

**Concordia University Dept. Engineering**  
**ENGR 371 Mid-Term Exam Sec. CC Summer 2008**

Name: \_\_\_\_\_

ID: \_\_\_\_\_

Allowed Materials: Original copy of the official course textbook (3<sup>rd</sup> or 4<sup>th</sup> edition)

No other written material allowed.

Department approved calculator

No other electronic devices permitted (including cell phones and other wireless devices)

Instructions: i) Attempt all questions.

ii) Show all of your work in the workbook.

iii) Put a box around your final answer.

iv) Put your name and ID on both the answer book and the mid-term question sheet.

v) You will have 80 minutes to complete the exam. At the end you must stop writing and return your exam to the invigilator.

**Question 1)** An aircraft engine has a dual ignition system made of two separate, but identical ignition systems. The ignition systems are connected in parallel so that in case one ignition system, fails, the other can take over. Each ignition system has a reliability of “ $x$ ” (e.g. the likelihood of the ignition system working is “ $x$ ”). The engine requires that the starter motor (reliability 0.95) *and* the fuel pump (reliability 0.8) *and* the exhaust system (reliability of 0.9) and the dual ignition system *all* work. The system is required to have an overall engine reliability of 0.62244. (Show all of your work and put a box around your final answer):

- a) Construct a well labelled block diagram of the system (2 Marks).
- b) Compute the value of the reliability of the ignition system required to satisfy the requirements given above(4 Marks).

**Question 2)** Using the engine design from question 1 and assuming that an aircraft needs at least half of the engines to operate (show all of your work and put a box around your final answer): Hint, uses all of the decimal places to get an accurate answer. (Show all of your work and put a box around your final answer):

- A) Compute the probability that a two-engine aircraft will operate? (2 Marks)
- B) Compute the probability that a four-engine aircraft will operate? (2 Marks)
- C) Determine which aircraft is the more reliable (will be more likely to operate). Validate your answer. (2 Marks)

**Question 3)** An assessment of 4000 computer hard drives indicated that 3.5% of the 100 GB drives malfunctioned after 5 years, and 2.5 % of the 150 GB hard drives malfunctioned after 5 years and 1.5% of the 200 GB hard drives malfunctioned after 5 years. There are an equal number of 100 GB and 200 GB hard drives while there are twice as many 150 GB hard drives. For parts a) and b) you can assume that 5 years have passed. (Show all of your work and put a box around your final answer):

- a) Determine the likelihood that given a hard drive failure that is a 200 GB hard drive (2 Marks).
- b) Determine the likelihood that a hard drive fails (2 Marks).

**Question 4)** Every week the biomedical engineering department of a hospital must allocate their four member technical staff members for different tasks. One of the tasks is routine maintenance of medical equipment. For any given week the number of technicians needed for maintenance is given by the following probability density: the probability of needing no technicians is 0.2, of needing one technician is 0.3, of needing two technicians is 0.3, of needing three technicians is 0.1 and of needing four technicians is 0.1. (Show all of your work and put a box around your final answer)

- a) Compute the expected number of technicians needed for maintenance (1 Mark).
- b) Compute the variance of the number of technicians needed for maintenance (1 Mark).
- c) Compute the probability that when 25% of the technicians are sick (i.e. unavailable) there will be enough staff to perform routine maintenance (1 Mark).
- d) Compute the cumulative distribution function for the above probability density function (1 Mark).

## Formulas

Permutations of subsets size  $r$  from a set of  $n$  elements:  $\frac{n!}{(n-r)!}$

Combinations of subsets size  $r$  from a set of  $n$  elements:  $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Addition Rule:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Baye's Rule:  $P(E_k|B) = \frac{P(B|E_k)P(E_k)}{\sum P(E_k|B)P(E_k)}$

Total Probability:  $P(B) = \sum_{k=1}^n P(B|E_k)P(E_k)$

Mean:  $\mu = E(X) = \sum_{all\ x} xf(x)$

Variance:  $\sigma^2 = V(X) = E((X - \mu)^2)$

Binomial Distribution:  $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

Geometric Distribution:  $f(x) = p^1 (1-p)^{x-1}$

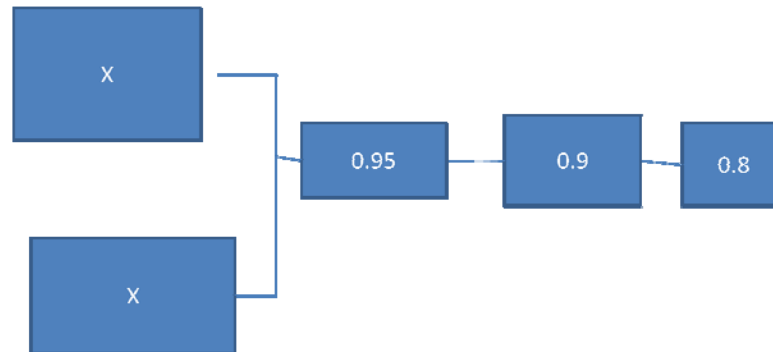
Negative Binomial Distribution:  $f(x) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$

Poisson Distribution:  $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

Uniform Distribution (n element):  $f(x)=1/n$

## Answers

### • Figure Q1



Q1 a)

b)  $x=0.7$

Q2) a)  $\text{prob}(X=1 \text{ or } X=2)=0.8574$  (Binomial  $n=2$ ,  $p=0.62244$ )

b)  $\text{prob}(X=2 \text{ or } X=3 \text{ or } X=4) = 0.8457$  (Binomial  $n=2$ ,  $p=0.62244$ )

c) Most reliable system has the highest probability of functioning 2 engine has  $P(\text{working}) = 0.8574$  while the 4 engine plane has  $P(\text{working}) = 0.8457$ .

Q3) a)  $P(\text{failure} | 200 \text{ GB})$  which is 0.15

b)  $P(\text{failure})=0.025$

Q4) a)  $E(x)=1.6$

b)  $V(X) = 1.44$

c)  $P(X < 4)=0.9$

d)  $X=0, F(0)=0.2; X=1, F(1)=0.5; X=2, F(2)=0.8, X=3, F(3)=0.9, X=4, F(4)=1.0$