Midterm Exam Engr 233

February 27, 2006: $\boxed{\textbf{Return this page}}$

A: Time allowed: 1h15min.

[10 points] Problem 1.
Find the normal component of the acceleration a_N for a particle that moves as described by the vector-valued function

$$\vec{r}(t) = \left\langle 3t, t^2 - 2, t \right\rangle$$

[10 points] Problem 2
Find the work done by the force field $\vec{F}(x, y) = -y^2\mathbf{i} - xy\mathbf{j}$ along the curve

$$\vec{r}(t) = \left\langle 2t, t^2 \right\rangle$$

during the time from $t = 0$sec to $t = 2$sec.

[10 points] Problem 3.
Recall that given any scalar function $f(x, y, z)$ we have the identity

$$curl(grad(f)) = 0\ .$$

Compute the *curl* of the vector field below and **explain in words** why it is not the gradient of any function

$$\vec{F}(x, y, z) = \left\langle y^2, 2xy, x \right\rangle$$

[10 points] Problem 4. Find an equation for the tangent plane to the following surface at the indicated point

$$x^3y - z^2 + xyz = -5\ , \qquad (-1, 2, 1)$$

[10 points] Problem 5.
Given the curve

$$\vec{r}(t) = \langle t, 2\cos(t), 3\sin(t) \rangle$$

find the equation of the tangent line at the point $(\pi, -2, 0)$.

[10 points] Problem 6.
One of the following integrals is independent of the path; find which one and evaluate it

$$\int_{(\pi,2)}^{(2,0)} x\cos(xy)\mathrm{d}x + y\cos(xy)\mathrm{d}y$$

$$\int_{(\pi,2)}^{(2,0)} y\cos(xy)\mathrm{d}x + x\cos(xy)\mathrm{d}y$$

$\cos\theta = \vec{a}\bullet\vec{b}/(\|\mathrm{a}\|\|\mathrm{b}\|)$

$\mathrm{comp_b a} = \|\mathrm{a}\|\cos\theta = \mathrm{a}\cdot\hat{\mathrm{b}}$

$\mathrm{proj_b a} = (\vec{\mathrm{a}}\cdot\hat{\mathrm{b}})\hat{\mathrm{b}}$

Area of a parallelogram = $\| a \times b \|$

Volume of a parallelepiped = $|a \bullet (b \times c)|$

$$\text{Equation of a line}: \vec{r} = \vec{r}_2 + t(\vec{r}_2 - \vec{r}_1) = \vec{r}_2 + t\vec{a}$$

$$\text{Equation of a plane}: a\,\mathrm{x} + b\,\mathrm{y} + c\mathrm{z} + d = 0$$

$$\text{also}: \ [(\vec{r}_2 - \vec{r}_1)\times(\vec{r}_3 - \vec{r}_1)]\bullet(\vec{r} - \vec{r}_1) = 0$$

$$\frac{d\vec{r}(s)}{dt} = \frac{d\vec{r}}{ds}\frac{ds}{dt}$$

$$\text{Length of a curve}: s = \int_{t_1}^{t_2} |\vec{r}\,'(t)|\,dt$$

Curvature of a smooth curve:

$$\kappa = \left\|\frac{d\vec{\mathrm{T}}}{ds}\right\| = \left\|\frac{d^2\vec{\mathrm{r}}}{ds^2}\right\| = \frac{|\vec{\mathrm{T}}'|}{|\vec{\mathrm{r}}'|} = \frac{\|\mathrm{r}'(t)\times\mathrm{r}''(t)\|}{\|\mathrm{r}'(t)\|^3}$$

Acceleration:

$$\vec{\mathrm{a}}(t) = \kappa v^2\hat{\mathrm{N}} + \frac{dv}{dt}\hat{\mathrm{T}} = a_N\hat{\mathrm{N}} + a_T\hat{\mathrm{T}}$$

$$\hat{\mathrm{N}} = \frac{d\mathrm{T}/dt}{\|d\mathrm{T}/dt\|}$$

$$\hat{\mathrm{T}} = \frac{\mathrm{r}'(t)}{\|\mathrm{r}'(t)\|}$$

$$a_T = \frac{dv}{dt} = \frac{\|\mathrm{v}\bullet\mathrm{a}\|}{\|\mathrm{v}\|} \ \ \& \ \ a_N = kv^2 = \frac{\|\mathrm{v}\times\mathrm{a}\|}{\|\mathrm{v}\|}$$

The Binormal $\hat{\mathrm{B}} = \hat{\mathrm{T}}\times\hat{\mathrm{N}}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} \ \ \& \ \ \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

$$\nabla = \mathrm{i}\frac{\partial}{\partial x} + \mathrm{j}\frac{\partial}{\partial y} + \mathrm{k}\frac{\partial}{\partial z}$$

$$D_u(F) = \nabla F\bullet\hat{u}, \ \hat{u} = \text{unit vector}$$

Equation of Tangent Plane: $\vec{n}_o\bullet(\vec{r} - \vec{r}_o) = 0, \ \vec{n}_o = \nabla F \text{ at P}$

$$\text{equation of normal line to a surface}: \ \vec{n}_o\times(\vec{r} - \vec{r}_o) = 0, \ \vec{n}_o = \nabla F \text{ at P}$$

$$\text{Also}: x = x_o + t\,F_x(x_o, y_o, z_o), \ \ y = y_o + t\,F_y(x_o, y_o, z_o),$$

$$\mathrm{z} = \mathrm{z}_o + t\,F_z(x_o, y_o, z_o)$$

$$W = \int_C \vec{F}\bullet d\vec{r} = \int_c F_1(x,y,z)dx + \int_c F_2(x,y,z)dy + \int_c F_3(x,y,z)dz$$

$$\int_C F(x,y)ds = \int_a^b F(f(t), g(t))\sqrt{[f']^2 + [g']^2}\,dt = \int_a^b F(x, f(x))\sqrt{1 + [f']^2}\,dx$$