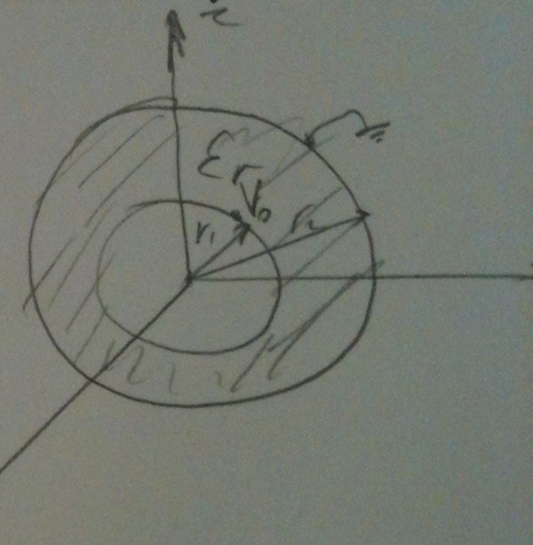
Problem 5: [18 marks]

A capacitor consists of two partial spherical conducting shells of radii r_1 *and* r_2 respectively and limited to $0 \le \theta \le \alpha$ $0 \le \varphi \le 2\pi$ .The region between the two conductors is filled with a dielectrics of ε_r Neglecting edge effects and assuming a potential V_0 at $r = r_1$ and a zero potential a $r = r_2$:

a) Make a good sketch of the capacitor.
b) Electric potential V in the dielectric region is a function of what variable?
c) Solve Laplace equation in the dielectric region to obtain Electric potential V in the dielectric region
d) AT what points the solution is invalid?
e) Write an equation representing equipotential surfaces .

Also determine:

f) The electric field intensity E in the dielectric region.
g) The surface charge density on the inner surface of the dielectric region $(r = r_1)$.
h) The surface charge density on the outer surface of the dielectric region $(r = r_2)$.
i) The capacitance
j) The leakage conductance assuming that the dielectric region has conductivity σ .
k) The energy stored in the electric field in the capacitor.



b) $\cancel{E}$ V is a function of r.

c) $\nabla^2 V = 0.$

$V = Ar + B.$

$V_0 = Ar_1 + B.$ $\quad V_0 = Ar_1 - Ar_2$

$0 = Ar_2 + B.$ $\quad B = -Ar_2$ $\quad V_0 = A(r_1 - r_2)$

d) $r = 0.$

e) $\frac{4}{3}\pi r^3$

f) $E = \frac{r^3}{3\pi\varepsilon_0\varepsilon_r}$

i) $C = \frac{Q}{V}$

j))

k) $W = \frac{1}{2}\int \rho\, ds.$

Problem 6: [14 marks]

I)

a) Write point form of all Maxwell equations for <u>static fields</u> and also for <u>time varying fields.</u>

b) Show why and how displacement current needs to be taken into consideration in one of the Maxwell's Equations. Obtain the expression for displacement current.

II)

Given that the electric field intensity vector E in the free space region is :

$\vec{E} = 100Cos(y)Sin(kz - \omega t)\vec{a}_x \quad [V/m]$

a) The corresponding time varying magnetic field intensity H.

b) The displacement current density vector J_d.

a) $B \times H = 0.$

$D \cdot B = ?$

Problem 1: [2×5 marks]

Multiple choice questions. Please circle the correct answer. Negative marks (50% of the full mark) are assigned for wrong answers. The minimum mark for problem 1 is zero.

1) If the strength of the magnetic field H is 10 (A/m) at a distance 2 cm away from an infinitely long wire with uniform current, what is at the distance 1 cm from the wire?

a) 20 (A/m) b) 10 (A/m) c) 5 (A/m) d) 2.5 (A/m)

$H = \int \frac{I\, dl \times a_z}{4\pi |r - r'|^2}$

2) Two magnetic media have an interface at x=0. If $\mu_{r1} = 1$ *and* $\mu_{r2} = 4$ and the magnetic field in region 1 is $\vec{H}_1 = -4\vec{a}_x + 12\vec{a}_y - 8\vec{a}_z$, the magnetic field $\vec{H}_2$ is:

$B = \mu H$

a) $-4\vec{a}_x + 12\vec{a}_y - 8\vec{a}_z$

b) $-4\vec{a}_x + 3\vec{a}_y - 2\vec{a}_z$

c) $-\vec{a}_x + 12\vec{a}_y - 8\vec{a}_z$

d) $-\vec{a}_x + 3\vec{a}_y - 2\vec{a}_z$

K

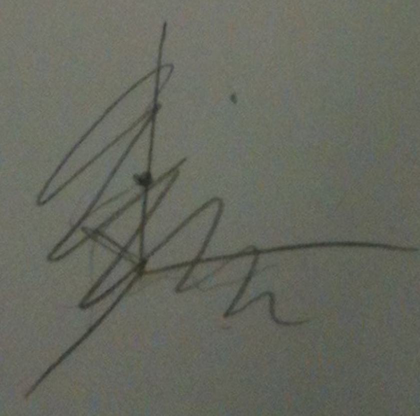
3) The z=0 plane carries a uniform surface current density of -20$\boldsymbol{a}_y$ (mA/m). At the point (2,3,-5)the magnetic field intensity is:

a) -10 $\boldsymbol{a}_y$ mA/m

b) -10 $\boldsymbol{a}_x$ mA/m

c) +10 $\boldsymbol{a}_x$ mA/m

d) -20 $\boldsymbol{a}_z$ mA/m



4) In a parallel-plate capacitor filled with paraffin ($\varepsilon_r = 2.2$), the charge on each plate is 10nC.What is the electric flux density D magnitude, if the area of each plate is $10^{-4}\,m^2$ (assuming uniform field distribution)?

$Q = 10\,nC$

$10^{-4}\,m^2$

$10^{-9}/10^{-4}$

10^{-5}

a) $10^{-4}\varepsilon_r\varepsilon_0\ \ C/m^2$
b) $10^{-4}\varepsilon_r\ \ C/m^2$
c) $10^{-4}/\varepsilon_r\ \ C/m^2$
d) $10^{-4}\ \ C/m^2$

5) Assume a rigid line of current, $I = 2\pi$ [A], coinciding with the y-axis in free space. (The current is flowing in the negative y-direction). If a uniform magnetic field $H = H_0(+\vec{a}_y)$ were applied, the line of current would experience a force:

a) In the $-\vec{a}_x$ direction
b) In the $+\vec{a}_x$ direction
c) In the $+\vec{a}_y$ direction
d) In the $+\vec{a}_z$ direction
e) In the $-\vec{a}_z$ direction
f) Zero

Problem 2: [12 marks]

An infinite sheet of current characterized by the surface current density vector $K = (\frac{A}{\rho^2})\vec{a}_\phi \; [A/m]$ (where A is a constant) occupies all of the z=0 plane except for a whole $\rho < \rho_0 \; [m]$ around the origin.

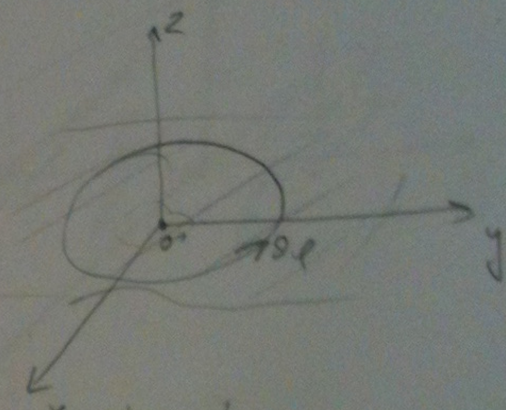
a) Use Biot-savart's law to develop the expression for the magnetic field intensity vector, $\vec{H}$, at the origin.

Show all steps in the derivation including the value for $\boldsymbol{r}, \boldsymbol{r}'$ and $\boldsymbol{r} - \boldsymbol{r}'$.

Hint: $\vec{H} = \int_s \frac{\vec{K} ds \times (\vec{r} - \vec{r}')}{4\pi |\vec{r} - \vec{r}'|^3}$

b) Determine the expression for the total current I flowing in the $\vec{a}_\phi$ direction.

a)



$$H = \int_s \frac{k\, ds \times (\vec{r} - \vec{r}')}{4\pi |\vec{r} - \vec{r}'|^3}$$

$\vec{r} = 0 \qquad (r - r') = -\rho a_\rho$

$\vec{r}' = \rho a_\rho \qquad ds = \rho d\rho d\phi \, \hat{a}_z$

$|\vec{r} - \vec{r}'| = \rho \qquad ds \times (\vec{r} - \vec{r}')$

$$\begin{vmatrix} a_\rho & -a_\phi & a_z \\ 0 & 0 & \rho d\rho d\phi \\ -\rho a_r & 0 & 0 \end{vmatrix} = -\hat{a}_\phi \rho^2 d\rho d\phi$$

$$H = \int_{\phi=0}^{2\pi} \int_{\rho_0} \frac{\frac{A}{\rho^2} a_\phi \cdot (-\rho^2 d\rho d\phi)}{4\pi \rho^3} = \frac{-A}{4\pi} a_\phi \iint \frac{1}{\rho^3} d\rho d\phi = \frac{-A}{2} \int_{\rho_0}^{\infty} \frac{1}{\rho^3} d\rho$$

$$\boxed{= \frac{A}{\rho^2} a_\phi \Big|_{\rho_0}^{\infty}} = 0 - \frac{A}{\rho_0^2} a_\phi$$

$I = \int J \cdot ds$

Problem 3: [14 marks]

The electrostatic potential in a dielectric material of relative permittivity ε_r is given as $V = x^2 y - 2z^2$ Volt. The dielectric material exists in $-5 \le y \le 0$; and the region $y \le -5$ is Perfect conductor and $y \ge 0$ is free space.

(a) Determine the electric field intensity $\vec{E}$ everywhere;

(b) Determine the electric flux density $\vec{D}$ everywhere;

(c) Determine the polarization $\vec{P}$ everywhere; $\approx E \quad \varepsilon_r \times E$

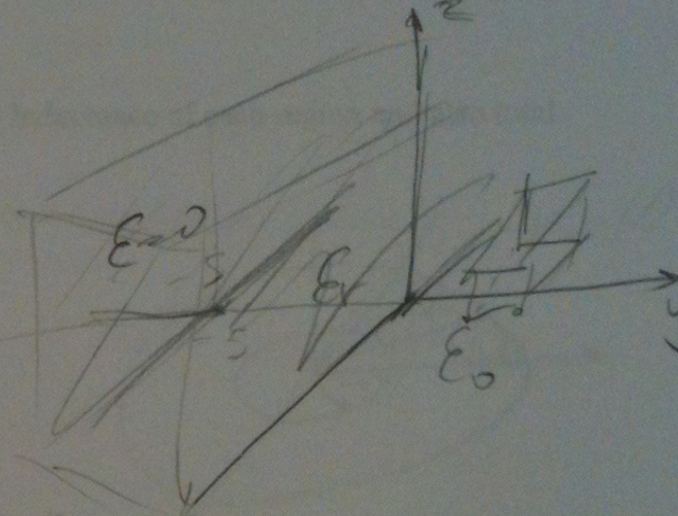
(d) Determine the volume charge density ρ_v in region $-5 \le y \le 0$;

(e) Determine the polarization volume charge density in the dielectric region;

(f) Determine the polarization surface charge density at dielectric-free space interface;

$E = -\nabla V$. $\quad V = -\nabla \cdot E$

$D = \varepsilon E$



a) $E = -\nabla V$.

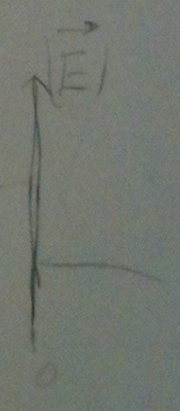
$\nabla V = 2xy + x^2 y - 4z$

$E = -2xy \hat{x} + x^2 y + 4z$

b) $\vec{D} = \varepsilon E$

$y < -5 \quad D = 0$. $\quad -5 \le y \le 0 \quad D = \varepsilon_r \varepsilon_0 E$

$y \ge 0 \quad D = \varepsilon_0 E$.



$D = \varepsilon E$.

$Q = \int \rho_v \, dv$. $\quad E = \dfrac{Q}{4\pi\varepsilon r^2}$

$\nabla Q = \rho_v$

$E = -\nabla V = \dfrac{Q}{4\pi\varepsilon r^2}$

$\dfrac{-\nabla V}{4\pi\varepsilon_r{}^2} = Q$

$\dfrac{-\nabla^2 V}{4\pi\varepsilon r^2} = \rho_v$

Problem 4: [18 marks]

A long coaxial cable has the inner conductor of a radius of 1 mm and an outer conductor of a radius of 4mm. The inner conductor is non-magnetic ($\mu_r = 1$).The inner conductor has a uniformly distributed total Current I flowing in the axial direction. The outer conductor carry the same current, flowing in the opposite direction.

i) The space between $\pi/2 \le \phi \le \pi$ and $1 \le r \le 4$ is filled with a material with permeability of $\mu_r = 10$. The rest of the space is free space.

a) Find H everywhere

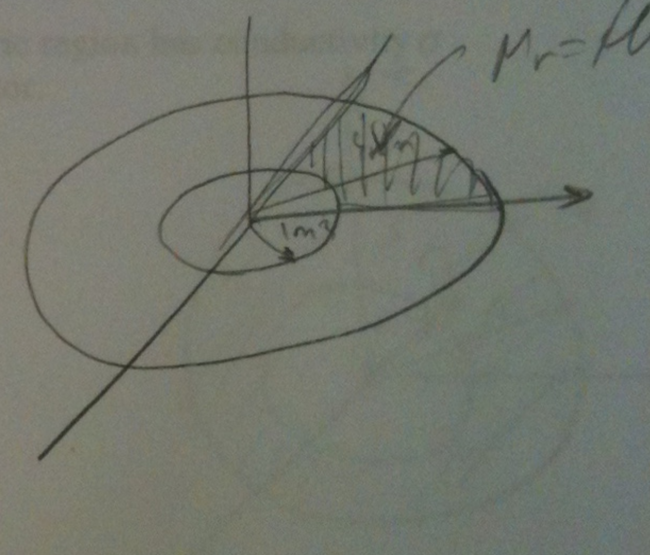
b) Find B everywhere

ii) Now for the same coaxial cable , we change the material of all the region between the two conductor to $\mu_r = 10$.(now the region $1 \le r \le 4$ and $0 \le \varphi \le 2\pi$ is filled with $\mu_r = 10$)

a) Find flux crossing each region

b) Using part ii)a) of this question, find Inductance of each region and also total inductance.

$$H = \int_S \frac{I\, dl \times (r - r')}{4\pi\, (r - r')^3}$$



$B = H \cdot \mu$