MAT 1341G Diagnostic Test

Winter 2016

January 25, 2:30pm

Professor: Charles Starling

Family Name: Key

First Name: ________________

Student Number: ________________

DGD TA: ☐ TA1
☐ TA2
☐ TA3

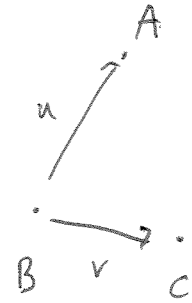
θ	$\cos\theta$	$\sin\theta$
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1

Question	Response
1	D
2	B
3	B
4	A
5	B
6	E
7	F
8	B
9	E
10	C
11	A
12	E
Total	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are permitted. The use of calculators, cell phones, or similar devices is not permitted. All cybernetic implants not necessary for life-support must be disabled at the beginning of the exam.
3. Read each question carefully – you will save yourself time and unnecessary grief later.
4. All questions are multiple choice, are worth 1 point each and no part marks will be given. **Please record your answers in the spaces on this page provided next to the question numbers above.**
5. Where it is possible to check your work, do so.
6. Good luck! Bonne chance!

1. The area of the triangle with vertices $A = (0, -1, 0)$, $B = (1, 0, 2)$ and $C = (-1, 2, 3)$ is



$u = A - B = (-1, -1, -2)$

$v = C - B = (-2, 2, 1)$

A. $3\sqrt{5}$

B. $\dfrac{3\sqrt{5}}{2}$

C. $5\sqrt{2}$

D. $\dfrac{5\sqrt{2}}{2}$

E. 25

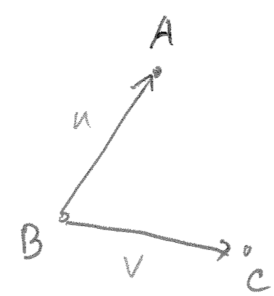
F. None of the above are correct

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & -2 \\ -2 & 2 & 1 \end{vmatrix} = (3, 5, -4)$$

$$A = \frac{1}{2}\|u \times v\| = \frac{1}{2}\sqrt{3^2 + 5^2 + (-4)^2} = \frac{1}{2}\sqrt{50} = \frac{5}{2}\sqrt{2}$$

A B C

2. An equation for the plane passing through the points $(1, 0, -1)$, $(1, 1, 0)$, and $(2, -1, 1)$ is:



A. $x - 2y + z = 0$

B. $3x + y - z = 4$

C. $x - 3y + z = 0$

D. $3x - y - z = 2$

E. $-x + y + z = -1$

F. None of the above are correct

$u = A - B = (0, -1, -1)$

$V = C - B = (1, -2, 1)$

$$u \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & -1 \\ 1 & -2 & 1 \end{vmatrix} = (-3, -1, 1)$$

$-3x - y + z = d$ plug in A

$-3 - 0 + (-1) = d \Rightarrow d = -4$

$-3x - y + z = -4$ or

$3x + y - z = 4$

3. What is the polar form of $\frac{1+i}{\sqrt{3}+i}$? $= \frac{re^{i\theta}}{se^{i\varphi}}$

A. $\frac{\sqrt{2}}{2}e^{i(7\pi/12)}$

B. $\frac{\sqrt{2}}{2}e^{i(\pi/12)}$

C. $\frac{2}{\sqrt{2}}e^{i(5\pi/12)}$

D. $\frac{2}{\sqrt{2}}e^{-i(7\pi/12)}$

E. $\frac{2}{\sqrt{2}}e^{-i(5\pi/12)}$

F. $\frac{\sqrt{2}}{2}e^{-i(\pi/12)}$

$1+i$: $r = \sqrt{1^2+1^2} = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\sin\theta = \frac{1}{\sqrt{2}}$ $\Rightarrow \theta = \frac{\pi}{4}$

$\sqrt{3}+i$: $s = \sqrt{\sqrt{3}^2+1^2} = \sqrt{4} = 2$

$\cos\varphi = \frac{\sqrt{3}}{2}$ $\sin\varphi = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{6}$

$$\Rightarrow \frac{1+i}{\sqrt{3}+i} = \frac{re^{i\theta}}{se^{i\varphi}} = \frac{\sqrt{2}e^{i(\pi/4)}}{2e^{i(\pi/6)}} = \frac{\sqrt{2}}{2}e^{i\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}$$

$$= \frac{\sqrt{2}}{2}e^{i(\pi/12)}$$

4. Consider the complex number $z = 1+i$. Then $z^8 =$

A. 16

B. $4(1+i)$

C. $1+i$

D. $16i$

E. $8(1+i)$

F. 32

$z = 1+i = \sqrt{2}e^{i\pi/4}$ (from #3)

$$z^8 = \left(\sqrt{2}e^{i\pi/4}\right)^8 = (\sqrt{2})^8 e^{i(\pi/4)\cdot 8}$$

$$= \cancel{16}\ 2^4\left(e^{2\pi i}\right) \qquad e^{2\pi i} = 1$$

$$= 16$$

5. Find the **real part** of the complex number

$$z = \frac{1+3i}{1-i} \quad =$$

A. 3

B. -1

C. 2

D. -2

E. -3

F. 1

$$z = \frac{1+3i}{1-i} = \frac{1+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+3i)(1+i)}{1^2+1^2}$$

$$= \frac{1}{2}(1-3+3i+i)$$

$$= \frac{1}{2}(-2+4i)$$

$$= -1 + 2i \qquad \text{so } \operatorname{Re}(z) = -1$$

6. Let u, v, and w be nonzero vectors in $\mathbb{R}^3$. Which of the following statements are true?

(1) The expression $(u \cdot w)v - (u \cdot v)w$ is a scalar F, it is a vector

(2) The expression $(u \times v) \times (u \cdot w)$ is not defined. T, becauze u·w is a scalar

(3) $u \cdot (v \times w)$ is a vector in $\mathbb{R}^3$. F, it is a dot product

(4) $\|u \times v\| = \|u\|\|v\| \cos\theta$ where θ is the angle between u and v. F, should be $\sin\theta$

(5) If $u \cdot (v \times w) = 0$, then the three vectors are coplanar. T

(6) The vectors u and $u \times v$ are parallel. F, they are perpendicular

A. (3) only

B. (4) and (5)

C. (2) only

D. (1) and (5)

E. (2) and (5)

F. (1) and (6)

7. Consider the two vectors $u = (1, 1, 1)$ and $v = (2, 1, 3)$ in $\mathbb{R}^3$. Find the orthogonal projection of u onto the vector v, i.e. find $\text{proj}_v(u)$.

A. $\frac{4}{7}(1, 1, 1)$

B. $\frac{2}{7}(1, , 1, 1)$

C. $\frac{4}{7}(2, 1, 3)$

D. $\frac{3}{7}(1, 1, 1)$

E. $\frac{2}{7}(2, 1, 3)$

F. $\frac{3}{7}(2, 1, 3)$

$$\text{proj}_v(u) = \frac{u \cdot v}{\|v\|^2} v = \frac{2+1+3}{2^2+1^2+3^2}(2,1,3)$$

$$= \frac{6}{14}(2,1,3)$$

$$= \frac{3}{7}(2,1,3)$$

8. Consider the lines L_1 and L_2 with the following parametric equations:

$$L_1 \; : \; x = s - 1, \; y = 2s - 3, \; z = -3s + 2, \qquad s \in \mathbb{R}$$

$$L_2 \; : \; x = -2t, \; y = -4t - 1, \; z = 3t + 2, \qquad t \in \mathbb{R}$$

Which **one** of the following statements is true?

A. L_1 and L_2 intersect at the point $(0, 1, 2)$.

B. L_1 and L_2 intersect at the point $(-2, -5, 5)$.

C. L_1 and L_2 do not intersect and are parallel.

D. L_1 and L_2 intersect at the point $(-1, -3, 2)$.

E. L_1 and L_2 do not intersect and are not parallel.

F. L_1 and L_2 are perpendicular.

$L_1 = (-1,-3,2) + s(1,2,-3)$
$L_2 = (0,-1,2) + t(-2,-4,3)$

$L_1 = L_2$
$(-1,-3,2) + s(1,2,-3) = (0,-1,2) + t(-2,-4,3)$
$(-1+s, -3+2s, 2-3s) = (-2t, -4t-1, 3t+2)$

① $-1+s = -2t$
② $-3+2s = -4t-1$
③ $2-3s = 3t+2$

① $s = 1-2t$

into ② $-3+2(1-2t) = -4t-1$
$-3+2-4t = -4t-1$
$-1 = -1$ ok.

into ③ $2-3(1-2t) = 3t+2$
$2-3+6t = 3t+2$
$3t = 3$
$t = 1 \Rightarrow s = -1$

$L_1 = (-1,-3,2) + (-1)(1,2,-3) = (-2,-5,5)$ | $L_2 = (0,-1,2) + (-2,-4,3) = (-2,-5,5)$

9. Let L be the line in $\mathbb{R}^3$ given by the following parametric equations:

$$x = -2t + 1, \; y = t, \; z = -t + 2, \qquad t \in \mathbb{R}.$$

Then the equation of the plane which contains the point $(1, -1, -2)$ and the line L is

A. $5x - 8y - 2z = 17$

B. $5x + 8y + 2z = -7$

C. $x - z = 3$

D. $y + z = -3$

E. $5x + 8y - 2z = 1$

F. None of the above are correct.

$L = (1,0,2) + t(-2,1,-1)$

$(1,0,2)$ and $(1,-1,-2)$ on the plane

$\Rightarrow u = (1,0,2) - (1,-1,-2) = (0,1,4)$

is $\parallel$ to plane

$\therefore$ $n = u \times d$ is normal

$$u \times d = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 4 \\ -2 & 1 & -1 \end{vmatrix} = (-5,-8,2) \Rightarrow -5x - 8y + 2z = C$$

$-5 + 8 - 4 = C \Rightarrow C = -1$

$-5x - 8y + 2z = -1$

$5x + 8y - 2z = 1$

10. The volume of the parallelepiped formed by the vectors $u = (1, 0, -2)$, $v = (0, 1, 1)$ and $w = (1, 4, 0)$ is

A. 1

B. 4

C. 2

D. 0

E. 3

F. None of the above are correct.

$V = |u \cdot (v \times w)|$

$$(v \times w) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 4 & 0 \end{vmatrix} = (-4, 1, -1)$$

$u \cdot (v \times w) = (1,0,-2) \cdot (-4,1,-1)$

$= -4 + 0 + 2 = -2$

$\therefore V = 2$

11. Let L be the line formed by the intersection of the two planes $y+z=1$ and $x-y=-2$. Which of the following vectors is a direction vector for L?

A. $(1, 1, -1)$

B. $(-1, 1, -1)$

C. $(1, 0, -1)$

D. $(1, -1, -1)$

E. $(0, 1, -1)$

F. None of the above are correct.

$\parallel$ to both planes

$\Rightarrow$ $\perp$ to both normals

$y+z=1$: $n_1 = (0,1,1)$

$x-y=-2$: $n_2 = (1,-1,0)$

$$n_1 \times n_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = (1,1,-1)$$

$d = (-2,0,3)$, through $(1,-3,2)$.

12. The equations $x = -2t + 1$, $y = -3$, $z = 3t + 2$ (where $t \in \mathbb{R}$) represent...

A. A plane in $\mathbb{R}^3$ through $(1, -3, 2)$ with normal vector $n = (-2, 0, 3)$.

B. A plane in $\mathbb{R}^3$ through $(-2, 0, 3)$ with normal vector $n = (1, -3, 2)$.

C. A plane in $\mathbb{R}^3$ through $(-1, 3, -2)$ with normal vector $n = (2, 0, -3)$.

D. A line in $\mathbb{R}^3$ through the points $(1, -3, 2)$ and $(-2, 0, 3)$.

E. A line in $\mathbb{R}^3$ through $(1, -3, 2)$ with direction vector $d = (-2, 0, 3)$.

F. A line in $\mathbb{R}^3$ through $(-2, 0, 3)$ with direction vector $d = (1, -3, 2)$.

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