$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \underline{\nabla} \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \qquad \underline{F} = Q(\underline{E} + \underline{v} \times \underline{B})$$

$$\underline{\nabla} \cdot \underline{D} = \rho_v \qquad \underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \cdot \underline{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\underline{D} = \varepsilon_0 \underline{E} + \underline{P} \qquad \underline{H} = \frac{1}{\mu_0}\underline{B} - \underline{M} \qquad \underline{J} = \sigma \underline{E}$$

$\mu_0 = 4\pi \times 10^{-7}$ (H/m) $\qquad \varepsilon_0 = 8.85 \times 10^{-12} \approx \frac{10^{-9}}{36\pi}$ (F/m)

Algebraic

(1) $\mathbf{F} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{F}$

(2) $\mathbf{F} \times \mathbf{G} = -\mathbf{G} \times \mathbf{F}$

(3) $\mathbf{F} \cdot (\mathbf{G} + \mathbf{H}) = \mathbf{F} \cdot \mathbf{G} + \mathbf{F} \cdot \mathbf{H}$

(4) $\mathbf{F} \times (\mathbf{G} + \mathbf{H}) = \mathbf{F} \times \mathbf{G} + \mathbf{F} \times \mathbf{H}$

(5) $\mathbf{F} \times (\mathbf{G} \times \mathbf{H}) = \mathbf{G}(\mathbf{H} \cdot \mathbf{F}) - \mathbf{H}(\mathbf{F} \cdot \mathbf{G})$

(6) $\mathbf{F} \cdot (\mathbf{G} \times \mathbf{H}) = \mathbf{G} \cdot (\mathbf{H} \times \mathbf{F}) = \mathbf{H} \cdot (\mathbf{F} \times \mathbf{G})$

Integral

(7) $\oint_s \mathbf{F} \cdot d\mathbf{s} = \int_v \nabla \cdot \mathbf{F}\, dv$

(8) $\oint_\ell \mathbf{F} \cdot d\boldsymbol{\ell} = \int_s (\nabla \times \mathbf{F}) \cdot d\mathbf{s}$

(9) $\oint_s f(\nabla g) \cdot d\mathbf{s} = \int_v [f\, \nabla^2 g + (\nabla f) \cdot (\nabla g)]\, dv$

(10) $\oint_s [f\, \nabla g - g\, \nabla f] \cdot d\mathbf{s} = \int_v (f\, \nabla^2 g - g\, \nabla^2 f)\, dv$

Differential

(11) $\nabla(f + g) = \nabla f + \nabla g$

(12) $\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

(13) $\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

(14) $\nabla(fg) = f\nabla g + g\nabla f$

(15) $\nabla \cdot (f\mathbf{F}) = \mathbf{F} \cdot \nabla f + f(\nabla \cdot \mathbf{F})$

(16) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$

(17) $\nabla \times (f\mathbf{F}) = (\nabla f) \times \mathbf{F} + f(\nabla \times \mathbf{F})$

(18) $\nabla \cdot \nabla f = \nabla^2 f$

(19) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$

(20) $\nabla \times (\nabla f) = 0$

(21) $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

(22) $\nabla \times (f\, \nabla g) = \nabla f \times \nabla g$

$$\underline{A} \cdot \underline{B} = |\underline{A}||\underline{B}| \cos\theta$$

$$\underline{A} \times \underline{B} = |\underline{A}||\underline{B}| \sin\theta\, \hat{a}_\perp$$

$$\textbf{grad}\, f \equiv \mathbf{a}_1 \frac{1}{h_1}\frac{\partial f}{\partial u_1} + \mathbf{a}_2 \frac{1}{h_2}\frac{\partial f}{\partial u_2} + \mathbf{a}_3 \frac{1}{h_3}\frac{\partial f}{\partial u_3}$$

$$\textbf{grad}\, f = \mathbf{a}_x \frac{\partial f}{\partial x} + \mathbf{a}_y \frac{\partial f}{\partial y} + \mathbf{a}_z \frac{\partial f}{\partial z}$$

$$\textbf{grad}\, f = \mathbf{a}_\rho \frac{\partial f}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial f}{\partial \phi} + \mathbf{a}_z \frac{\partial f}{\partial z}$$

$$\textbf{grad}\, f = \mathbf{a}_r \frac{\partial f}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial f}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\text{div}\, \mathbf{F} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial(F_1 h_2 h_3)}{\partial u_1} + \frac{\partial(F_2 h_1 h_3)}{\partial u_2} + \frac{\partial(F_3 h_1 h_2)}{\partial u_3}\right]$$

$$\text{div}\, \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\text{div}\, \mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\text{div}\, \mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(F_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\textbf{curl}\, \mathbf{F} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\mathbf{curl\,F} = \frac{\mathbf{a}_1}{h_2h_3}\left[\frac{\partial(F_3h_3)}{\partial u_2} - \frac{\partial(F_2h_2)}{\partial u_3}\right] + \frac{\mathbf{a}_2}{h_3h_1}\left[\frac{\partial(F_1h_1)}{\partial u_3} - \frac{\partial(F_3h_3)}{\partial u_1}\right] + \frac{\mathbf{a}_3}{h_1h_2}\left[\frac{\partial(F_2h_2)}{\partial u_1} - \frac{\partial(F_1h_1)}{\partial u_2}\right]$$

$$\mathbf{curl\,F} = \frac{\mathbf{a}_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(F_\phi\sin\theta) - \frac{\partial F_\theta}{\partial\phi}\right] + \frac{\mathbf{a}_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}(rF_\phi)\right] + \frac{\mathbf{a}_\phi}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]$$

$$\mathbf{curl\,F} = \mathbf{a}_\rho\left[\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right] + \mathbf{a}_\phi\left[\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right] + \mathbf{a}_z\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\phi) - \frac{1}{\rho}\frac{\partial F_\rho}{\partial\phi}\right]$$

$$\nabla\cdot(\nabla f) = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial f}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3h_1}{h_2}\frac{\partial f}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial f}{\partial u_3}\right)\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\nabla^2 f \equiv \nabla\cdot\nabla f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla\cdot(\nabla f) \equiv \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

NB. "CURL-CURL" NOT REQUIRED.

RECTANGULAR TO CIRCULAR CYLINDRICAL

$A_\rho = A_x\cos\phi + A_y\sin\phi$ (1-74a)

$A_\phi = -A_x\sin\phi + A_y\cos\phi$ (1-74b)

$A_z = A_z$ (1-74c)

in which

$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad z = z$ (1-75)

CIRCULAR CYLINDRICAL TO RECTANGULAR

$A_x = A_\rho\cos\phi - A_\phi\sin\phi$ (1-76a)

$A_y = A_\rho\sin\phi + A_\phi\cos\phi$ (1-76b)

$A_z = A_z$ (1-76c)

in which

$\rho = \sqrt{x^2+y^2}, \quad z = z$ (1-77)

$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}$ (1-78)

RECTANGULAR TO SPHERICAL

$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ (1-79a)

$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ (1-79b)

$A_\phi = -A_x\sin\phi + A_y\cos\phi$ (1-79c)

in which

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$ (1-80)

SPHERICAL TO RECTANGULAR

$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ (1-81a)

$A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ (1-81b)

$A_z = A_r\cos\theta - A_\theta\sin\theta$ (1-81c)

in which

$r = \sqrt{x^2+y^2+z^2}$

$\cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}, \quad \sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$ (1-82)

$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}$ (1-82)