CONCORDIA UNIVERSITY
Department of Economics

ECON 301 INTERMEDIATE MICROECONOMIC THEORY I
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Summer 2015

PROBLEM SET 1 SOLUTION

Name: **I.D:**

Points Total: points

1. TRUE/FALSE (1 point each - 4 points total)

a)If someone has the utility function $U = 1{,}000 + 2\min\{x, y\}$, then x and y are perfect complements for that person.

ANS: T

b)Ambrose has an indifference curve with equation $x_2 = 20 - 4x^{1/2}{}_1$. When Ambrose is consuming the bundle (4, 16), his marginal rate of substitution is 25/4.

ANS: F

c)Wanda Lott has the utility function $U(x, y) = \max\{x, y\}$. Wanda's preferences are convex.

ANS: F

d)If one utility function is a monotonic transformation of another, then the former must assign a higher utility number to every bundle than the latter.

ANS: F

2. A consumer has a utility function of the form U(x, y) = *x* a + *y* b, where both *a* and *b* are nonnegative. What additional restrictions on the values of the parameters *a* and *b* are imposed by each of the following assumptions?
a. Preferences are quasilinear and convex, and *x* is a normal good.
b. Preferences are homothetic.
c. Preferences are homothetic and convex.
d. Goods *x* and *y* are perfect substitutes.

ANS:
a. $a = 1$ and b is between 0 and 1.
b. $a = b$.
c. $a = b$ and a is between 0 and 1.
d. $a = b = 1$.

3. Mac Rowe doesn't sweat the petty stuff. In fact, he just cannot detect small differences. He consumes two goods, *x* and *y*. He prefers the bundle (x, y) to the bundle (x', y') if and only if $(xy - x'y' > 1)$. Otherwise he is indifferent between the two bundles.
a. Show that the relation of indifference is not transitive for Mac. (Hint: Give an example.)
b. Show that the preferred relation is transitive for Mac.

ANS:
a. Consider the bundles $A = (1, 1)$, $B = (1, 1.75)$, and $C = (1, 2.5)$. Then *A* is indifferent to *B* and *B* to *C*, but *C* is preferred to *A*.
b. To see that strict preference is transitive, suppose we have any three bundles, (x, y), (x', y') and (x'', y''). If the first is preferred to the second and the second to the third, then $xy - x'y' > 1$ and $x'y' - x''y'' > 1$. Simple algebra shows that $xy - x''y'' > 1$. Therefore the first must be preferred to the third.

4. Max has the utility function $U(x, y) = x(y + 1)$. The price of *x* is \$2 and the price of *y* is \$1. Income is \$10. How much *x* does Max demand? How much *y*? If his income doubles and prices stay unchanged, will Max's demand for both goods double?

ANS:
To set his MRS equal to the price ratio, Max sets $(y + 1)/x = 2$. His budget constraint is $2x + y = 10$. Solve these two equations to find that $x = 11/4$ and $y = 9/2$. If his income doubles and prices stay unchanged, his demand for both goods does not double. A quick way to see this is to note that if quantities of both goods doubled, the MRS would not stay the same and hence would not equal the price ratio, which has stayed constant.

5. Use separate graphs to sketch two indifference curves for people with each of the following utility functions:

a. U(x, y) = x+2y.

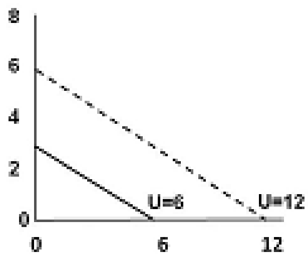
b. U(x, y) = min{x,2y}.

c. U(x, y) = max{x,2y}.

ANS:

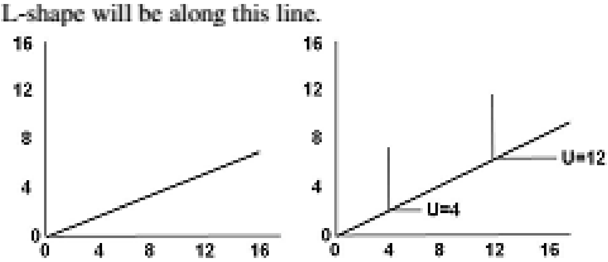
(a) U(x, y) = $x + 2$y.

Pick a utility level, say U(x,y) = k = x + 2y. Then we can write y = -1/2 x + 1/2 k. This is a line with slope -1/2 and y-intercept 1/2 k. If we set k = 6, we get the solid line labeled U = 6, and if we set k = 12, we get the dashed line.



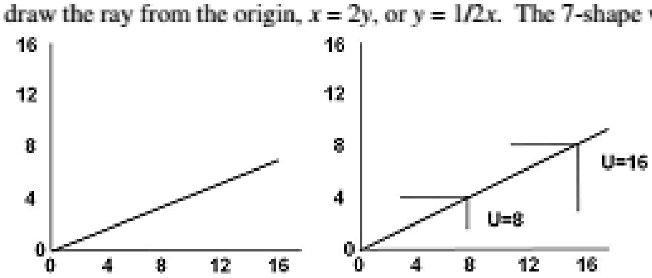
(b) U(x, y) = min{x,2y}.

This is perfect compliments, the L-shape. Here we draw the ray from the origin, x = 2y, or y = 1/2x. The L-shape will be along this line.



(c) U(x, y) = max{x,2y}.

In this instance, the shape is like a 7 as opposed to an L, and we simply choose the larger. Again, we draw the ray from the origin, x = 2y, or y = 1/2x. The 7-shape will be along this line.



6. Max has a utility function $U(x,y) = 2xy + 1$. The prices of x and y are both \$1 and Max has an income of \$20.

a. How much of each good will he demand?

b. A tax is placed on x so that x now costs Max \$2 while his income and the price of y stay the same. How much of good x does he now demand?

c. Would Max be as well off as he was before the tax if when the tax was imposed, his income rose by an amount equal to \$1 times the answer to part (b)?

ANS:

(a) How much of each good will he demand?

Assume that Max is a rational individual, so he will maximize his utility. From the above information, his budget constraint is $x + y = 20$. We can solve for $y = 20 - x$ and substitute into the utility function to get $U(x, y) = 2x(20 - x) + 1$. To maximize utility, we take the derivative with respect to x, set the derivative equal to zero, and solve for x. $dU/dx = 40 - 4x = 0$. Then, $x = 10$. Substituting back into the budget constraint, $x + y = 20$, we find that $y = 10$.

(b) A tax is placed on x so that x now costs Max \$2 while his income and the price of y stay the same. How much of good x does he now demand?

The new budget constraint is $2x + y = 20$. Using the same process as in part (a), we can solve for $y = 20 - 2x$ and substitute into the utility function to get $U(x, y) = 2x(20 - 2x) + 1$. To maximize utility, we take the derivative with respect to x, set the derivative equal to zero, and solve for x.

$dU/dx = 40 - 8x = 0$. Then, $x = 5$. (Substituting back into the budget constraint, $2x + y = 20$, we find that $y = 10$.)

(c) Would Max be as well off as he was before the tax if when the tax was imposed, his income rose by an amount equal to \$1 times the answer to part (b)?

Now, Max's income rises by \$1 times 5 to 25. The new budget constraint is $2x + y = 25$. Using the same process as in part (a), we can solve for $y = 25 - 2x$ and substitute into the utility function to get $U(x, y) = 2x(25 - 2x) + 1$. To maximize utility, we take the derivative with respect to x, set the derivative equal to zero, and solve for x. $dU/dx = 50 - 8x = 0$. Then, $x = 6.25$. Substituting back into the budget constraint, $2x + y = 25$, we find that $y = 12.5$.

To determine if Max is as well off as he was before the tax, part (a),

$U(x, y) = 2xy + 1 = 2(10)(10) + 1 = 201$.

After the tax, part (c), $U(x, y) = 2xy + 1 = 2(6.25)(12.5) + 1 = 157.25$

Clearly, Max is not as well off as he was before the tax if when the tax was imposed, his income rose by an amount equal to \$1 times the answer to part (b). Therefore, the answer is NO.