y - 2y^2 - \frac{2}{3}y^3 + \frac{1}{4}y^4 \Big|_0^2$$

$$= 16 - 8 - \frac{16}{3} + 4 - 0 = 12 - \frac{16}{3} = \boxed{\frac{20}{3}}$$

4.

top is plane $z = 3 - 3x - \frac{3}{2}y$ so z goes from 0 to $3 - 3x - \frac{3}{2}y$

 then y goes from 0 to $2 - 2x$
and x from 0 to 1

$$\text{then } V = \int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} dz dy dx$$

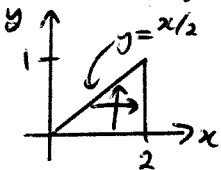
$$= \int_0^1 \int_0^{2-2x} (3-3x-\frac{3}{2}y) dy dx = \int_0^1 \left((3-3x)(2-2x) - \frac{3}{4}(2-2x)^2 \right) dx$$

(3)

$$= \int_0^1 (6 - 6x - 6x + 6x^2 - \frac{3}{4}(4 - 4x - 4x + 4x^2)) dx$$

$$= \int_0^1 (3 - 6x + 3x^2) dx = 3x - 3x^2 + x^3 \Big|_0^1 = \boxed{1}$$

5. $\int_1^4 \int_0^1 \int_{2y}^2 \frac{2\cos(x^2)}{\sqrt{z}} dz dy dx = \left(\int_1^4 \frac{dz}{\sqrt{z}}\right) \left(\int_0^1 \int_{2y}^2 2\cos(x^2) dx dy\right)$



x goes $2y$ to 2
then y 0 to 1

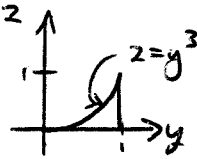
so y goes 0 to $x/2$
then x 0 to 2

$$= (2\sqrt{z} \Big|_1^4) \left(\int_0^1 \int_0^{x/2} 2\cos(x^2) dy dx \right)$$

$$= 2(2-1) \int_0^2 x \cos(x^2) dx$$

$$= \sin(x^2) \Big|_0^2 = \boxed{\sin(4)}$$

6. $\int_0^1 \int_{3\sqrt{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} dx dy dz$



$$= \left(\int_0^{\ln 3} e^{2x} dx\right) \left(\int_0^1 \int_{3\sqrt{z}}^1 \frac{\pi \sin(\pi y^2)}{y^2} dy dz\right)$$

$$= \left(\frac{1}{2} e^{2x} \Big|_0^{\ln 3}\right) \left(\int_0^1 \int_0^{y^3} \frac{\pi \sin(\pi y^2)}{y^2} dz dy\right)$$

y $3\sqrt{z}$ to 1
 z 0 to 1
so z from 0 to y^3
and y 0 to 1

$$= \frac{1}{2}(9-1) \int_0^1 \frac{\pi \sin(\pi y^2)}{y^2} (y^3 - 0) dy$$

$$= 4 \int_0^1 \pi y \sin(\pi y^2) dy = -2\cos(\pi y^2) \Big|_0^1$$

$$= -2(\cos \pi - \cos 0) = \boxed{4}$$