MATH 1104B
TEST 1
Sept. 28, 2012

TIME: 50 MIN.
NAME:
ST.NO.:

Q1) Which of the following augmented matrices are in echelon form
(Circle the correct answer)

a) $\begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 4 & 3 & -3 & 5 \\ 0 & 0 & 0 & 5 \\ 0 & 1 & 8 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (3 Marks)

Q2) Which of the following augmented matrices are in reduced echelon form
(Circle the correct answer)

a) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ (3 Marks)

Q3) Find the general solution of the linear system whose augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -3 & -5 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right]$$ (10 Marks)

$$\left[\begin{array}{ccc|c} 1 & -3 & -5 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right] \overset{R_1' = R_1 + 3R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 9 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$\Rightarrow$ x_3 is a free variable.

Let $x_3 = t \in \mathbb{R}$.

$\Rightarrow x_2 + x_3 = 3 \Rightarrow x_2 + t = 3$

$\Rightarrow x_2 = 3 - t$

$\Rightarrow x_1 - 2x_3 = 9 \Rightarrow x_1 - 2t = 9$

$\Rightarrow x_1 = 9 + 2t$

Thus, $X = \left\{ \begin{bmatrix} 9+2t \\ 3-t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}$.

Q4) **Find the inverse of the matrix A, if it exists, then check your answer**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$ **(10 marks)**

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\text{ops}}{\overset{\text{Row}}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{10} & -\frac{5}{10} & \frac{5}{10} \\ 0 & 1 & 0 & -\frac{3}{10} & -\frac{5}{10} & -\frac{1}{10} \\ 0 & 0 & 1 & \frac{7}{10} & \frac{5}{10} & \frac{-1}{10} \end{array}\right]$$

$$\| $$

$$[I \mid A^{-1}]$$

Thus, $A^{-1} = \frac{1}{10}\begin{bmatrix} -5 & -5 & 5 \\ -3 & -5 & -1 \\ 7 & 5 & -1 \end{bmatrix}$

Check

$$AA^{-1} = \frac{1}{10}\begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} -5 & -5 & 5 \\ -3 & -5 & -1 \\ 7 & 5 & -1 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3 .$$

Q5) **Solve the system by reducing it to (RREF)**

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 7x_2 - 2x_3 = 5 \qquad \textbf{(10 Marks)}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & 7 & -2 & 5 \\ -2 & 3 & 3 & 3 \end{array}\right] \xrightarrow{\text{Row Ops}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & -7/3 \end{array}\right]$$

Thus, $x_1 = -3 \quad x_2 = \frac{4}{3} \quad x_3 = -\frac{7}{3}$.

Q6) **If** $A = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ **. Show that** $A^3 = I$ **and use it to find** A^{-1} **. (9 marks)**

Consider $A^2 = AA = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$.

$A^3 = A^2A = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$\Rightarrow A^2A = I_2$

Thus, A^2 must be A's inverse!

by def'n of matrix inverse.

MATH 1104B

TEST2

Oct.12, 2012

TIME: 50 MIN.
NAME:
ST.NO.:

Q1) Use Cramer's rule to solve the system

$$x_1 + 2x_2 + 3x_3 = 14$$
$$2x_1 + x_2 + 2x_3 = 10 \quad \textbf{(15 marks)}$$
$$3x_1 + 4x_2 - 3x_3 = 2$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 4 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 14 \\ 10 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 4 & -3 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{matrix} = (1)(1)(-3) + (2)(2)(3) + (3)(2)(4) - (3)(1)(3) - (4)(2)(1) - (-3)(2)(2) = 28$$

$$|A_1(b)| = \begin{vmatrix} 14 & 2 & 3 \\ 10 & 1 & 2 \\ 2 & 4 & -3 \end{vmatrix} \begin{matrix} 14 & 2 \\ 10 & 1 \\ 2 & 4 \end{matrix} = 28 \Rightarrow x_1 = \frac{|A_1(b)|}{|A|} = \frac{28}{28} = 1$$

$$|A_2(b)| = \begin{vmatrix} 1 & 14 & 3 \\ 2 & 10 & 2 \\ 3 & 2 & -3 \end{vmatrix} \begin{matrix} 1 & 14 \\ 2 & 10 \\ 3 & 2 \end{matrix} = 56 \Rightarrow x_2 = \frac{|A_2(b)|}{|A|} = \frac{56}{28} = 2$$

$$|A_3(b)| = \begin{vmatrix} 1 & 2 & 14 \\ 2 & 1 & 10 \\ 3 & 4 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{matrix} = 84 \Rightarrow x_3 = \frac{|A_3(b)|}{|A|} = \frac{84}{28} = 3$$

Check

$(1) + 2(2) + 3(3) = 14$ ✓

$2(1) + (2) + 2(3) = 10$ ✓

$3(1) + 4(2) - 3(3) = 2$ ✓

Q2) Find A if $\left(2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}\right)^T=(4A^T)^{-1}$ **(7 Marks)**

$$\left(2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}\right)^T=(4A^T)^{-1}$$

take the transpose of both sides.

$$\Rightarrow 2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}=\left((4A^T)^{-1}\right)^T$$

$(A^T)^T = A$, $(A^T)^{-1} = (A^{-1})^T$

$$\Rightarrow 2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}=\left((4A^T)^T\right)^{-1}$$

$$\Rightarrow 2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}=\left(4^T(A^T)^T\right)^{-1}$$

$$\Rightarrow 2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}=(4A)^{-1}$$

$$\Rightarrow 2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}-5A^{-1}=\frac{1}{4}A^{-1}$$

$$\Rightarrow \left(\frac{1}{4}+5\right)A^{-1}=2\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{8}{21}\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}$$

$$\Rightarrow A=\left(\frac{8}{21}\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}\right)^{-1}=\left(\frac{8}{21}\right)^{-1}\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}^{-1}$$

$$=\frac{21}{8}\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}^{-1}$$

$$\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}^{-1}=\frac{1}{(2)(3)-(-1)(1)}\begin{bmatrix}3 & -1\\1 & 2\end{bmatrix}=\frac{1}{7}\begin{bmatrix}3 & -1\\1 & 2\end{bmatrix}$$

$$=\frac{21}{8}\left(\frac{1}{7}\begin{bmatrix}3 & -1\\1 & 2\end{bmatrix}\right)$$

$$=\frac{3}{8}\begin{bmatrix}3 & -1\\1 & 2\end{bmatrix}$$

Q3) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and det(A)= 7. Find the determinant of

$$B = \begin{bmatrix} 2a & 2b & 2c \\ g & h & i \\ 2d+2a & 2e+2b & 2f+2c \end{bmatrix}$$ (8 Marks)

$$|B| = \begin{vmatrix} 2a & 2b & 2c \\ g & h & i \\ 2d+2a & 2e+2b & 2f+2c \end{vmatrix}$$

theorem 3.3.2 on p.113.

$$= \begin{vmatrix} 2a & 2b & 2c \\ g & h & i \\ 2d & 2e & 2f \end{vmatrix}$$

theorem 3.3.3 on p.113.

$$= 2\begin{vmatrix} a & b & c \\ g & h & i \\ 2d & 2e & 2f \end{vmatrix}$$

theorem 3.3.3 on p.113.

$$= 2\cdot 2\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$$

theorem 3.3.1 on p.113

$$= -2\cdot 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= -4\,|A|$$

$$= -4(7) = -28.$$

Q4) Let A and B be two 3x3 matrices, and det A=4 and det B = -3. Find

$\det(2A^2B^TA^{-1}B)$ (7 Marks)

$$\det(2A^2B^TA^{-1}B) = 2^3(\det A)^2(\det B)(\det A)^{-1}(\det B)$$

Theorem 3.5 on p.120

Theorem 3.6 on p.121

$A^2 = AA$ and Theorem 3.5 on p.120.

Theorem 3.9 on p.124

Theorem 3.8 on p.122

$$= 2^3(4)^2(-3)(4)^{-1}(-3) = 288$$

Q5) Let $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$. Write A and its inverse as a product of elementary matrices. (8 Marks)

$$[A|I] = \left[\begin{array}{cc|cc} 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow E_1^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right] \xrightarrow{R_2' = \frac{1}{2}R_2} E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \Rightarrow E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} & 0 \end{array}\right] \xrightarrow{R_1' = R_1 - R_2} E_3 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Rightarrow E_3^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 0 \end{array}\right] = [I|A^{-1}]$$

$$\boxed{\begin{aligned} A^{-1} &= E_3E_2E_1 \\ A &= E_1^{-1}E_2^{-1}E_3^{-1} \end{aligned}}$$

$I = E_3E_2E_1A$

by defn of "Row-equivalent"

MATH 1104B
TEST3
Oct.26, 2012

TIME: 50 MIN.
NAME:
ST.NO.:

Q1) **Find the adjoint of the following matrix , and use it to find its inverse**

$$A=\begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix} \quad . \text{ (15 marks)}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

$$= \frac{1}{\det(A)} \left(\text{cofactor}(A)\right)^T$$

$$= \frac{1}{\det(A)} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \frac{1}{25} \begin{bmatrix} 14 & 3 & -1 \\ -9 & 7 & 6 \\ -12 & 1 & 8 \end{bmatrix}^T$$

$$= \frac{1}{25} \begin{bmatrix} 14 & -9 & -12 \\ 3 & 7 & 1 \\ -1 & 6 & 8 \end{bmatrix}$$

$$\det(A) = \left|\begin{matrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{matrix}\right| \begin{matrix} 2 & 0 \\ -1 & 4 \\ 1 & -3 \end{matrix}$$

$$= (2)(4)(5) + (0)(-2)(1) + (3)(-1)(-3) - (1)(4)(3) - (-3)(-2)(2) - (5)(-1)(0)$$

$$= 25$$

$$C_{11} = \left|\begin{matrix} 4 & -2 \\ -3 & 5 \end{matrix}\right| = 14$$

$$C_{12} = - \left|\begin{matrix} -1 & -2 \\ 1 & 5 \end{matrix}\right| = 3$$

$$C_{13} = + \left|\begin{matrix} -1 & 4 \\ 1 & -3 \end{matrix}\right| = -1$$

$$C_{21} = - \left|\begin{matrix} 0 & 3 \\ -3 & 5 \end{matrix}\right| = -9$$

$$\vdots$$

Q2) **For what Value(s) of h the following vectors are linearly dependent .**

$$\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix}$$ **(10 Marks)**

<u>Def</u> Linearly independent, $V_1, V_2, ---, V_n$.
means $C_1V_1 + C_2V_2 + --- + C_nV_n = 0$ (*)
$\Rightarrow C_1 = C_2 = --- = C_n$. (trivial solution)

Thus, $C_1\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} + C_2\begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix} + C_3\begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix} = 0$ (**)

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ -1 & -5 & 5 & 0 \\ 4 & 7 & h & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & h-6 & 0 \end{array}\right]$$

<u>Def</u> Linearly dependent, $V_1, V_2, ---, V_n$
means if (*), then $\exists (C_1, C_2 \dots C_n) \neq (0 0 --- 0)$

Thus, (**) has non-trivial solution
iff
$h-6=0$
iff
$h=6$.

Q3) Determine whether the vector(8,0,5) is a linear combination of the vectors (1,2,3) , (0,1,4), and (2,-1,1). (13 Marks)

$$\exists\ (c_1\, c_2\, c_3) \text{ s.t } c_1\begin{pmatrix}1\\2\\3\end{pmatrix} + c_2\begin{pmatrix}0\\1\\4\end{pmatrix} + c_3\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}8\\0\\5\end{pmatrix} \overset{?}{=}$$

$$\begin{bmatrix}1 & 0 & 2\\2 & 1 & -1\\3 & 4 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}8\\0\\5\end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c}1 & 0 & 2 & 8\\2 & 1 & -1 & 0\\3 & 4 & 1 & 5\end{array}\right] \overset{\text{Row ops.}}{\sim} \left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\0 & 1 & 0 & -1\\0 & 0 & 1 & 3\end{array}\right]$$

$$\Rightarrow c_1 = 2,\quad c_2 = -1,\quad c_3 = 3.$$

<u>Check</u>

$$2\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}0\\1\\4\end{pmatrix} + 3\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}2-0+6\\4-1-3\\6-4+3\end{pmatrix} = \begin{pmatrix}8\\0\\5\end{pmatrix} \checkmark$$

Q4) Consider the subset $W = \left\{ \begin{bmatrix} a \\ 2a \\ 3a \end{bmatrix}, a \in R \right\}$ **of** R^3**. Determine if W is a subspace of** R^3**.** **(7marks)**

We will prove this by using subspace test!
(Theorem 4.5 on p. 162)

required-to-show

1) R.T.S. : closure under addition
i.e. if $w_1, w_2 \in W$, then $w_1 + w_2 \in W$.

<u>Proof</u>. Let $w_1 = \begin{bmatrix} \alpha \\ 2\alpha \\ 3\alpha \end{bmatrix} \in W$. Thus, $\alpha \in \mathbb{R}$.

Let $w_2 = \begin{bmatrix} \beta \\ 2\beta \\ 3\beta \end{bmatrix} \in W$. Thus, $\beta \in \mathbb{R}$.

Consider $w_1 + w_2 = \begin{bmatrix} \alpha \\ 2\alpha \\ 3\alpha \end{bmatrix} + \begin{bmatrix} \beta \\ 2\beta \\ 3\beta \end{bmatrix} = \begin{bmatrix} (\alpha+\beta) \\ 2(\alpha+\beta) \\ 3(\alpha+\beta) \end{bmatrix} \in W$

because $\alpha + \beta \in \mathbb{R}$
and is of the same form
as elements in W.

2) R.T.S: closure under scalar multiplication
i.e. if $w \in W$, $c \in \mathbb{R}$, then $cw \in W$.

<u>Proof</u>. Let $w = \begin{bmatrix} \alpha \\ 2\alpha \\ 3\alpha \end{bmatrix} \in W$. Thus, $\alpha \in \mathbb{R}$.

Take $c \in \mathbb{R}$. Then $cw = c \begin{bmatrix} \alpha \\ 2\alpha \\ 3\alpha \end{bmatrix} = \begin{bmatrix} (c\alpha) \\ 2(c\alpha) \\ 3(c\alpha) \end{bmatrix} \in W$.

Math1104B
TEST4
Nov.16, 2012

TIME: 50 MIN.
NAME:
ST.NO.:

Q1) Let $A = \begin{bmatrix} 1 & -3 & 2 & 5 \\ -2 & 6 & 0 & -3 \\ 4 & -12 & -4 & -1 \end{bmatrix}$.

1) **Find a basis for Col(A)**
2) **Find a basis for Nul(A) (15 Marks)**
3) **Find a basis for Row(A)**

$$A = \begin{bmatrix} 1 & -3 & 2 & 5 \\ -2 & 6 & 0 & -3 \\ 4 & -12 & -4 & -1 \end{bmatrix} \overset{\text{row ops.}}{\sim} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & -3 & 2 & 5 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = \text{REF of } A.$$

1) Since the 1^{st} & 3^{rd} columns are the pivot columns of REF of A, the 1^{st} and 3^{rd} columns of A form a basis for col(A). Thus,

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} \right\} \text{ is a basis for col}(A).$$

2) We see that x_2 and x_4 are free-variables.
Let $x_2 = \ \in \mathbb{R}$
$x_4 = \ \in \mathbb{R}$. $\Rightarrow$ $x_1 = 3s - (\frac{3}{2})t$
$x_3 = -\frac{7}{4}t$

Thus, $Ax = 0 \Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3/2 \\ 0 \\ -7/4 \\ 1 \end{bmatrix} t$

Hence, $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 0 \\ -7/4 \\ 1 \end{bmatrix} \right\}$ is a basis for nul(A).

3). Take the non-zero rows in REF of A. They form a basis for row(A). Hence, $\{[1\ -3\ 2\ 5], [0\ 0\ 4\ 7]\}$ is a basis for row(A)

Q2) Diagonalize the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ **(10 Marks)**

To show that A is diagonalizable, we will show that A has 2 distinct eigenvalues. Then, by Theorem 7.6 (on p. 358), A is diagonalizable.

$$\det(A - \lambda I) = 0 \iff \det\left(\begin{bmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}\right) = 0$$

$$\iff (1-\lambda)(2-\lambda) - 12 = 0$$

$$\iff 2 - 3\lambda + \lambda^2 - 12 = 0$$

$$\iff \lambda^2 - 3\lambda - 10 = 0$$

$$\iff (\lambda - 5)(\lambda + 2) = 0$$

$$\iff \lambda = 5 \text{ and } -2.$$

Thus, A has 2 distinct eigenvalues, namely $\lambda_1 = 5$ and $\lambda_2 = 2$. By Theorem 7.6, A is diagonalizable!

Note: Check this!

For $\lambda_1 = 5$, the eigenvector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = P_1$.

For $\lambda_2 = -2$, the eigenvector is $\begin{bmatrix} -4 \\ 3 \end{bmatrix} = P_2$.

Thus, we have $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix} = P^{-1}AP$ where $P = [P_1\ P_2]$

Q3) Let $v_1 = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -6 \\ 3 \end{bmatrix}$ **and** $p = \begin{bmatrix} 1 \\ 14 \\ -9 \end{bmatrix}$. **Determine if p is in Col A, where** $A = [v_1 \quad v_2 \quad v_3]$. **(10 marks)**

The question is asking if P can be written as a linear combination of V_1, V_2, and V_3.

Let

$$X_1 V_1 + X_2 V_2 + X_3 V_3 = P. \quad (*)$$

If we can find such X_1, X_2, and X_3, then $p \in Col(A)$.

$(*)$ implies $\begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ -9 \end{bmatrix}$

Thus, $\left[\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 6 & 3 & 3 & -9 \end{array}\right] \underset{\sim}{\text{Row ops}} \left[\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

We see that this system is consistent. Thus $\exists$ $(X_1\ X_2\ X_3)$ s.t we have $(*)$. Actually, we have infinitely many $(X_1\ X_2\ X_3)$'s! Pick one! check.

Thus, $p \in col(A)$.

Q4) Determine the dimension of the subspace H of R^3 spanned by vectors v_1, v_2, v_3,

$$v_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 6 \\ -7 \end{bmatrix}.$$ **(10 marks)**

$H = \text{span}\{v_1, v_2, v_3\}$.

To determine dim(H), find a basis of H and the # of vectors in the basis is equal to dim(H).

Let $A = [v_1 \ v_2 \ v_3] \xrightarrow{\text{Row ops}} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 5 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \text{REF of } A$.

Thus, 1st & 2nd colums are the pivot columns. So, $\left\{ \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix} \right\}$ forms a basis of H. Thus, dim(H) = 2.